Exam 1 ALTERNATE Due: Thursday 12 March, 2026

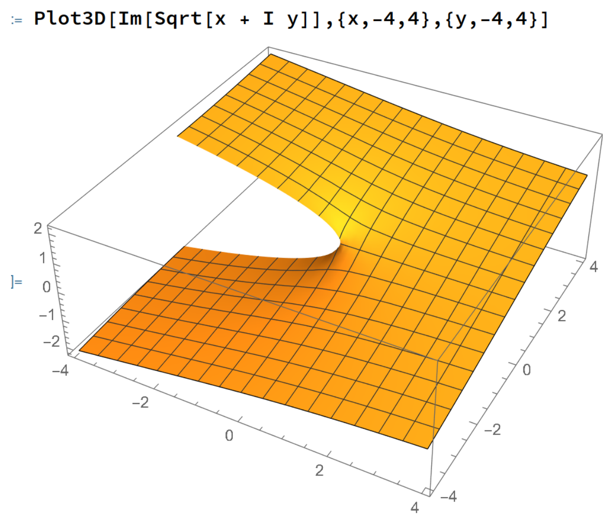
Each problem is 20 points.

PROBLEM		SCORE
1	Complex Variables	
2	Eigenvectors	
3	Coupled motion	
4	Fourier Transform	
5	Group Theory	
6	Oscillations	
Total	out of 120	

Exam 1 ALTERNATE Due: Thursday 12 March, 2026

Each problem is 20 points.

Problem 1:



We found that the complex square root function ($\sqrt{z}$) has a discontinuity along the negative axis (with the conventional definition). This is also the case for the n-th root $(z^{1/n})$.

a) Let's start with the cube root $(z^{1/3})$.
Let's assume the branch cut is along the negative axis. Compute the discontinuity across the branch cut as a function of r.
b) Sketch this for r=[-4,0]

c) **GRADS ONLY**: In general, compute the discontinuity for the n-th root $(z^{1/n})$.

Problem 2:

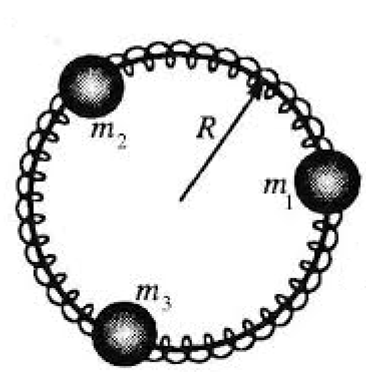
a) For the given matrix, M find the eigenvalue and eigenvectors.
b) Normalize the eigenvectors.
c) **GRADS ONLY:** Construct the matrix B from the normalized eigenvectors. Show that $B.M.B^T$ diagonalizes M.

$$\begin{pmatrix} 14 & -3 \\ -3 & 6 \end{pmatrix}$$

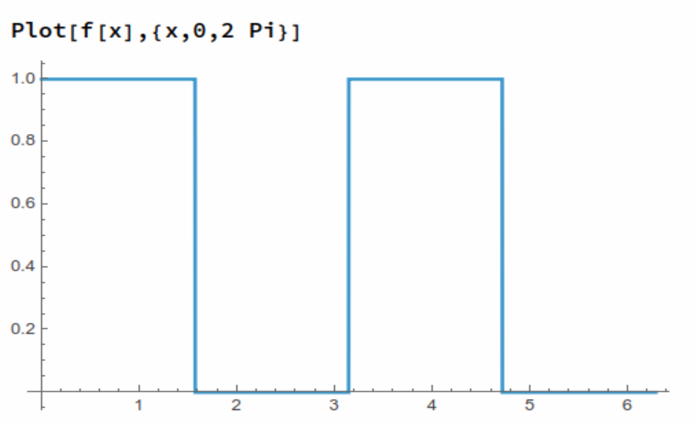
Hint: Before you normalize the math should NOT be too complicated.
After you normalize, there is a radical denominator,
but you can factor that out when you check the diagonalization properties.

Problem 3:

Consider 3 masses that can slide freely on a circular wire of radius R.
You may wish to label their positions $x_i = R\,\theta_i$, and thus $v_i = x_i' = R\,\theta_i'$,
a) Write down the kinetic energy T and potential energy V for the system in terms of θ_i and θ_i'. Assume all springs have equal k.
b) Determine the T and V matrices for the system.



Problem 4:
On the interval $[0,2\pi]$, consider function $f(x) = 1$ for $x=[0,\pi/2]$ and $x=[\pi,3\pi/2]$, and zero elsewhere. Compute a[0] and b[n]



$$a_n = \frac{1}{\pi}\int_0^{2\pi} dx \, \cos(nx) \, f(x)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} dx \, \sin(nx) \, f(x)$$

Problem 5: Group Theory:
Key properties of groups are i) closure, ii) associativity, iii) identity element, iv) inverse element.

a) On the Rubik's cube, if R represents a 1/4 turn of the yellow face clockwise, we found this generates a cyclic group of order 4: $\{e, R, R^2, R^3, R^4{=}e\}$. A valid 2-element sub-group is $\{e, R^2\}$.
QUESTION: Why is $\{R,R^3\}$ NOT a valid 2-element subgroup.

b) For the SU(2) Pauli matrices, we found the commutator was: $[\sigma_j, \sigma_k] = 2\, i\, \varepsilon_{jkl}\, \sigma_l$.

For SU(3), there is a similar relation. We won't compute in general, but will only do one case. Compute $[\lambda_1,\lambda_6] = c\,\lambda_\#$, and determine which lambda matrix it yields ($\lambda_\#$) and the coefficient "c."

Matrices [edit]

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Problem 6: Oscillations:

Give me as much information as you can about {$\mathbf{x_0}$, $\mathbf{v_0}$, $\boldsymbol{\beta}$,$\mathbf{w_0}$, $\mathbf{w_D}$, $\mathbf{Q_0}$}

Figure a)

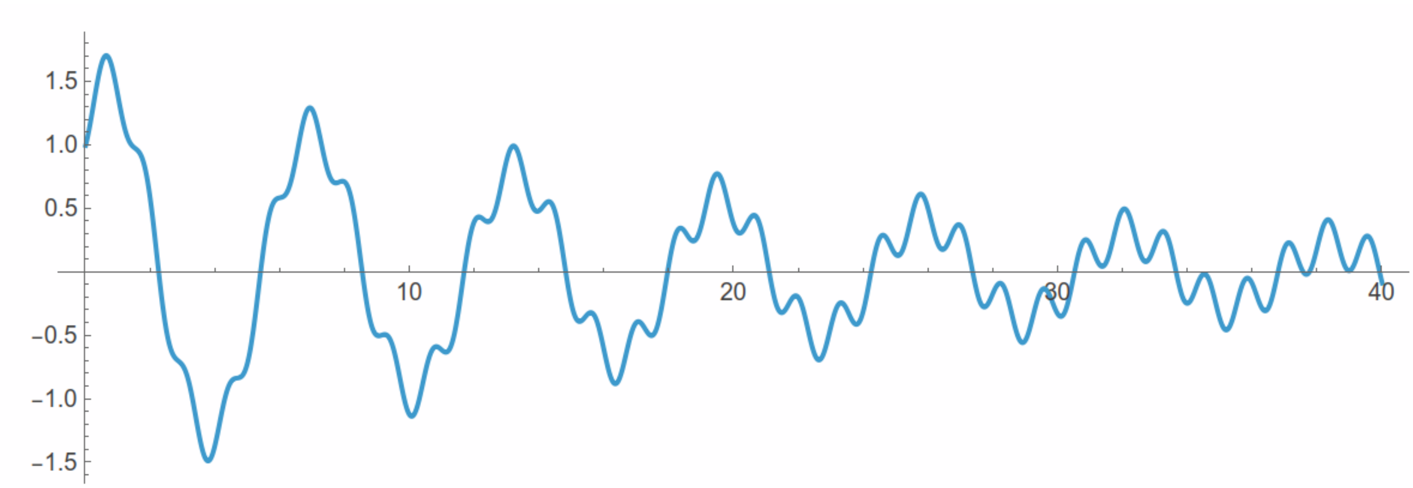


Figure b)

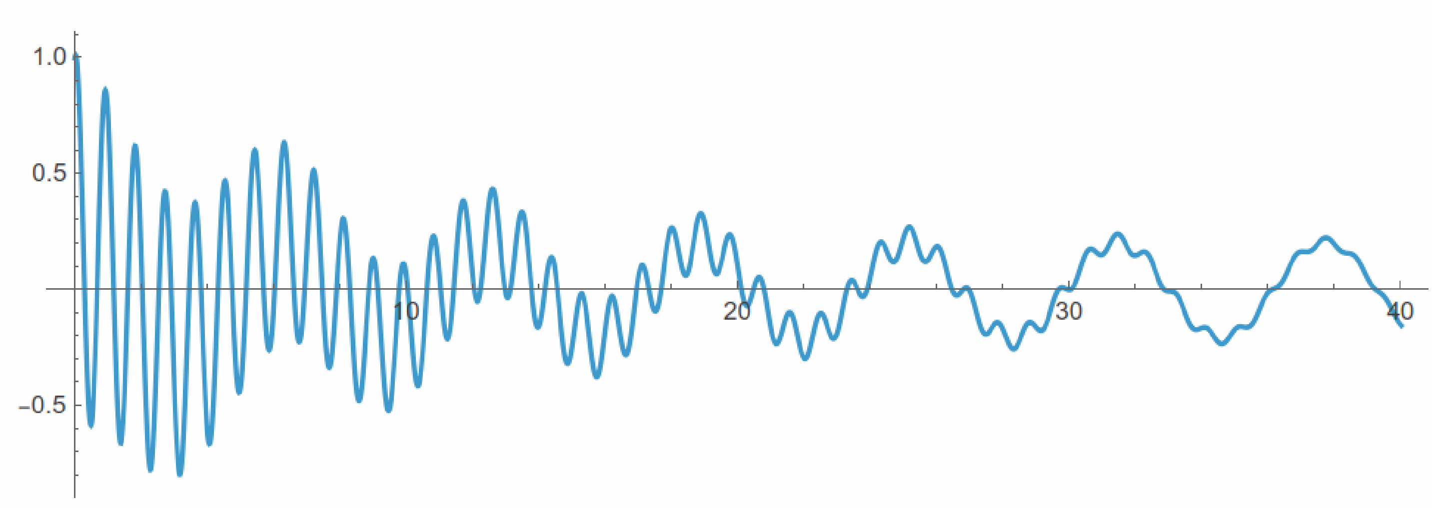


Figure c)

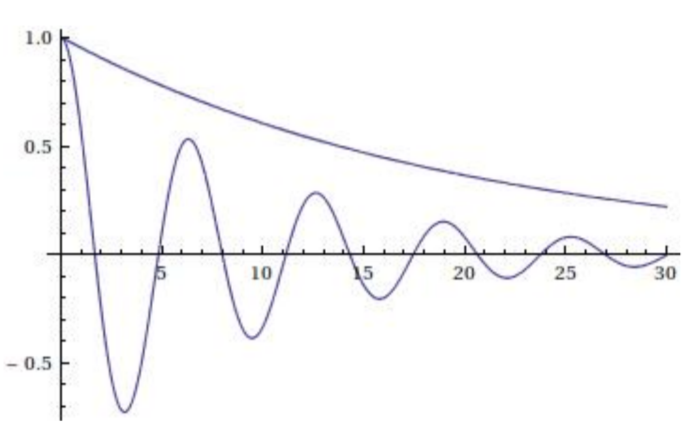


Figure d)