Measurement and Measurement Error

Rather Lengthy Introduction

Physics makes very general yet quite detailed statements about how the physical universe works. These statements are organized or grouped together in such a way that they provide a model or a kind of coherent picture about **how and why** the universe works the way it does. These sets of statements are called “theories” and are much more than a simple list of “facts and figures” like you might find in an almanac or a telephone book. (Even though almanacs and telephone books are quite useful!) A good physics theory is far more interested in principles than simple “facts”. Noting that the moon appears regularly in the night sky is far less interesting than understanding **why** it does so.

We have confidence that a particular physics theory is telling us something interesting about the physical universe because we are able to test **quantitatively** its predictions or statements about the universe. In fact, all physics (and scientific) theories have this “put up or shut up” quality to them. For something to be called a physics “theory“ in the first place, it **must** make sufficiently quantitative statements about the universe which can be then quantitatively tested. These tests are called “experiments”. The statement, “My girlfriend is the most charming woman in the world,” however true it may be, has no business being in a physics theory because it simply cannot be quantitatively tested. If the experimental measurements are performed correctly and the observations are inconsistent with the theory, then at least some portion of the theory is wrong and needs to be modified. On the other hand, consistency between the theoretical predictions and the experimental observations never decisively proves a theory true. It is always possible that a more comprehensive theory can be developed which will also be consistent with these same experimental observations. Physicists, and scientists in general, always give a kind of provisional assent to their theories, however successfully they might agree with experimental observation. Comfortably working with uncertainty is a professional necessity for them.

It is clear then that making careful quantitative measurements are important if we want to claim that a particular physics theory explains something about the world around us. Measurements represent some physical quantity. For example, 2.1 meter represents a distance, 7 kilograms represents a mass and 9.3 seconds represents a time. Notice that each of these quantities has a number like 9.3 and a unit, “seconds”. The number tells you the amount and the unit tells you the thing that you are talking about, in this case seconds. **Both the number and the unit** are required to specify a measured quantity.

There is a certain inherent inaccuracy or variation in the measurements we make in the laboratory. This inherent inaccuracy or variation is called experimental “error” and in no way is meant to imply incompetence on the part of the experimenter. It merely reflects the fact that our measuring instruments are not perfect. This lack of perfection in our measuring procedure is to be contrasted with mistakes like adding two numbers incorrectly or incorrectly writing down a number from an instrument. Those mistakes have nothing to do with what I mean by experimental error and everything to do with the competence of the experimenter! Understanding and quantifying measurement error is important in experimental science because it is a measure of how seriously we should believe (or not believe) our theories abut how the world works.

When we make measurements in the laboratory, we should distinguish between the **precision** and the **accuracy** of these measurements. If I measure my mass to be 120.317 kilograms, that is a very *precise* measurement because it is very specific. It also happens to be a very *inaccurate* measurement because I am not quite that fat. My mass is considerably less, something like 65 kilograms. So, when we say that we have made a precise measurement we can also say that we have made a very specific measurement. When we say we have made an accurate measurement we can also say that we have made a correct measurement.

The precision of a measurement is indicated by the so-called number of significant figures that it contains. In our mass example, the quantity 120.317 kilograms has 6 significant figures. This is rather precise as measurements go and is considerably more precise than anything you will measure in this course. Again, the fact that a measurement is precise does not make it accurate, just specific. In any event, it is useful to be familiar with how to recognize the number of significant figures in a number. The technique is this:

1. The leftmost non-zero digit is the most significant digit;
2. If there is no decimal point, the rightmost non-zero digit is the least significant;
3. If there is a decimal point, the rightmost digit is the least significant, even if it is zero.

4. All digits between the least significant and the most significant (inclusive) are themselves significant.

For example, the following four numbers have 4 significant figures:
2,314   2,314,000   2.314   9009   9.009   0.000009009   9.000

No physical measurement is completely exact or even precise. Accuracy and precision are *always* limited by the apparatus or the skill of the observer. For example, often physical measurements are made by reading a scale of some sort (ruler, thermometer, dial gauge, etc.). The fineness of the scale markings is limited and the width of the scale lines is greater than zero. In **every** case the final figure of the reading must be estimated and is therefore somewhat inaccurate.

For example, a length measurement of 2.500 meters (4 significant figures) is one digit less uncertain that a measurement of 2.50 meters (3 significant figures). If your measuring instrument is capable of 4 digit precision then you can distinguish between an item 2.500 meters long and one 2.501 meters long. If your instrument were precise to only 3 significant figures, then both items would appear to be 2.50 meters long and you could not distinguish between them. With such a ruler, the finest scale markings are typically 1 millimeter (0.1 cm) apart. You can estimate to 0.5 millimeter, possibly 0.3 millimeter at the best. An estimate of an uncertainty of +/- .03 centimeters is appropriate.

In general, precision and accuracy are limited by two types of error: **random errors** and **systematic errors**. Several measurements of the same thing may yield slightly different values. This is due to random errors. This means that the *variations* in your measurements have no fixed or predictable pattern. Your measured value could be a bit high or a bit low of the correct value. In fact, with random errors an individual measurement of an object is just as likely to be a bit too high as to be a bit too low. For example, using a ruler often produces random errors because you will not consistently get the ruler’s zero line **exactly** aligned with the edge of the measured object every time you make a measurement.

The other general kind of error is a systematic error. These types of errors are often insidious. Measurements may display good precision yet be very inaccurate. Often, measurements afflicted with systematic errors are usually *consistently* too high or too low from the accurate value. The measurements are “systematically” wrong. For example, weighing yourself with a scale that has a bad spring inside of it will yield weight measurements that are inaccurate, either too high or too low, even though the weight measurements may seem reasonable. For example, you might notice that you seem to gain weight after Thanksgiving holiday and lose weight after being ill. These observations may in fact be true but the actual value for your weight will be inaccurate. It is often very difficult to catch systematic errors. To do so requires that you understand very carefully the workings of your measuring devices.

Taking the mean (average) value for a set of measurements is a good way of reducing the effects of random errors. The mean value of the set of measurements is obtained by summing them all up and dividing by the number of measurements that went into the sum. $A = \Sigma m / n$, where n is the number of measurements and m represents each individual value. As you increase the number of measurements, the mean value of these measurements will more closely resemble the actual value of the quantity you are trying to measure, assuming there are no systematic errors present. This is because you are just as likely to measure a value that is slightly too high as one that is too low, so that the random errors will "average themselves out". The magnitude of the relative errors for the individual measurements give you an idea of the uncertainty in your value.

A measure of precision of each measurement is its deviation from the mean value:

$$\%P = | ((m-M) / M) | \times 100\%,$$

where m is a measured value and M represents the mean (or average) of all the measured values.

The uncertainty on the average value A is given by the formula

$$Delta(A) = \sqrt{\frac{\sum (m_i - A)^2}{n(n-1)}}$$

Averaging your measurements will not help you when there are systematic errors present. In fact, there is no standard way to deal with systematic measurements, which is why they are so troublesome.

Objectives

1. To see how measurements and error analysis are a part of experimental science.
2. To make some actual measurements and analyze the errors in them.
3. To observe and understand the difference between accuracy and precision.
4. To understand the nature of random and systematic errors.

Equipment

Candle**,** ruler, special ruler, aluminum dish, metal rods, material to be weighed (dry ice and alcohol) and balance scale.

Procedure

1. Measure the length of the metal rods.

Using **only** the special ruler, measure the length of each piece 6 times. When you are finished, average the values to get a better measure of the piece's true length. Next, use your plastic ruler to measure your pieces again. Measure 6 times as before, compute the average to refine your measured value and estimate the random uncertainty. Then make an "eyeball" estimate of your systematic uncertainties.

2. Measure the height of the candle flame.

Light your candle and let the flame burn steadily for a minute or so. Use the plastic ruler to measure the height of the flame. Make 10 measurements and. try not to melt the ruler. Hold the ruler a small distance away from the flame Record your measurements. Compute the average of your measurements. Compute and record the relative uncertainty of each measurement, %*P*.

3. Measure your reaction time.

Your reaction time is the time that passes between some external stimulus and your first action. We will use an old method to measure your reaction time. A falling ruler will suffice. This is what we do.

a. Have your partner hold the regular ruler vertically, holding it by the top and having the zero point toward the bottom.
b. Place your thumb and forefinger at the ruler's bottom, surrounding the zero point. Be prepared to pinch the ruler as if it were to fall.
c. Your partner will drop the ruler without warning.
d. Pinch and grab the falling ruler as fast as you can. Record the distance the ruler fell. This will tell you your reaction time.
e. Compute your reaction time using Galileo's formula.

$$\text{Distance} = 9.81\ \text{m/s}^2 \cdot (\text{time})^2/2$$

Make 5 measurements and record the corresponding reaction times. Record **your** reaction times on **your** data sheet. Do not mix your times with your partners. This means you will make 5 measurements per person. Compute and record the relative uncertainty of each measurement of your reaction time. Be sure that both you **and** your lab partner have your reaction times measured.

4. Measure the diameter of a closed curve.

Measure the diameter of the large closed curve. Measure this curve diameter across 6 *different* diameters of the curve. Record your measurements. Compute the average diameter. Calculate the %P value for each measurement. Estimate the uncertainty as the largest %P value.

5. Measure the mass of a cold material.

Go to the instructor's table with your partner, where you will be given a cup containing some alcohol and some crushed dry ice. Using the balance on the instructor's table, measure and record the mass of the cup 6 times, at 1 minute intervals. **Warning: The dry ice and alcohol mixture is quite cold. If you stick your fingers in the mixture you will feel much pain.**

Measurement and Measurement Error

Name: ______________________ **Section**: ___________________

Abstract:

Analysis:

1. Rod lengths

Rod 1	Special Ruler	Plastic Ruler
Measurement 1	__________	__________
Measurement 2	__________	__________
Measurement 3	__________	__________
Measurement 4	__________	__________
Measurement 5	__________	__________
Measurement 6	__________	__________
Average Value	__________	__________
Uncertainty	__________	__________

Rod 2	Special Ruler	Plastic Ruler
Measurement 1	___________	___________
Measurement 2	___________	___________
Measurement 3	___________	___________
Measurement 4	___________	___________
Measurement 5	___________	___________
Measurement 6	___________	___________
Average Value	___________	___________
Uncertainty	___________	___________

a. What are the possible sources of error in this measurement?

b. How well did your measurements with the special ruler agree with those done with the plastic ruler? If there was a disagreement, what kind of error was it? Random or systematic? What caused this error?

2. Candle Flame

	Flame height	%*P*
Measurement 1	____________	____________
Measurement 2	____________	____________
Measurement 3	____________	____________
Measurement 4	____________	____________
Measurement 5	____________	____________
Measurement 6	____________	____________
Measurement 7	____________	____________
Measurement 8	____________	____________
Measurement 9	____________	____________
Measurement 10	___________	____________
Average Value	____________	____________
Uncertainty	____________	____________

a. What are the possible sources of error in this measurement?

b. What might you do to get a better measurement of the flame’s height?

3. Your reaction time

	Distance	Time	%*P* (time)
Measurement 1	____________	_________	_________
Measurement 2	____________	_________	_________
Measurement 3	____________	_________	_________
Measurement 4	____________	_________	_________
Measurement 5	____________	_________	_________
Average value	____________	_________	_________

a. What are the possible sources of error in this measurement?

4. Closed Curve Diameter

	Curve diameter	%*P*
Measurement 1	_____________________	________________
Measurement 2	_____________________	________________
Measurement 3	_____________________	________________
Measurement 4	_____________________	________________
Measurement 5	_____________________	________________
Measurement 6	_____________________	________________
Average value	_____________________	________________

Uncertainty %P _____________________

a. What are possible sources of error in this measurement?

b. What do the measurements tell you about the curve's diameter?

5. Mass of cold material.

Measurement 1	______________________
Measurement 2	______________________
Measurement 3	______________________
Measurement 4	______________________
Measurement 5	______________________
Measurement 6	______________________

a. What are possible sources of error in this measurement?

b. Do you see any pattern in the measured masses?

6. Which of your measurements (metal rods, curve diameter, flame height, etc.) was the most uncertain? Why was it so?

7. Which of your measurements (metal rods, curve diameter, flame height, etc.) was the least uncertain? Why was this so?

8. Which measurements (metal rods, curve diameter, flame height, etc.), if any, suffered from systematic error?

Conclusions: