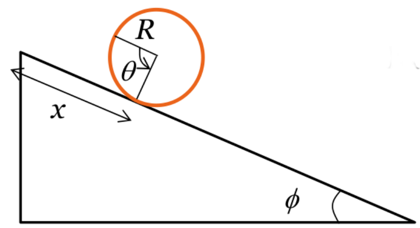
Homework #9: Phys 4321:   Prof. Olness   Spring 2026

1) Find the shortest distance between two points located on the surface of a cylinder.

2) Solve the brachistochrone as outlined in the book (your choice). Add in all the intermediate steps. (Most books leave out quite a bit. I want to see the trig substitutions and integration done by hand!) Plot the resulting curves.

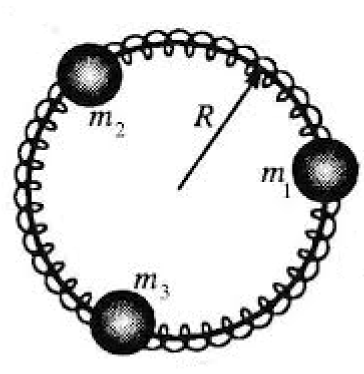
3) Consider light passing from one medium to another with indices of refraction of {n1,n2}. Use Fermat's principle to minimize the time and find the resulting law of refraction.

4) Consider a hoop of mass m and moment of inertia $I=m R^2$ sliding down an incline of angle ϕ a distance x (along the incline). The hoop rolls WITHOUT slipping.. Do this using the Lagrange equations.
a) First do this WITHOUT the Lagrange multiplier λ ,
by using: $R\theta - x=0$.
b) Second, do this WITH the Lagrange multiplier λ , and use the constraint equation: $R\theta - x=0$.



Problem 5:

Consider 3 masses that can slide freely on a circular wire of radius R.
You may wish to label their positions $x_i = R\,\theta_i$, and thus $v_i=x_i' = R\,\theta_i'$,
a) Write down the kinetic energy T and potential energy V
for the system in terms of θ_i and θ_i'.
Assume all springs have equal k.
b) Using the Lagrangian L=T-V, compute the
equations of motion and put into matrix form.



FOR GRADS ONLY:

Goldstein, 2nd edition, Chapter 2, Exercise 12

A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R. The only external force is gravity. If the smaller cylinder starts rolling from rest on top of the larger cylinder, find (using Lagrange multipliers) the point at which the hoop falls off the cylinder.