

## Pythagorean Tuning

*All tones found by pure 5ths*

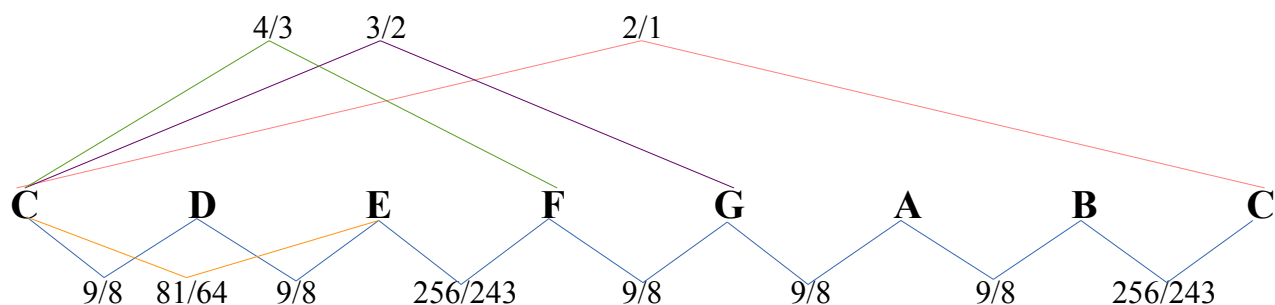
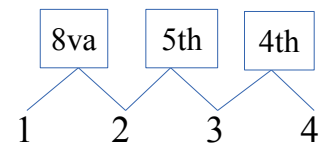
### Relationship in ratios:

OCTAVE:  $2/1$

FIFTH:  $3/2$

FOURTH:  $4/3$

Or



### Pythagorean Third: C-E

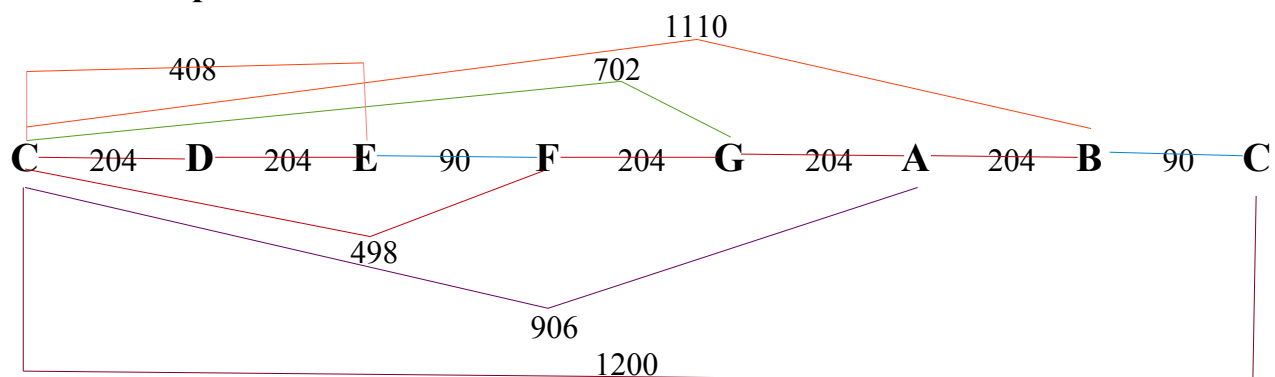
$C \times 3/2 = G \rightarrow G \times 3/2 = D \rightarrow D \times 1/2 = D \rightarrow D \times 3/2 = A \rightarrow A \times 3/2 = E \rightarrow E \times 1/2 = E$

Calculated out:

$3/2 \times 3/2 = 9/4 \rightarrow 9/4 \times 1/2 = 9/8 \rightarrow 9/8 \times 3/2 = 27/16 \rightarrow 27/16 \times 3/2 = 81/32 \rightarrow 81/32 \times 1/2 = 81/64$

**E = 81/64**

### Relationship in cents:



### Pythagorean Comma:

Difference between B# arrived at by twelve 5ths (702¢) and C arrived at by seven octaves (1200¢).

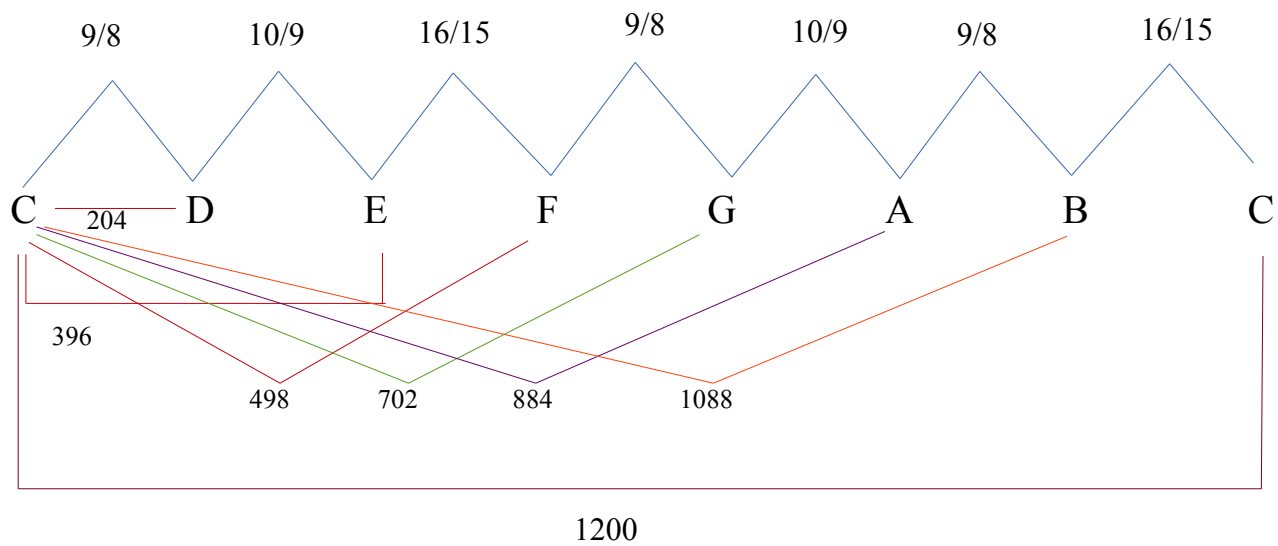
Comma = 24¢

## *Just Temperament*

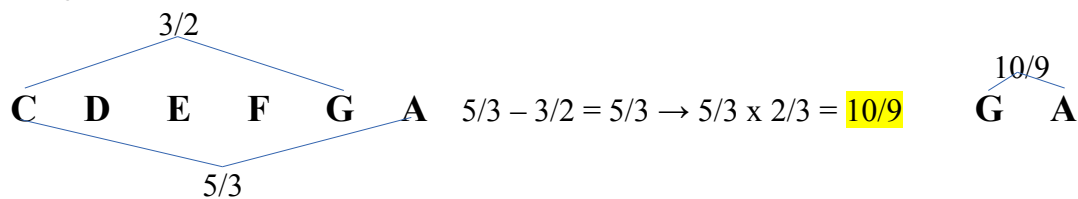
*Based on pure 5<sup>th</sup> plus pure 3<sup>rd</sup>*

### Relationship in ratios:

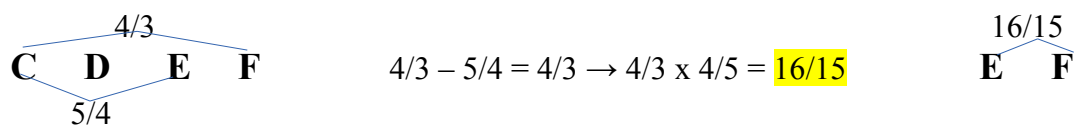
OCTAVE: 2/1  
FIFTH: 3/2  
FOURTH: 4/3  
THIRD: 5/4  
MAJ SECOND: 9/8 (less preferred 10/9)  
MIN SECOND: 16/15



### Major Second: G-A



### Minor second: E-F



## **Mean Tone Tuning**

*Based on altering successive 5<sup>ths</sup> to get pure 3<sup>rd</sup>*

### **Finding Pure 3<sup>rd</sup>:**

- Spreading 1/4 Didymean Comma (80/81 or 22 cents) over each of the four 5<sup>th</sup>
- 1/4 of 22 cents is 5½ cents
- So each 5<sup>th</sup> is 696½ cents instead of 702 cents

**Pythagorean:** 702      702      702      702      = 2808

C      G      D      A      E

**Mean tone:** 696½      696½      696½      696 ½      = 2786

2768 – 2400 (two octaves) = 386

*(Note: 2808 – 2786 = 22 cents too wide)*

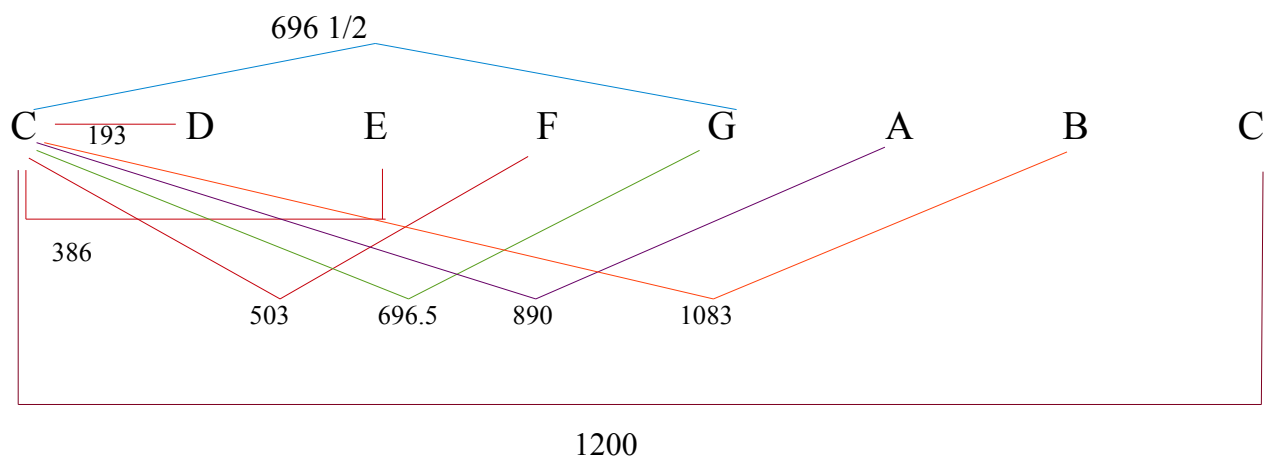
**Pure 3<sup>rd</sup> = 386¢**

OCTAVE: 1200¢

FIFTH: 696½¢

THIRD: 386¢

SECOND: 386/2 (meantone)\* = 193¢



# Comparison in Cents Among Various Tuning/Temperament s

Physics 1320  
Professors Olness and Tunks

	<u>Pythagorean</u>	<u>Just</u>	<u>Mean Tone*</u>	<u>Equal</u>
<b>C</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>C#</b>	<b>114</b>	<b>92</b>	<b>76</b>	<b>100</b>
<b>D</b>	<b>204</b>	<b>204</b>	<b>193</b>	<b>200</b>
<b>Eb</b>	<b>294</b>	<b>316</b>	<b>310</b>	<b>300</b>
<b>E</b>	<b>408</b>	<b>386</b>	<b>386</b>	<b>400</b>
<b>F</b>	<b>498</b>	<b>498</b>	<b>503</b>	<b>500</b>
<b>F#</b>	<b>612</b>	<b>590</b>	<b>579</b>	<b>600</b>
<b>G</b>	<b>702</b>	<b>702</b>	<b>696.5</b>	<b>700</b>
<b>G#</b>	<b>816</b>	<b>816</b>	<b>772</b>	<b>800</b>
<b>A</b>	<b>906</b>	<b>884</b>	<b>890</b>	<b>900</b>
<b>Bb</b>	<b>996</b>	<b>996</b>	<b>1007</b>	<b>1000</b>
<b>B</b>	<b>1110</b>	<b>1088</b>	<b>1083</b>	<b>1100</b>
<b>C</b>	<b>1200</b>	<b>1200</b>	<b>1200</b>	<b>1200</b>