PHYS 1105 **Last edit: May 25, 2017**
SMU Physics Dept.

Simple Harmonic Motion

Goal

- To determine the spring constant k and effective mass m_{eff} of a real spring.

Equipment

Tapered spring, straight spring, apparatus rod, clamp, mass set, mass hanger, stop watch.

The Big Picture

Simple harmonic motion (SHM), or sinusoidal motion with a constant oscillation frequency, is a mathematical idealization (like point masses and frictionless surfaces) but one that is very useful in approximating real systems such as the pendulum of a grandfather clock, musical instruments of all types and tuning forks, electronic quartz watches which use a resonating crystal, light waves including lasers, radio transmitters and receivers, and many others.

For example, a tuning fork is a U-shaped piece of metal that when struck will vibrate at specific frequencies, the fundamental note and higher frequencies called overtones. Typically tuning forks are designed so that the overtones decay quickly and then only produce the fundamental note stamped into the fork. The frequency of the fork depends, among other factors, on the mass and the elasticity of the metal used. Before the advent of electronic tuners, musicians used tuning forks to tune their instruments. The frequency of the fork depends very slightly on temperature because the elasticity of the metal changes with temperature, but not on other factors like humidity which affect the tuning of stringed instruments, so forks are a good standard to which an orchestra can calibrate.

There is a tiny tuning fork made of the mineral crystal quartz inside electronic watches and clocks. Quartz is “piezoelectric” which means that a voltage applied to the crystal will cause the crystal to flex. The shape and size of the quartz crystal are chosen to produce a

Figure 1: Three tuning forks of different pitch mounted on resonance boxes. From Ref 2.

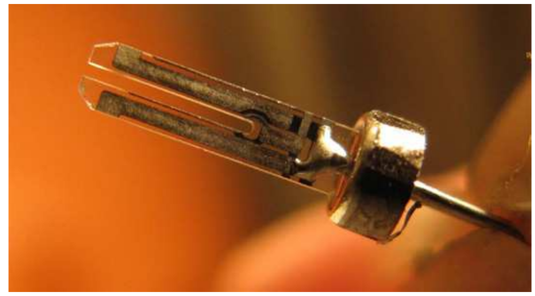

Figure 2: Magnified image of a quartz crystal cut into the shape of a tuning fork used in quartz clocks. From Ref 3.

frequency of 2^{15} vibrations per second. An electronic circuit in the timepiece applies voltage to the crystal and monitors voltage across the crystal as it oscillates in a feedback loop to keep accurate time.

Theory

Why is SHM ubiquitous in so many systems across diverse disciplines? Any system in stable equilibrium can be represented as a point near the bottom of a graph of potential energy. Remember that "equilibrium" means that the net force is zero, and "stable" means that if the system is disturbed slightly, it will return to equilibrium. Figure 3 is a graph of a system in stable equilibrium. A good model to think about is a marble rolling around the bottom of a bowl.

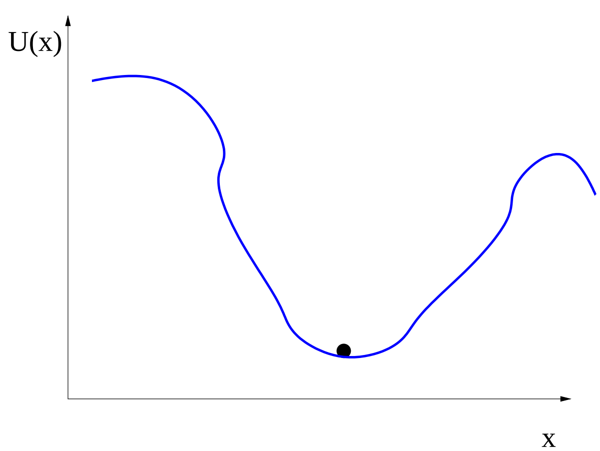

Figure 3: Potential energy U plotted versus displacement x for a system in stable equilibrium. The dot represents the current state of the system.

As long as the displacement from equilibrium (the minimum on the potential energy curve) is small, the curve can be approximated by a parabola $U(x) = kx^2 + b$, as in Figure 4

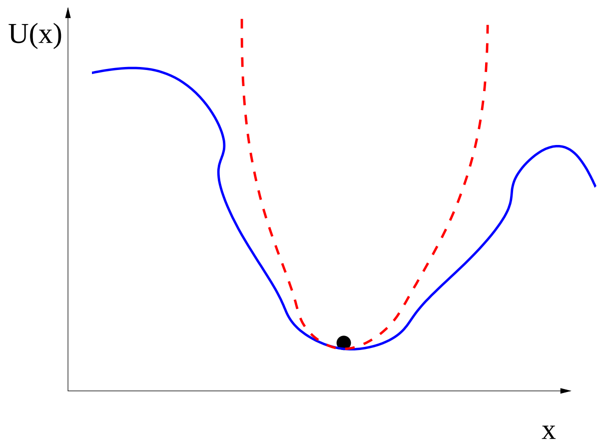

Figure 4: A parabolic approximation (dashed) is overlaid on the previous potential energy curve.

A parabolic potential energy gives rises to a linear restoring force

$$F = -\frac{dU}{dx} = -kx,$$

just as in the case of Hooke's law for a spring.

We can go a bit further. According to Newton's second law, the net force is equal to the mass multiplied by the acceleration, and the acceleration is just the second time derivative of the displacement

$$F = ma = m\frac{d^2x}{dt^2}.$$

So now we have a differential equation for the displacement $x(t)$ as a function of time

$$-kx(t) = m\frac{d^2x(t)}{dt^2}.$$

This is one of the first and most important differential equations that you will learn to solve eventually, but for now we can simply tell you the solution which you can verify by plugging into the equation above. The solution is

$$x(t) = A\sin(\omega t + C),$$

where $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency of oscillation, A is the amplitude of the oscillation (how far the system gets from equilibrium), and C is a constant related to when you start watching the system oscillate (you can safely set $C = 0$ by choosing the start time appropriately).

In summary, then: small displacements from stable equilibrium result in sinusoidal motion with constant frequency of oscillation; that is, simple harmonic motion.

Procedure

1. Attach one of the two springs to the metal rod. The tapered spring should be attached with the narrow end up. (Why?) Attach the mass hanger to the bottom of the spring and load it with some mass. You must use enough mass to achieve smooth oscillation, but **you must NOT exceed the elastic limit of the spring!** For the tapered spring, the limit is about 500 grams plus the hanger; for the straight spring, the limit is about 1000 grams plus the hanger.
2. Ease the loaded mass hanger to its new equilibrium position. Pull down gently on the hanger and release it to start the oscillation. The amplitude of oscillation only needs to be about 5 cm.
3. Record the attached mass (remember the 50-gram hanger) and the time for one complete oscillation (up and down). It is very difficult to time one oscillation – how can we improve precision?
4. Remember to record error estimates with all of your measurements.
5. Use at least eight different masses spread over the allowable range (between smooth oscillation and the maximum values given above). If you choose masses too close together, your best fit line will be imprecise.
6. Change springs and repeat the experiment.

Analysis

1. Write the equation relating mass m, the spring constant k, and the period T for an ideal massless Hooke's law spring loaded with a mass undergoing simple harmonic motion.
2. The world of the physics laboratory is not ideal – real springs have their own mass which oscillates with the load. In the equation you have written above, replace the mass m with the sum of the load mass m_{load} and the effective mass of the real spring m_{eff}.
3. Which variables in this last equation are easy to measure in lab?
4. What combination of these variables would you plot to produce a graph that is a straight line?
5. What variable which is not easy to measure directly in lab can be derived from the slope of the best-fit straight line graph? Determine it. No error propagation is required. (Mathematica may be useful.)
6. What variable which is not easy to measure directly in lab can be derived from the y-intercept of the best-fit straight line graph? Determine it. No error propagation is required. (Mathematica may be useful.)
7. Why should the narrow end of the tapered spring be up?
8. Identify at least two sources of statistical error.

9. Identify at least two sources of systematic error.

Literature

1. Halliday, Resnick, & Walker, *Fundamentals of Physics* Vol. 1 (10th ed.), Ch 15: Oscillations, Wiley, 2014.

2. "Tuning Forks"
`http://americanhistory.si.edu/science/tuningfork.htm`, accessed 24 May 2017.

3. "Simple Harmonic Motion"
`https://sites.google.com/site/simpleharmonicmotion73/`, accessed 24 May 2017.