Lecture 21
Calorimetry

• Basic principles
  – Interaction of charged particles and photons
  – Electromagnetic cascades
  – Nuclear interactions
  – Hadronic cascades

• Homogeneous calorimeters

• Sampling calorimeters
Reminder

- Ionisation
- Bremsstrahlung
- Photoelectric effect
- Compton effect
- Pair production

\[ \gamma \]

\[ \frac{dE}{dx} \]

\[ E \]

\[ e^+ / e^- \]
Critical Energy, $E_c$ – losses due to ionisation and Bremsstrahlung are equal

\[ \frac{dE}{dx}(E_c)_{\text{Brems}} = \frac{dE}{dx}(E_c)_{\text{ion}} \]

For muons

\[ E_c \approx E_c^{\text{elec}} \left( \frac{m_\mu}{m_e} \right)^2 \]

Critical Energy, $E_c$ for electrons

\[ E_{c(e^-)} \text{ in Fe}(Z=26) = 22.4 \text{ MeV} \]

\[ E_{c(m)} \text{ in Fe}(Z=26) \sim 1 \text{ TeV} \]
Electromagnetic cascades (showers)

Electron shower in a cloud chamber with lead absorbers

Consider only Bremsstrahlung and pair production.
Assume: $X_0 = \lambda_{\text{pair}}$

$N(t) = 2^t \quad E(t)/\text{particle} = E_0 \cdot 2^{-t}$

Process continues until $E(t) < E_c$

$$t_{\text{max}} = \frac{\ln E_0 / E_c}{\ln 2} \quad N_{\text{total}} = \sum_{t=0}^{t_{\text{max}}} 2^t = 2^{(t_{\text{max}}+1)} - 1 \approx 2 \cdot 2^{t_{\text{max}}} = 2 \frac{E_0}{E_c}$$

After $t = t_{\text{max}}$ the dominating processes are ionization, Compton effect and photo effect $\rightarrow$ absorption.
Electromagnetic cascades

**Longitudinal** shower development: \[ \frac{dE}{dt} \propto t^\alpha e^{-t} \]

Shower maximum at \[ t_{\text{max}} = \ln \left( \frac{E_0}{E_c} \right) \frac{1}{\ln 2} \]

95% containment \[ t_{95\%} \approx t_{\text{max}} + 0.08Z + 9.6 \]

Size of a calorimeter grows logarithmically with \( E_0 \)

**Transverse** shower development: 95% of the shower cone is located in a cylinder with radius 2 \( R_M \)

Moliere radius \[ R_M = \frac{21 \text{ MeV}}{E_c} X_0 \quad [g/cm^2] \]

Longitudinal and transverse development scale with \( X_0, R_M \)

Example: \( E_0 = 100 \text{ GeV} \) in lead glass

\( E_c = 11.8 \text{ MeV} \Rightarrow t_{\text{max}} \approx 13, \ t_{95\%} \approx 23 \)

\( X_0 \approx 2 \text{ cm}, \ R_M = 1.8 \cdot X_0 \approx 3.6 \text{ cm} \)

Example:
- Electron with \( 6 \text{ GeV/c} \)
- Energy deposition in different materials
- Longitudinal energy deposit scaling with depth
Energy resolution

Intrinsic limit

Total number of track segments

\[ N^{\text{total}} \propto \frac{E_0}{E_c} \]

Resolution

\[ \frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}} \]

Spatial and angular resolution scale like \(1/\sqrt{E}\)

Relative energy resolution of a calorimeter improves with \(E_0\)

\[ \frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{\sqrt{E}} \oplus \frac{c}{E} \]

stochastic term  constant term  noise term

Quality factor!

inhomogenities  electronic noise
bad cell inter-calibration  radioactivity
non-linearities  pile up
The interaction of energetic hadrons (charged or neutral) is determined by inelastic nuclear processes.

\[ \lambda_a = \frac{A}{N_A \sigma_{inel}} \]

**Nuclear interaction**

At high energies (>1 GeV) the cross-sections depend only weakly on the energy and on the type of the incident particle (p, p, K…)

\[ p_t \approx 0.35 \text{ GeV/c} \]

\[ \text{multiplicity} \sim \ln(E) \]

Excitation and breakup of nucleus \( \rightarrow \) nucleus fragments + secondary particles

In analogy to \( X_0 \) we define a **hadronic absorption length**
## Radiation and absorption length

<table>
<thead>
<tr>
<th>Material</th>
<th>Z</th>
<th>A</th>
<th>( \rho \text{ [g/cm}^3] )</th>
<th>( X_0 \text{ [g/cm}^2] )</th>
<th>( \lambda_a \text{ [g/cm}^2] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen (gas)</td>
<td>1</td>
<td>1.01</td>
<td>0.0899 (g/l)</td>
<td>63</td>
<td>50.8</td>
</tr>
<tr>
<td>Helium (gas)</td>
<td>2</td>
<td>4.00</td>
<td>0.1786 (g/l)</td>
<td>94</td>
<td>65.1</td>
</tr>
<tr>
<td>Beryllium</td>
<td>4</td>
<td>9.01</td>
<td>1.848</td>
<td>65.19</td>
<td>75.2</td>
</tr>
<tr>
<td>Carbon</td>
<td>6</td>
<td>12.01</td>
<td>2.265</td>
<td>43</td>
<td>86.3</td>
</tr>
<tr>
<td>Nitrogen (gas)</td>
<td>7</td>
<td>14.01</td>
<td>1.25 (g/l)</td>
<td>38</td>
<td>87.8</td>
</tr>
<tr>
<td>Oxygen (gas)</td>
<td>8</td>
<td>16.00</td>
<td>1.428 (g/l)</td>
<td>34</td>
<td>91.0</td>
</tr>
<tr>
<td>Aluminium</td>
<td>13</td>
<td>26.98</td>
<td>2.7</td>
<td>24</td>
<td>106.4</td>
</tr>
<tr>
<td>Silicon</td>
<td>14</td>
<td>28.09</td>
<td>2.33</td>
<td>22</td>
<td>106.0</td>
</tr>
<tr>
<td>Iron</td>
<td>26</td>
<td>55.85</td>
<td>7.87</td>
<td>13.9</td>
<td>131.9</td>
</tr>
<tr>
<td>Copper</td>
<td>29</td>
<td>63.55</td>
<td>8.96</td>
<td>12.9</td>
<td>134.9</td>
</tr>
<tr>
<td>Tungsten</td>
<td>74</td>
<td>183.85</td>
<td>19.3</td>
<td>6.8</td>
<td>185.0</td>
</tr>
<tr>
<td>Lead</td>
<td>82</td>
<td>207.19</td>
<td>11.35</td>
<td>6.4</td>
<td>194.0</td>
</tr>
<tr>
<td>Uranium</td>
<td>92</td>
<td>238.03</td>
<td>18.95</td>
<td>6.0</td>
<td>199.0</td>
</tr>
</tbody>
</table>

For \( Z > 6 \): \( \lambda_a > X_0 \)
Hadronic cascades

Various processes involved. Much more complex than electromagnetic cascades.

hadronic
\[ \downarrow \]
charged pions, protons, kaons ....
breaking up of nuclei (binding energy),
neutrons, neutrinos, soft g’s
muons .... \( \rightarrow \) invisible energy

\[ n(\pi^0) \approx \ln E(GeV) - 4.6 \]

for 100 GeV pp collision: \( n(\pi^0) \approx 18 \)

\[ \]


**Shower development**

Longitudinal shower shape

\[ t_{\max} (\lambda_t) \approx 0.2 \ln E[GeV] + 0.7 \]
\[ t_{95\%} (cm) \approx a \ln E + b \]

For Iron: \( a = 9.4, b = 39 \) \( \lambda_a = 16.7 \text{ cm} \)
\( E = 100 \text{ GeV} \) \( \rightarrow t_{95\%} \approx 80 \text{ cm} \)

Lateral shower shape

The shower consists of core + halo. 95% containment in a cylinder of radius \( \lambda \).

Hadronic showers are much longer and broader than electromagnetic ones.
Types of calorimeters

Homogeneous calorimeters: (e.g., crystals)

⇒ Detector = absorber
⇒ good energy resolution
⇒ limited spatial resolution (particularly in longitudinal direction)
⇒ only used for electromagnetic calorimetry

Sampling calorimeters:

⇒ Detectors and absorber separated ⇒ only part of the energy is sampled
⇒ limited energy resolution
⇒ good spatial resolution
⇒ used both for electromagnetic and hadron calorimetry
Homogeneous calorimeters

Signal = photons (scintillation or Cherenkov radiation).
Readout via photomultiplier, -diode/triode.….

- Scintillators (crystals)

<table>
<thead>
<tr>
<th>Scintillator</th>
<th>Density [g/cm$^3$]</th>
<th>$X_0$ [cm]</th>
<th>Light Yield $\gamma$/MeV (rel. yield)</th>
<th>$\tau_1$ [ns]</th>
<th>$\lambda_1$ [nm]</th>
<th>Rad. Dam. [Gy]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI (Tl)</td>
<td>3.67</td>
<td>2.59</td>
<td>$4 \times 10^4$</td>
<td>230</td>
<td>415</td>
<td>$\geq 10$</td>
<td>hydroscopic, fragile</td>
</tr>
<tr>
<td>CsI (Tl)</td>
<td>4.51</td>
<td>1.86</td>
<td>$5 \times 10^4$ (0.49)</td>
<td>1005</td>
<td>565</td>
<td>$\geq 10$</td>
<td>Slightly hygroscopic</td>
</tr>
<tr>
<td>CSI pure</td>
<td>4.51</td>
<td>1.86</td>
<td>$4 \times 10^4$ (0.04)</td>
<td>10</td>
<td>310</td>
<td>$10^3$</td>
<td>Slightly hygroscopic</td>
</tr>
<tr>
<td>BaF$_2$</td>
<td>4.87</td>
<td>2.03</td>
<td>$10^4$ (0.13)</td>
<td>0.6</td>
<td>220</td>
<td>&lt; 500</td>
<td></td>
</tr>
<tr>
<td>BGO</td>
<td>7.13</td>
<td>1.13</td>
<td>$8 \times 10^3$</td>
<td>36</td>
<td>310</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>PbW0$_4$</td>
<td>8.28</td>
<td>0.89</td>
<td>$\approx 100$</td>
<td>10</td>
<td>440</td>
<td>$10^4$</td>
<td>light yield =f(T)</td>
</tr>
</tbody>
</table>

Light yield relative to NaI(Tl) readout with PM (bialkali photocathode)
## Homogeneous calorimeters

### Cherenkov radiators

<table>
<thead>
<tr>
<th>Material</th>
<th>Density [g/cm$^3$]</th>
<th>$X_0$ [cm]</th>
<th>n</th>
<th>Light yield [p.e./GeV] (rel. p.e.)</th>
<th>$\lambda_{cut}$ [nm]</th>
<th>Rad. Dam. [Gy]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF-5</td>
<td>4.08</td>
<td>2.54</td>
<td>1.67</td>
<td>600 $(1.5 \times 10^{-4})$</td>
<td>350</td>
<td>$10^2$</td>
<td></td>
</tr>
<tr>
<td>SF-6</td>
<td>5.20</td>
<td>1.69</td>
<td>1.81</td>
<td>900 $(2.3 \times 10^{-4})$</td>
<td>350</td>
<td>$10^2$</td>
<td></td>
</tr>
<tr>
<td>PbF$_2$</td>
<td>7.66</td>
<td>0.95</td>
<td>1.82</td>
<td>2000 $(5 \times 10^{-4})$</td>
<td></td>
<td>$10^3$</td>
<td>Not available in quantity</td>
</tr>
</tbody>
</table>

Light yield relative to NaI(Tl) readout with PM (bialkali photocathode)
Sampling calorimeters

Absorber + detector separated $\Rightarrow$ sampling fluctuations

$\text{Detectable track segments}$

$$N = \frac{T_{\text{det}}}{d}$$

$$= F(\xi) \frac{E}{E_c} X_0 \frac{1}{d}$$

$$\frac{\sigma(E)}{E} \propto \sqrt{\frac{N}{N}} \propto \sqrt{\frac{1}{E} \sqrt{\frac{d}{X_0}}}$$

MWPC, streamer tubes
warm liquids:
  - TMP = tetramethylpentane,
  - TMS = tetramethysilane
cryogenic noble gases:
  - mainly LAr (LXe, LKr)
scintillators, scintillation fibres, silicon detectors
ATLAS LAr Calorimeter

Accordion geometry absorbers immersed in Liquid Argon

Liquid Argon (90K)
+ lead-steel absorbers (1-2 mm)
+ multilayer copper-polyimide readout boards
1 GeV E-deposit $\rightarrow 5 \times 10^6$ e$^-$

Accordion geometry minimizes dead zones.
Liquid Ar is intrinsically radiation hard.
Readout board allows fine segmentation (azimuth, pseudo-rapidity and longitudinal)

Spatial and angular uniformity $\approx 0.5\%$
Spatial resolution $\approx 5\text{mm} / E^{1/2}$
Pointing
CMS hadron calorimeter

Cu absorber + scintillator
2 × 18 wedges (barrel)
+ 2 × 18 wedges (endcap) ≈ 1500 T absorber

Scintillators fill slots and are read out via fibres by HPDs (hybrid photodiodes)

Test beam resolution for single hadrons

$$\frac{\sigma_E}{E} = \frac{65\%}{\sqrt{E}} \oplus 5\%$$
Energy Measurement

How we determine the energy of a particle from the shower?
– Detector response must have linearity i.e., signal proportional to the energy deposit
• The average calorimeter signal vs. the energy of the particle is different for homogenous and sampling calorimeters
• Hadronic showers may include electromagnetic component from $\pi^0$’s
• Detector resolution is controlled by fluctuations, i.e., event to event variations of the signal.

In general EM calorimeters have linear responses while hadronic calorimeters do not.

Sources of non-linearity:
- saturation of the medium (gas, crystal, scintillator)
- non-linearity of detectors (PMT, Photodiodes, electronics)
- leakage of the signal outside the detector
Homogeneous calorimeters - crystals or liquid Xe
Scintillation proportional to the total electron energy

Advantages:
- excellent energy resolution -> best statistical precision for mean energy W required to produce a signal eg., visible photon in a crystal
  \[ \frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}} \]
- uniform response -> good linearity

Disadvantages:
- limited segmentation
- high cost
CMS EM Calorimeter
barrel + endcaps 77,000 PbWO$_4$ crystals
Energy resolution – 1% at 30 GeV
**Sampling calorimeters**
Sandwich of dense material to induce showering interspaced with a detector (scintillator counting tracks, LAr counting ionization,..)
Advantage – good spatial segmentation, both lateral and in depth
Disadvantage – only see part of the shower

\[ f_{\text{sampling}} = \frac{E_{\text{visible}}}{E_{\text{deposited}}} \]

ATLAS
LAr
Energy resolution

\[ \sigma_E = a\sqrt{E} \oplus bE \oplus c \]

\[ \frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E} \]

a – stochastic term due to
intrinsic statistical shower fluctuations
sampling fluctuations
signal quantum fluctuations (e.g., photo-electron statistics)
b – constant term due to
inhomogeneities and imperfections
non-linearity of electronics
fluctuation of the energy lost in the absorber
c – noise term due to electronics noise, natural radioactivity, pile-up
Possible setups

Scintillators as active layer; signal readout via photo multipliers

Absorber  Scintillator
Light guide  Photo detector

Scintillators as active layer; wavelength shifter to convert light
Scintillator (blue light)
Wavelength shifter

Charge amplifier
Absorber as electrodes
HV

Active medium: LAr; absorber embedded in liquid serve as electrodes
Argon

Ionization chambers between absorber plates
Electrodes
Analogue signal
Hadron showers

Initiated by strong interactions
Characterized by hadronic interaction length
Contain electromagnetic components
Large complexity – requires simulation tools
Hadronic interactions cross sections

\[ \sigma_{tot} = \sigma_{el} + \sigma_{inel} \]
\[ \sigma_{el} \approx 10 \text{mb} \]
\[ \sigma_{inel} \approx A^{2/3} \]
\[ \sigma_{tot} (\text{nucl}) = \sigma_{tot} (\text{pp}) A^{2/3} \]

\( \sigma_{tot} (\text{pp}) \) increases with \( s \)
Energy resolution of hadron showers

Hadronic energy resolution of non-compensating calorimeters does not scale with $1/\sqrt{E}$ but as

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \left( \frac{E}{E_0} \right) \approx \frac{a}{\sqrt{E}} \oplus b$$

![Graph showing energy resolution as a function of $1/\sqrt{E}$](image)