

Appendix A: Significant Figures and Error Analysis

Every measurement of a physical quantity contains some amount of uncertainty or error. We often speak of a certain number or measurement as being precise or accurate when referring to its reliability. Although they are related, the concepts of **precision** and **accuracy** should not be confused. The precision of an instrument refers to the smallest interval which can be measured by the instrument. For example, an instrument which can measure millimeters is more precise than one which is limited to the measurement of centimeters. The precision is largely determined by the numerical agreement between repeated measurements. For example, you may measure a certain length three times and get the following data: 1.234 m, 1.232 m, 1.236 m. In this case, your measuring device is precise in the centimeter range and approaches precision in the millimeter range.

In contrast, accuracy refers to the agreement between the measurement taken and a standard or accepted value. To better understand these concepts, consider the measurement of a length with a standard ruler. The standard ruler can measure with a precision of up to 1 mm. The accuracy of the measurement, however, will depend on the manufacture of the ruler and its alignment with the length in question. If the length called 1 cm on the ruler is longer than the standard centimeter, or if the student fails to properly align the ruler, the measurement will have poor accuracy. **Systematic error** refers to problems such as the design of the ruler, while **random error** refers to such problems as mis-alignment.

Significant figures:

The number of significant figures in a quantity refers to the number of digits which have experimental significance. For instance, the actual or given value for the gravitational acceleration (g) can be written as

$$\begin{aligned}
 g &= 9.81 \text{ m/s}^2, \\
 g &= 981 \text{ cm / s}^2, \\
 \text{or} \quad g &= 9810 \text{ mm / s}^2.
 \end{aligned}$$

To write $g = 9.81 \text{ m/s}^2$ means that our value for g lies somewhere between 9.80 m/s^2 and 9.82 m/s^2 . The first two digits are certain, while the third is an **estimate**. All of the above values for g have three significant figures. If we had written $g = 9810. \text{ mm / s}^2$ (with a

decimal point at the end), the zero would have become significant, giving four significant figures. When counting significant figures, one must disregard all zeros which appear to the left of the first non-zero digit and all zeros which appear to the right of the last non-zero digit if no decimal point is present. If a decimal point is present, zeros to the right of the last non-zero digit are significant. Here are a few examples:

| Number | Significant Figures |
|--------|---------------------|
| 0.003 | 1 |
| 0.032 | 2 |
| 0.0320 | 3 |
| 320 | 2 |
| 320. | 3 |
| 320.0 | 4 |

When performing mathematical operations of multiplication or division on experimental data, the number of significant figures in the result must never exceed the number of "sig-figs" in the argument with the *least* number of sig-figs. For example, if you divide the number 221 (three sig-figs) by 72 (2 sig-figs), the result can only have two significant figures. If you perform this operation on a calculator, your result will be:

$$\frac{221}{72} = 3.069444$$

which has too many sig-figs. After adjusting for the correct number of significant figures, you would write:

$$\frac{221}{72} = 3.1$$

It is important to note that when performing a series of multiplications and or divisions, you should only round the final result, not the results from intermediate steps.

Error analysis:

Many times in your experiments, you will make repeated measurements. Due to a lack of precision and accuracy, the data will fluctuate. To get a better value from fluctuating data, we will calculate the average value or **mean**. Unfortunately, all information concerning the amount of fluctuation (a measurement of error and uncertainty) is lost after taking the mean. Therefore, we need a method of measuring the

reporting this deviation. This is accomplished through computing the **average deviation from the mean** (S).

Let us consider a data set of ten measurements of a quantity called x .

| n | x |
|-----|------|
| 1 | 1.23 |
| 2 | 1.24 |
| 3 | 1.22 |
| 4 | 1.22 |
| 5 | 1.26 |
| 6 | 1.27 |
| 7 | 1.22 |
| 8 | 1.27 |
| 9 | 1.22 |
| 10 | 1.23 |

If one calculates the mean on a calculator, the result will be 1.238. This number has four significant figures, so we need to round the result to three. To express the mean value of x , one writes $x_{ave} = 1.24$. Each of the measurements of x can be represented by a variable and a subscript. For example, $n = 1, x = 1.23$, can be written as $x_1 = 1.23$, and $n = 2, x = 1.24$, can be written as $x_2 = 1.24$. The total number of measurements is N . In this set, $N = 10$. In this notation, the formula for the mean can be written as:

$$x_{ave} = \frac{1}{N} \sum_{n=1}^N x_n$$

where the symbol Σ designates a sum, in this case, a sum of all the measurements of x from $n = 1$ to $n = 10$.

Many numbers in the set deviate from the mean value of $x = 1.24$. The amount of deviation is given by the absolute value of the difference between the individual measurement and the average value. For a certain x_n , this deviation can be written:

$$S_n = |x_n - x_{ave}|$$

Inserting this formula into the formula for the mean, we get the following equation for the average deviation from the mean:

$$S_{ave} = \frac{1}{N} \sum_{n=1}^N |x_n - x_{ave}|$$

$$S_{ave} = \frac{1}{N} \sum_{n=1}^N S_n$$

In conclusion, to calculate the average deviation from the mean, you should use the following steps:

1. Calculate the mean value, x_{ave} .
2. Calculate the individual deviations from the mean, $|x_n - x_{ave}|$
3. Average these individual deviations.

In the case of the data given above, the average deviation from the mean is 0.018. This is a measurement of random error. In writing a report for an experiment, you would report the value for x as:

$$x = 1.24 \pm 0.018$$

In some cases, you may wish to report the **percent error**. The percent error is a measure of the actual error, and is given by the following formula:

$$\% \text{ error} = \frac{|\text{actual value} - \text{experimental value}|}{\text{actual value}} \times 100\%$$

