Vectors and the Inclined Plane

Introduction:

This experiment is designed to familiarize you with the concept of force as a vector quantity. The inclined plane will be used to demonstrate how one force vector, the weight, can be decomposed into two component forces, one parallel to the plane and one perpendicular.

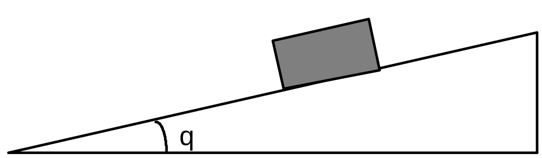
Theory:

By this time, you should already be somewhat familiar with the concept of vectors and scalars from the lecture part of the class. One of the most important vector quantities in physics is the *force vector* **F**. This means that if several forces are at work, each pulling or pushing on an object from a different direction, that the net or *resultant* force can be determined simply by adding up the vectors representing the individual forces by the rules of vector addition.

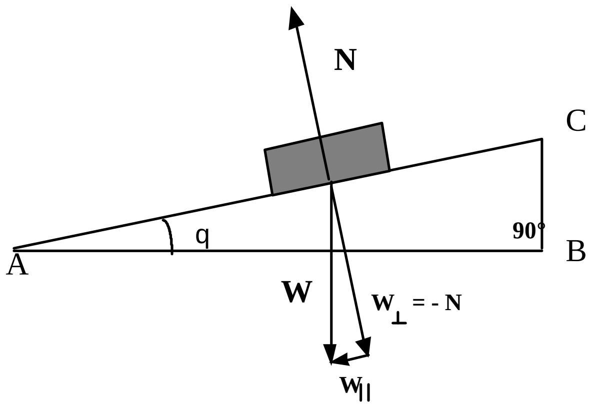
One of the most fundamental physical laws governing the behavior of objects subject to forces is **Newton's First Law:** *the velocity of an object subject to a* ***net force*** *of zero will remain constant.* This means that if an object is initially at rest (velocity of zero), and no net force acts on this object, it will remain at rest.

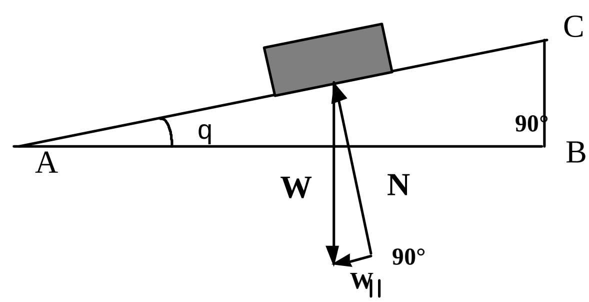
Consider as an example a person standing on a table. We all know that gravity exerts a downward force on the person (called the weight) as a result of the person's mass. With respect to Newton's First Law, how can this person remain at rest? The answer lies in the statement that the *net force* must be zero. Another force, equal and opposite to the gravitational force must be acting on the person so that their sum, the net force, will be zero. This force, which the table exerts on the person, is called the **normal force** and is the force exerted by a rigid object which prohibits penetration of the object. This force is always equal and opposite to the opposing force and perpendicular to the surface of the object in such a way that the sum of the normal and opposing forces is always zero. If it were not zero, the person in our example would accelerate through the table and hit the floor or in the opposite sense be repelled off the surface of the table and hit the ceiling.

Now we will consider the case of an object which is placed on an *inclined plane*, a flat (planar) surface which is at some angle q with respect to level.



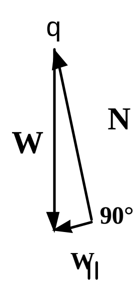
What are the forces at work on this object? If we neglect the friction force, the only two forces are the weight of the object caused by the gravitational force and the normal force exerted by the plane on the object.





On the left, the plane is shown with an inclination q . The force of the object's weight is represented by the vector **W**, and the normal force by the vector **N**. As you can see from the diagram on the right, the addition of **W** and **N** gives a resultant force $\mathbf{W}_{||}$, which is in a direction parallel to the plane[*]. In this sense, **N** can be considered as canceling the *component* of **W** perpendicular to the plane. $\mathbf{W}_{||}$ is the remaining component which is parallel to the plane.

Looking at the diagram on the right, we see that the vectors **W, N,** and $\mathbf{W}_{||}$ make a right triangle.



This triangle is similar to the right triangle A-B-C made from the plane of inclination q. The angle between vectors **W** and **N** is also q.

Now it is time to introduce the trigonometric function called the **sine**. The sine of an angle q in a **right triangle** is the ratio of the length of the side **opposite** to the angle and the length of the **hypotenuse**. For the right triangle of the inclined plane, A-B-C, the sine of q is defined as follows:

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[*] This is the force that makes your car roll down a hill.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC}$$

We also know that for the inclined plane, the triangle made by the vectors **W**, **N**, and $\mathbf{W}_{||}$ is similar to the triangle A-B-C. Therefore, by the definition of similar triangles,

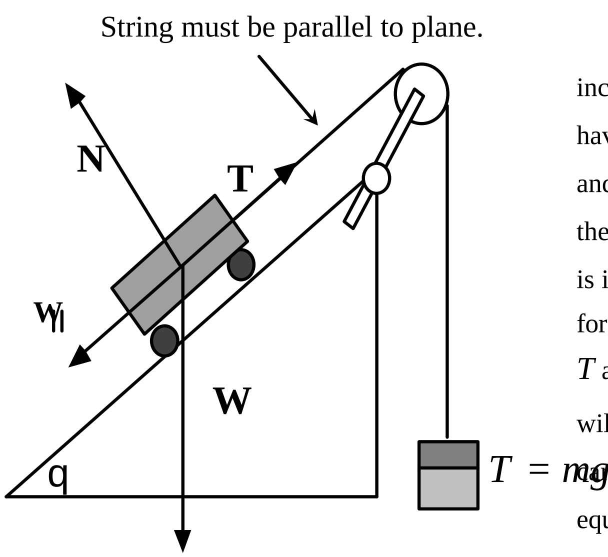
$$\frac{BC}{AC} = \sin\theta = \frac{|\mathbf{W}_{||}|}{|\mathbf{W}|}$$

where $|\mathbf{W}| = W$, the weight of the object and $|\mathbf{W}_{||}| = W_{||}$, the component of the weight parallel to the plane also called the down-hill force. Thus, with a little manipulation, we get the equation for the inclined plane:

$$W_{||} = W \sin\theta \qquad (1)$$

The magnitude of the resultant down-hill force is the weight multiplied by the sine of the angle of inclination of the plane.

Following is a diagram of the apparatus we will be using.



In the diagram to the left, we have an inclined plane of angle θ. On the inclined plane we have a rolling cart, which by means of string, pulley, and mass m, is attached to a force **T**. **T** is parallel to the inclined plane and has magnitude mg. This force is in the exact opposite direction from the down-hill force $\mathbf{W}_{||}$. If the mass m is adjusted such that mg and T are equal to $W_{||}$, then all forces in the diagram will cancel and the cart will not accelerate. When the cart is at rest or has constant velocity, the following equation will be satisfied:

$$W_{||} = T = mg \qquad (2)$$

Our objective is to test Equation (1) by means of Equation (2).

Procedure:

1. Use the angle scale to set the angle of the inclined plane to 15°.
2. Using slot weights, add a 200 g load to the cart.
3. Weigh the cart and load and record the total mass. Multiply the mass by g (the gravitational acceleration) to calculate the weight W.
4. Tie the cord to the cart and pass the cord over the pulley and attach a weight hook to the cord.
5. Add sufficient slot weights to the weight hook to achieve a rough balance. Adjust the pulley to make the cord parallel to the plane. This can be done by naked eye estimation or through the use of a ruler to make the cord equidistant from the plane.
6. Using slot weights as small as 1 g, adjust the hanging weight so that the cart will move up the plane at a very slow constant speed (zero acceleration).
7. Record the amount of hanging mass required to move the mass up the plane and the angle of the plane. Multiply this by g to get the tension T, which is the magnitude of the string force **T**. Record T and θ on the lab form.
8. Repeat 6 and 7, only this time with the cart moving at a slow constant speed *down* the plane.

Note: This process helps us to eliminate the effects of **static** and **kinetic friction**.

9. Repeat 6, 7, and 8 for angles 25° and 40°.

Calculations:

For each angle, calculate the average tension (T_{ave}) from the tension with upward motion (T_{up}) and the tension with downward motion (T_{down}).

On diagrams similar to the one on the opposite page, draw in friction vectors for the cart moving up and moving down the ramp. Assume, since your cart rolled at the same constant speed both up and down that the magnitude of the two friction vectors will be equal. Friction always acts in a direction opposite to motion. How would equation (2) be altered by the presence of friction, both up and down the ramp? (Hint: You will need this information for conclusion question 3.)

From the mass of the cart you should be able to calculate the weight (W), and the predicted down-hill force at the different angles, $W_{||} = W \sin\theta$.

Conclusions:

1. Write in your own words your results for this experiment.

2. What happens to the tension T_{ave} as the angle is increased? Compare the experimental average tension with the theoretical resultant down-hill force. Do their values correspond?

3. Where does friction enter into our system? Describe how your processing of the data reduced the influence of kinetic friction.

4. If you were to plot the tension versus the angle, what would the graph look like?

Error Analysis:

Why did we measure tension for the up-hill constant motion and down-hill constant motion? Take the *difference* between T_{up} and T_{down} and divide by two. What does this represent? Name some other sources for potential error.

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Name:__________________________

Abstract:

Data:

mass of cart & load:________

weight of cart & load: W = ________

angle θ	$W_{\parallel} = W \sin\theta$

	with cart moving up		with cart moving down		
angle θ	**hanging mass** m_{up}	**tension** T_{up}	**hanging mass** m_{down}	**tension** T_{down}	T_{ave}

Calculations:

Conclusions:

Error Analysis: