

Electric forces and electric charges in everyday life

Electric force & charge: The **electric force** is one of the main forces encountered in nature. The strength of the electric force varies depending on the object. When an object is able to interact by an electric force, we say that this object carries an **electric charge**. The charge can be transferred from one object to another. Furthermore, you will observe today that the electric force can be repelling or attractive, which can be explained by assuming that the **charges can be either positive or negative**. A charged object will repel another object with a like-sign charge, while it will attract another object with an opposite-sign charge. At the atomic level, the **positive charge is carried by** tiny particles called **protons**, and the **negative charge is carried by electrons**. In most situations, opposite charges cancel one another by forming **neutral** atoms and molecules that contain equal amount of positive and negative charges (so that the total charge of the atom is zero). Nevertheless, there are processes such as friction of two objects, deformation and rupture, in which the opposite charges are forced to separate, resulting in a nonzero net charge that can be observed.

Point charges and Coulomb's law. In general, the computation of the electric force requires lengthy calculations, but the mathematical description becomes very simple for **point charges**. If a charged object has a very small size compared to the distance r to the other charges, it essentially behaves as a charge located at a single point. The electric force between two point charges, q_1 and q_2 , is given by Coulomb's law, which states that the force $F_{1\leftarrow 2}$ that q_2 exerts on q_1 is given by

$$F_{1\leftarrow 2} = k q_1 q_2 / r^2,$$

where k is a proportionality constant. The direction of the force is along the line between the two charges. Like-signed charges repel, and unlike-signed charges attract. Here the concept of the point charge is important, because without this idealization the distance between the two charges and the direction of the force would be ambiguous. For two point charges, the distance r and the direction of the force are well defined, and Coulomb's law is accurate.

Superposition principle: When dealing with systems containing more than two charges, Coulomb's law still applies to each charge pair. This means that the force between any two charges in the system is independent of the presence of the other charges nearby.

The net force \mathbf{F} exerted on a charge Q is the summation of the pair-forces \mathbf{F}_i exerted by each of the other charges q_i :

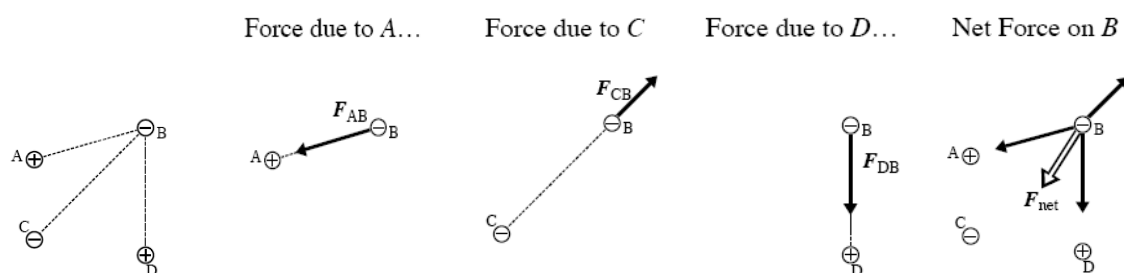
$$\mathbf{F} = \mathbf{F}_{Q\leftarrow q_1} + \mathbf{F}_{Q\leftarrow q_2} + \dots + \mathbf{F}_{Q\leftarrow q_N},$$

where each pairwise force is found by using the Coulomb's law,

$$\mathbf{F}_{Q\leftarrow q_i} = k \frac{Q q_i}{r_{Q\leftarrow q_i}^2} \hat{\mathbf{r}}_{Q\leftarrow q_i} \text{ for } i = 1, 2, \dots, N.$$

Keep in mind that each electric force is a vector (denoted by the bold face font in this write-up or by an arrow in your textbook). It is directed along the unit vector $\hat{\mathbf{r}}_{Q \leftarrow q_i}$ pointing from the source charge q_i to the test charge Q . The addition of several forces must be done under the rules of vector summation.

The following figure shows an example of applying the superposition principle stated above to a four particle system. To obtain the net force exerted on particle B, we first find the forces exerted on it by each of the other three particles using Coulomb's Law, and after that sum these three force vectors.



Charge distributions & volume charge density. The point charge model describes a certain amount of charge concentrated in a very small volume that could be seen as a point. To address situations when charge is spread over three dimensional objects whose sizes cannot be neglected, it is convenient to introduce the idea of a **volume charge density**, denoted by $\rho(x,y,z)$.

If dq is the charge contained in a small volume $dV=dx dy dz$ around a point P with coordinates x, y, z , then the volume charge density at this point is defined as

$$\rho(x,y,z)=dq/dV.$$

The volume charge density is reminiscent of the volume mass density $\rho_m(x,y,z)=dm/dV$ that was encountered in the determination of the center-of-mass in the mechanics class. However, $\rho(x,y,z)$ describes the distribution of electric charge and not of mass.

In the case of a **uniform charge distribution**, charge density is the same at all points, $\rho=q/V$ independently of x,y,z , where q and V are the total charge and volume. But in general its value varies at different space points inside charged objects.

Charge density can be used to compute the electric force between charged objects with non-negligible size. You will encounter many examples of such calculations in your textbook. In this lab, we will perform a few simple exercises to help you get used to manipulating the charge density. This experience will help you to solve various problems in the later parts of the course.

Linear charge density. In one common situation, the charged object has a large extension in one direction and small extensions in two other directions. For instance, a charged wire or rod often may have a significant length L and a much smaller diameter D . In this case, using a **linear charge density** $\lambda(x)$ instead of the volume charge density $\rho(x,y,z)$ is more convenient, where $\lambda(x)$ is defined as the charge per unit length x of the wire:

$$\lambda(x) = dq/dx.$$

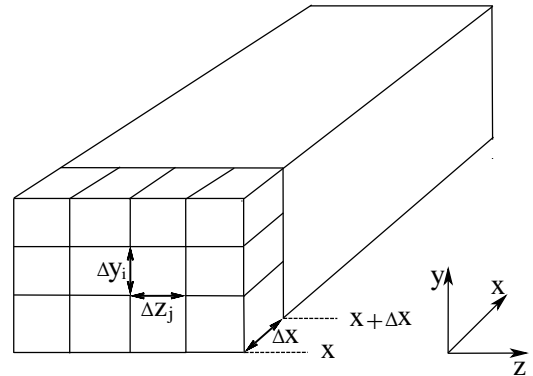
Just like $\rho(x,y,z)$, $\lambda(x)$ can be used to find the total electric force produced by the wire, but the calculations now depend only on one coordinate (x) instead of three (x,y,z). The calculations are simplified by using λ .

The linear density λ can be found from volume density ρ by integrating the volume charge distribution over the transverse directions y and z :

$$\lambda(x) = \iint \rho(x, y, z) dy dz \quad (1)$$

$\lambda(x)$ can always be found if $\rho(x,y,z)$ is known. It is not an independent function, but a simplified representation for the full 3-dimensional charge distribution when the dependence on y and z is small or irrelevant.

Derivation of Eq. (1). To see where this relation between λ and ρ comes from, consider a part of the wire shown in the figure and divide its volume into many small blocks $\Delta V(x,y_i,z_j) = \Delta x \Delta y_i \Delta z_j$. The charge of each block is $\Delta q(x,y_i,z_j) = \rho(x,y_i,z_j) \Delta x \Delta y_i \Delta z_j$. If we sum charge of all blocks in the interval between x and $x + \Delta x$, we get the charge $\Delta q(x)$ in the interval between x and $x + \Delta x$:



$$\Delta q(x) = \sum_i \sum_j \Delta q(x, y_i, z_j) = \sum_i \sum_j \rho(x, y_i, z_j) \Delta x \Delta y_i \Delta z_j.$$

Dividing both sides by Δx , we get

$$\lambda(x) \approx \frac{\Delta q(x)}{\Delta x} = \sum_i \sum_j \rho(x, y_i, z_j) \Delta y_i \Delta z_j.$$

This relation becomes Eq. (1) in the limit $\Delta x \rightarrow 0$, $\Delta y_i \rightarrow 0$, and $\Delta z_j \rightarrow 0$.