## MAGNETIC DIPOLE MOMENT

EQUIPMENT: Helmholtz Coils, multimeter (ammeter), power supply, 0-10 DCV meter, timer, cylindrical magnet, thread, vernier calipers, and general use items.

In this exercise we will introduce oscillations to a small permanent magnet located in the magnetic field of a Helmholtz coil to measure the magnetic dipole moment of the magnet and the horizontal component of Earth magnetic field.

A permanent magnet placed in an external magnetic field behaves as a compass needle and rotates to become aligned with the field. In absence of artificially created field, the equilibrium position is determined by the horizontal component of the Earth magnetic field $\mathrm{B}_{\mathrm{e}}$. A suspended magnet displaced from its equilibrium position oscillates about this position in simple harmonic motion with a period of oscillation depending on the strength of the field. In this laboratory exercise, in addition to the Earth magnetic field we will apply an external field created by a $2 \times 60$ turns Helmholtz coil.
The field generated by such coil is nearly uniform and given by the relation
$\mathrm{B}=\mathrm{C} I$, where I is the applied current and the constant C describes magnet properties

$$
\mathrm{C}=4 \pi 10^{-7} \frac{8 \mathrm{~N}}{\mathrm{R} \sqrt{125}}
$$

Here $\mathrm{N}=60$ is the number of turns, R is the average radius of the coil.
In general, the frequency of oscillations, $f$, depends on values of both (Earth and coil) fields, on the value of the magnetic torque, $\mu$, on the moment of inertia of the magnet, I, and on the elastic torque of the suspension string. In practice, we can neglect the dependence on the string elastic properties.

The moment of inertia of a cylindrical magnet suspended in its middle and rotating in horizontal plane is given by

$$
\mathrm{I}=\mathrm{ML}^{2} / 12
$$

where M is the mass and L is the length of the magnet.
The oscillation frequency dependence on the current is given by an equation

$$
\mathrm{f}^{2}=\left(\frac{\mathrm{C} \mu}{4 \pi^{2} \mathrm{I}}\right) \mathrm{i}+\frac{\mathrm{B}_{\mathrm{e}} \mu}{4 \pi^{2} \mathrm{I}}
$$

Notice that $f^{2}$ is a linear function of $i$. If $f^{2}$ is plotted as a function of $i$, the slope of the resulting straight line allows a calculation of $\mu$ and the $y$-intercept allows a calculation of Be.

In summary, the theory predicts a linear plot for $\mathrm{f}^{2}$ vs. I and there is the following meanings for the symbols involved.
$\mathrm{I}=\mathrm{ML}^{2} / 12$ - the moment of inertia of the cylindrical magnet, where M is the mass and L is the length of the magnet.
$\mu$ is the magnetic dipole moment of the magnet and $\mathrm{B}_{\mathrm{e}}$ is the horizontal component of the earth's magnetic field.

The constants, C and I, may be determined by direct measurement of the parameters involved, $R, M$ and $L$. With these known, a plot of $f^{2}$ vs. $i$ provides a slope from which $\mu$ may be determined and an intercept from which $\mathrm{B}_{\mathrm{e}}$ may be determined.

## PROCEDURE:

1) Use a ruler to measure the average radius of the coils ( R ). The average radius can be measured between the outer edge of the disc and the screws indicating inner radius of the coil. Each coil should be measured at least twice and the average found. Record these data for use in calculating C .
2) Use a triple-beam balance to measure $M$ and a vernier caliper to measure $L$ for the cylindrical magnet. Record these data for use in calculating the moment of inertia (I) of the magnet.
3) Suspend the magnet by thread so that it hangs in the central region of the coils. Adjust the thread so that the magnet hangs in a horizontal (level) position.
4) Connect the coils, power supply and ammeter ( $2,000 \mathrm{ma}, \mathrm{DCA}$ ), in a series circuit.
5) When the freely suspended magnet becomes stationary, it will point in a magnetic north south direction by definition of the north magnetic pole. When the magnet is stationary, gently rotate the coils until their axis (line joining their centers) is parallel to the magnet. Make final adjustments to the thread so that the magnet is level and lies along the axis of the coils.
6) In the theory description it was assumed that the earth's field and the field of the coils are aligned in the same direction. To satisfy this condition, give the magnet a small amplitude angular displacement and allow it to oscillate, turn the power supply on and slowly increase the voltage until the current through the coils is about 1Ampere. If the Earth magnetic field and the field generated by the Helmholtz coil are in the same direction, the frequency of oscillations will increase with increasing current. If the two fields are in opposite directions the magnet will spin around while current is increased. In such case, reverse the direction of the coil current by reversing (exchanging) the connections and repeat the check.
7) Now you need to measure frequency as function of current and generate $f^{2}$ vs. i plot. As a first step the frequency is needed when the coil current $i$ is zero. Turn the power supply off, set the magnet into angular oscillation in a horizontal plane with a "small" amplitude of about 10 degrees. Use the timer (stop watch) to find the total time ( t ) required for a selected number of oscillations ( n ) measuring the frequency f
$=n / t$. Choose $n$ yourself so that the relative error in time is small. This is a very important choice because f is squared and propagation of error will be large. In order to minimize human (systematic) error, the time should be larger than 10 seconds. Also, in timing oscillations, either extreme of the motion may be used as a reference, starting the clock at the zero count and stopping it at the nth count. Record $n$ and $t$ at $\mathrm{i}=0$ for later use. This data is not to be plotted but is to be used to check the accuracy of the intercept found from the plot.
8) Turn the power supply on and measure the frequency varying the current from 0.2 ampere to 0.8 ampere. Do not exceed a value of 1.2 ampere as such high current will damage the coils. Choose at least four well spaced points will for the plot. Record the number of oscillations, time and current and calculate the frequency squared.

ANALYSIS: Calculate $f^{2}$ for each i and plot $\mathrm{f}^{2}$ vs. i. Calculate C and I and use these to find $\mu$ from the slope then use $\mu$ with the $y$-intercept to find $\mathrm{B}_{\mathrm{e}}$. Compare the value of $B_{e}$ found from the $y$-intercept of the plot and the $B_{e}$ found using the measured $f$ when $i=$ 0 .

## Magnetic Dipole Moment

## Name

$\qquad$ Section $\qquad$


#### Abstract

:


## Data:

Coil radius $\quad \mathbf{R}_{1}=$ $\qquad$

$$
\mathbf{R}_{2}=
$$

Average radius $\mathbf{R}=$ $\qquad$
Coil Mass $\quad M=$ $\qquad$
Moment of inertia $I=$ $\qquad$

Oscillation data:

| $I, A$ | $n$ | $t$ | $T=t / n$ | $f=1 / T$ | $f^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |
| 0.2 |  |  |  |  |  |
| 0.4 |  |  |  |  |  |
| 0.6 |  |  |  |  |  |
| 0.8 |  |  |  |  |  |

## Calculations:

## Conclusion:

## Error analysis:

