Simple Pendulum

Introduction:

Simple harmonic motion takes place when the force on an object is opposite in direction and proportional to the displacement of the object. The force described by Hooke's Law ($\mathbf{F} = -k\,\mathbf{x}$) is this kind of force. When we dealt with the spring, we observed the case where the acceleration was zero. The position where the total force was zero is called the **equilibrium position**. You also know that if the mass on the spring is not placed carefully at this position with zero velocity, it will oscillate up and down. The more the spring is displaced from the equilibrium position, the greater the **restoring force** in the opposite direction (toward the equilibrium position). The interesting thing about the restoring force described by Hooke's law is that it produces the motion of harmonic oscillation. Harmonic motion has a constant **amplitude** and **frequency**. The amplitude (A) is simply the maximum displacement from the equilibrium position, and the frequency (f) is the number of oscillations (back and forth, up and down, et cetera) per unit time. The **period** (T) is the inverse of the frequency ($1/f$) and represents the time per single oscillation.

Another device besides the spring-mass which can produce simple harmonic motion is that of the **simple pendulum**. A simple pendulum consists of a point-mass (m) suspended from a fixed point by a rod or string of length (L) and of mass (approaching) zero. This mass is then allowed to swing freely under the force of gravity. The mass is in the equilibrium position when it hangs vertically from the support. The restoring force on the pendulum for small horizontal displacements (angular displacements of less that 10°, for instance) is given by the following equation:

$$\mathbf{F} = m\mathbf{a} = m\,(-\,g/L)\,\mathbf{x} \qquad (1)$$

Here m is the mass of the point-mass or bob, g is the magnitude of the gravitational acceleration, L is the length of the string or rod, and $\mathbf{x}$ is the horizontal displacement. Since m, g, and L are positive constants, we observe that this is of the same form as the Hooke's Law equation. This restoring force will therefore produce simple harmonic motion.

A real pendulum swinging with a small amplitude may be described as a simple pendulum if the following two conditions are met:

1. The length of the string or rod (L) is much greater than the size of the bob. In this case the bob can be treated as a point-mass located at the center of gravity of the bob.
2. The mass of the string or rod supporting the mass is much less than the mass of the bob.

The period of the pendulum will be given by the following equation:

$$T = 2\pi\sqrt{L/g} \qquad (2)$$

Notice that the mass of the bob does not appear. Squaring both sides of Equation (2) yields:

$$L / T^2 = \text{constant} \qquad (3)$$

Furthermore, solving for g yields

$$g = 4\pi^2 L/T^2 \qquad (4)$$

Our objective is to observe simple harmonic motion for a simple pendulum. We will take measurements of L and T for a real pendulum and then can use (3) to test the validity of the theory (L / T^2 = const.). By using Equation (4) we can determine an experimental value for g.

Procedure:

1. Clamp one end of a piece of thread to a rigid support in such a way that insures that the point of suspension remains fixed throughout a complete (but small-angle) cycle.
2. Attach the bob to the other end of the thread. Beginning with a pendulum length of slightly less than 100 cm. (from the point of suspension to the center of the bob), start the bob swinging through a small arc (about 5 ° on either side of vertical).
3. Use the timer to find the time for about 20 cycles at this L. Note that the timer should be started at the zero count and stopped at the nth count. (n is the number of cycles.) Record L, t, and n.

4. Change *L* to about 60 cm. Find the time for about 25 cycles. Record *L*, *t*, and *n* for this new arrangement.
5. Change *L* to about 30 cm. Find the time for about 30 cycles. Record *L*, *t*, and *n* for this new arrangement.

Analysis:

For each trial, compute the theoretical and experimental period. The theoretical period can be obtained from Equation (2), while the experimental period can be obtained from $\frac{t}{n}$. Compare these values. Use the experimental period and Equation (4) to compute g and record this in the table provided.

Conclusions:

Describe your results for this experiment.

1. Why was it desirable to use a greater *n* for smaller *L* values?
2. If you were to double the mass on the end of the pendulum, what would happen to the period?
3. Do you think the string / hanging mass approximates a simple pendulum?
4. If you have a pendulum clock which runs too slow, should you lengthen or shorten the pendulum to correct it?

Error Analysis:

Compare your value for *g* to the accepted value. (Do a % error calculation.) How do they compare. If the % error is large, what do you think caused this problem.

Simple Pendulum

Name:__________________________Station:______Section: _________

Abstract:

Data:

L	t	n	$T_{theoretical}$	$T_{experimental}$	g

g_{ave} = _________

Calculations:

Conclusions:

Error Analysis:

Data Collection / Calculations (40%)
Abstract / Conclusions (40%)
Error Analysis (20%)
Grade: