Lecture 18

Problem with units

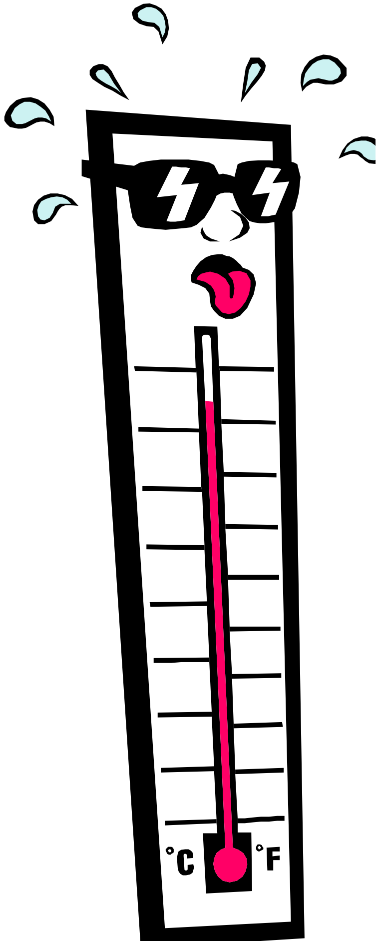
One of the typical problems is keeping track of the consistency of units in particle physics calculations.
The physical constants in quantum mechanics often have dimensions

Planck constant	$h = 6.626 \times 10^{-34}$ J s
reduced Planck constant	$\hbar = 1.054 \times 10^{-34}$ J s
velocity of light in vacuum	$c = 299\,792\,458$ m s^{-1}
electronvolt	eV = 1.6×10^{-19} J
barn	$b = 10^{-28}$ m^2

conversion factor	$\hbar c = 197.3$ MeV fm (femtometer = 10^{-15}m)

Calorimetry

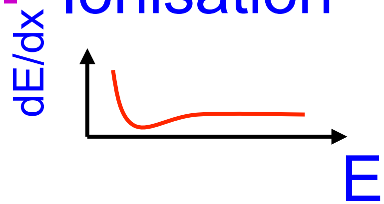
- Basic principles
  - Interaction of charged particles and photons
  - Electromagnetic cascades
  - Nuclear interactions
  - Hadronic cascades
- Homogeneous calorimeters
- Sampling calorimeters



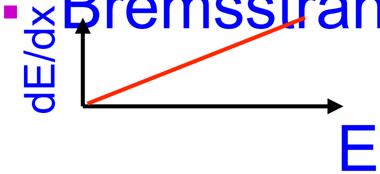
Reminder

e^+ / e^-

- Ionisation

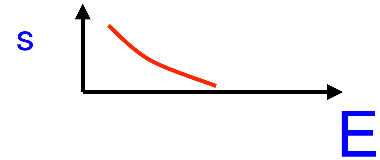


- Bremsstrahlung

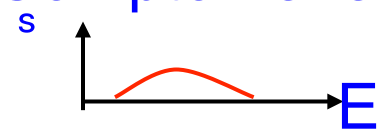


γ

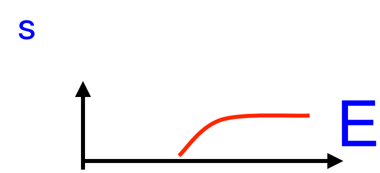
- Photoelectric effect



- Compton effect

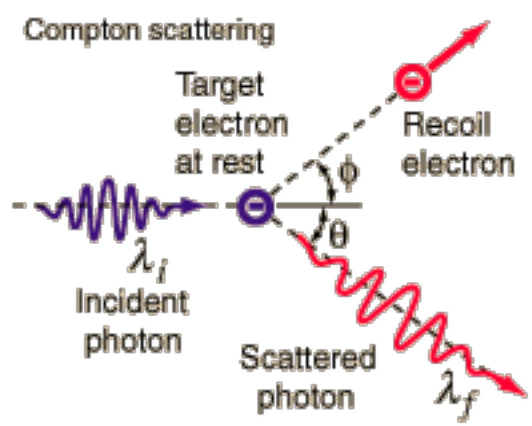


Pair production

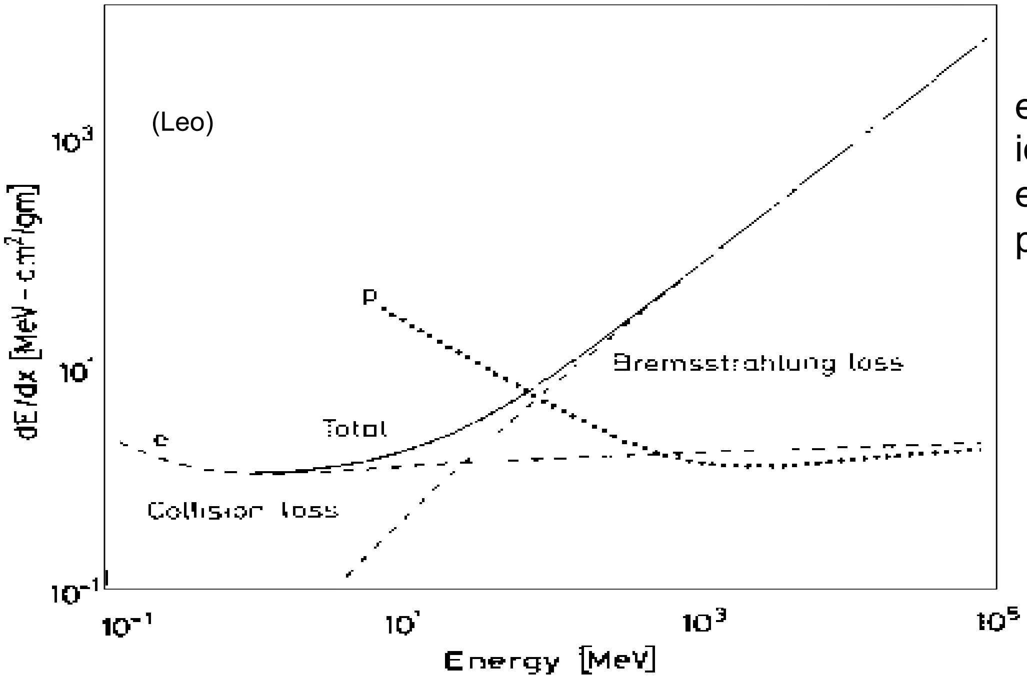


Compton scattering

Artur Compton observed an effect of inelastic scattering of photon on charged particle (usually electron). It results in a decrease of the energy of the photon − increase of the wavelength with art of the energy transferred to a recoiling electron. This is a low energy (X ray) effect and it is not important for high energy photons.



Interaction of charged particles



energy loss (radiative + ionization) of electrons and protons in copper

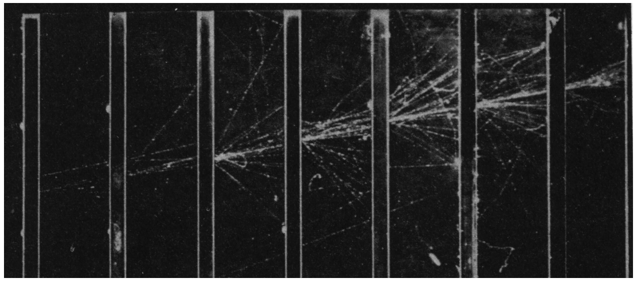
Critical Energy, E_c – losses due to ionisation and Bremsstrahlung are equal

$$\left.\frac{dE}{dx}(E_c)\right|_{Brems} = \left.\frac{dE}{dx}(E_c)\right|_{ion} \longrightarrow \quad E_c^{solid+liq} = \frac{610MeV}{Z+1.24} \qquad E_c^{gas} = \frac{710MeV}{Z+1.24}$$

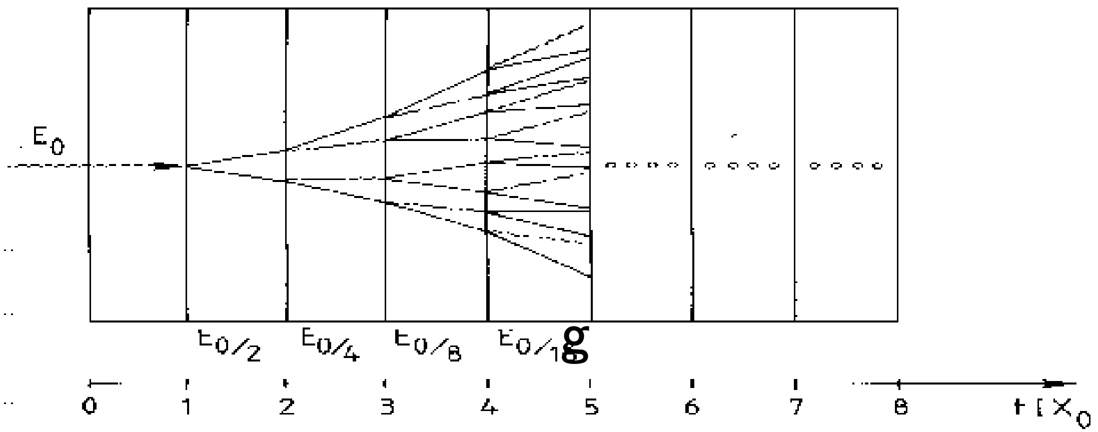
For muons $$E_c \approx E_c^{elec}\left(\frac{m_\mu}{m_e}\right)^2$$

$E_c(e^-)$ in Fe(Z=26) = 22.4 MeV
$E_c(m)$ in Fe(Z=26) ~ 1 TeV

Electromagnetic cascades (showers)



Electron shower in a cloud chamber with lead absorbers



Consider only Bremsstrahlung and pair production.
Assume: $X_0 = \lambda_{pair}$

$$N(t) = 2^t \qquad E(t)/particle = E_0 \cdot 2^{-t}$$

Process continues until $E(t) < E_c$

$$t_{max} = \frac{\ln E_0 / E_c}{\ln 2} \qquad N^{total} = \sum_{t=0}^{t_{max}} 2^t = 2^{(t_{max}+1)} - 1 \approx 2 \cdot 2^{t_{max}} = 2\frac{E_0}{E_c}$$

After $t = t_{max}$ the dominating processes are ionization, Compton effect and photo effect → absorption.

Electromagnetic cascades

Longitudinal shower development: $\frac{dE}{dt} \propto t^{\alpha} e^{-t}$

Shower maximum at $t_{max} = \ln \frac{E_0}{E_c} \frac{1}{\ln 2}$

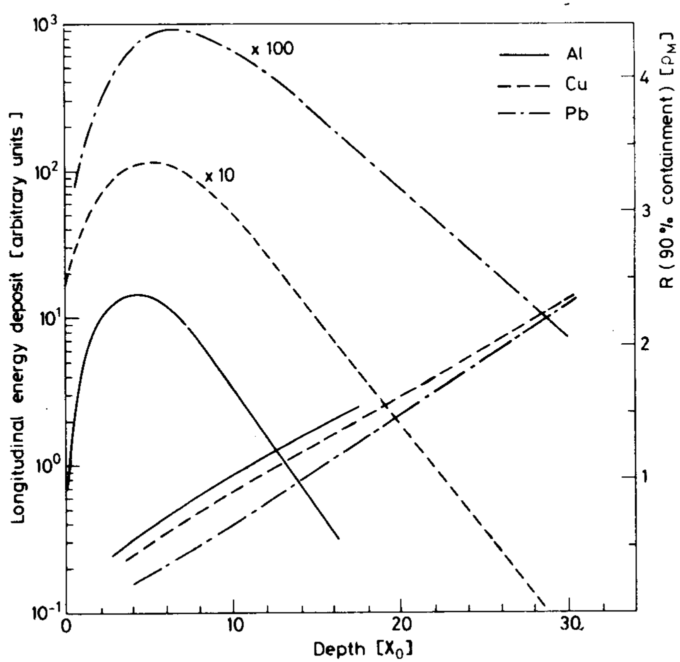
Size of a calorimeter grows logarithmically with E_0

95% containment $t_{95\%} \approx t_{max} + 0.08Z + 9.6$

Transverse shower development: 95% of the shower cone is located in a cylinder with radius 2 R_M

Moliere radius $R_M = \frac{21\,\text{MeV}}{E_c} X_0 \quad [g/cm^2]$

e^- 6 GeV/c

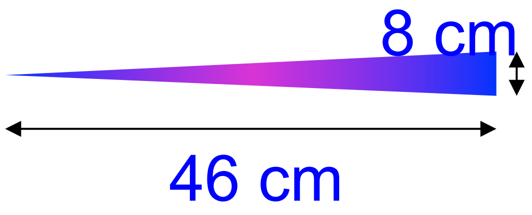


Longitudinal and transverse development scale with X_0, R_M

Example: E_0 = 100 GeV in lead glass

E_c=11.8 MeV → $t_{max} \approx 13$, $t_{95\%} \approx 23$

$X_0 \approx 2$ cm, $R_M = 1.8 \cdot X_0 \approx 3.6$ cm



Energy resolution

Intrinsic limit

Total number of track segments $N^{total} \propto \frac{E_0}{E_c}$

Resolution

$$\frac{\sigma(E)}{E} \propto \frac{\sigma(N)}{N} \propto \frac{1}{\sqrt{N}} \propto \frac{1}{\sqrt{E_0}}$$

Spatial and angular resolution scale like 1/√E

Relative energy resolution of a calorimeter improves with E_0

$$\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

Quality factor !

stochastic term **constant term** **noise term**

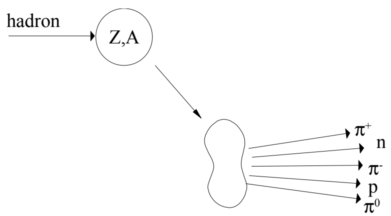
inhomogenities
bad cell inter-calibration
non-linearities

electronic noise
radioactivity
pile up

Nuclear interaction

The interaction of energetic hadrons (charged or neutral) is determined by inelastic nuclear processes.

p,n,p,K,…



multiplicity ~ $\ln(E)$

$p_t \approx 0.35$ GeV/c

Excitation and breakup of nucleus → nucleus fragments + secondary particles

At high energies (>1 GeV) the cross-sections depend only weakly on the energy and on the type of the incident particle (p, p, K…)

$$\sigma_{inel} \approx \sigma_0 A^{0.7} \qquad \sigma_0 \approx 35\ mb$$

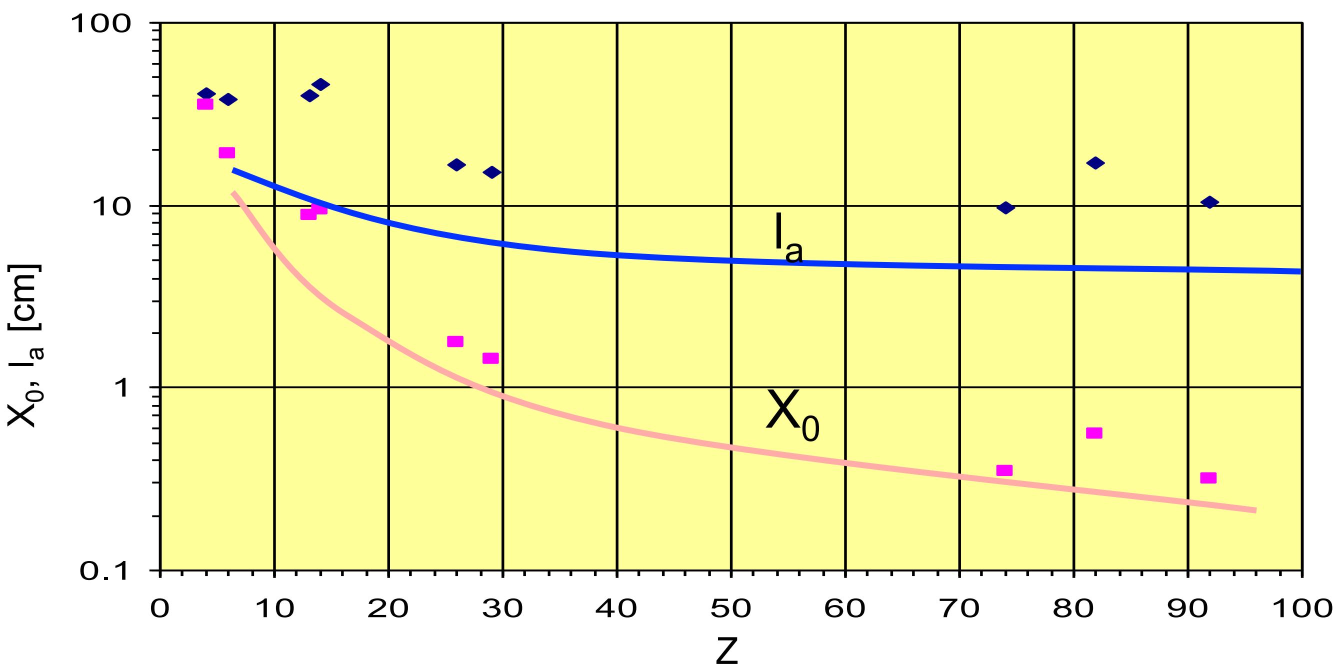
In analogy to X_0 we define a hadronic absorption length

$$\lambda_a = \frac{A}{N_A \sigma_{inel}}$$

Radiation and absorption length

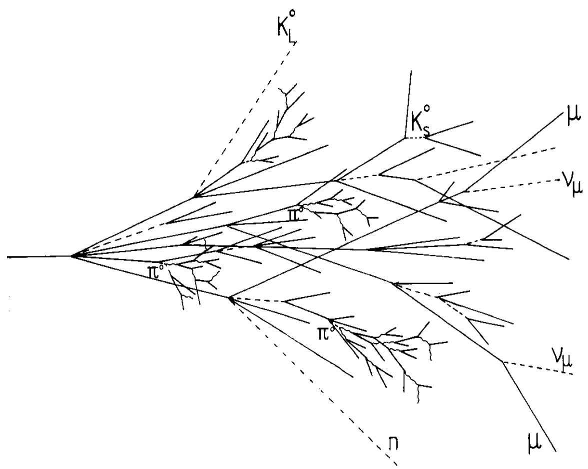
Material	Z	A	ρ [g/cm³]	X_0 [g/cm²]	λ_a [g/cm²]
Hydrogen (gas)	1	1.01	0.0899 (g/l)	63	50.8
Helium (gas)	2	4.00	0.1786 (g/l)	94	65.1
Beryllium	4	9.01	1.848	65.19	75.2
Carbon	6	12.01	2.265	43	86.3
Nitrogen (gas)	7	14.01	1.25 (g/l)	38	87.8
Oxygen (gas)	8	16.00	1.428 (g/l)	34	91.0
Aluminium	13	26.98	2.7	24	106.4
Silicon	14	28.09	2.33	22	106.0
Iron	26	55.85	7.87	13.9	131.9
Copper	29	63.55	8.96	12.9	134.9
Tungsten	74	183.85	19.3	6.8	185.0
Lead	82	207.19	11.35	6.4	194.0
Uranium	92	238.03	18.95	6.0	199.0

For Z > 6: $l_a > X_0$



Hadronic cascades

Various processes involved. Much more complex than electromagnetic cascades.



hadronic

↓

charged pions, protons, kaons ….
breaking up of nuclei (binding energy),
neutrons, neutrinos, soft g' s
muons …. → invisible energy

electromagnetic

↓

neutral pions → 2γ → electromagnetic cascade

number of neutral pions

$$n(\pi^0) \approx \ln E(GeV) - 4.6$$

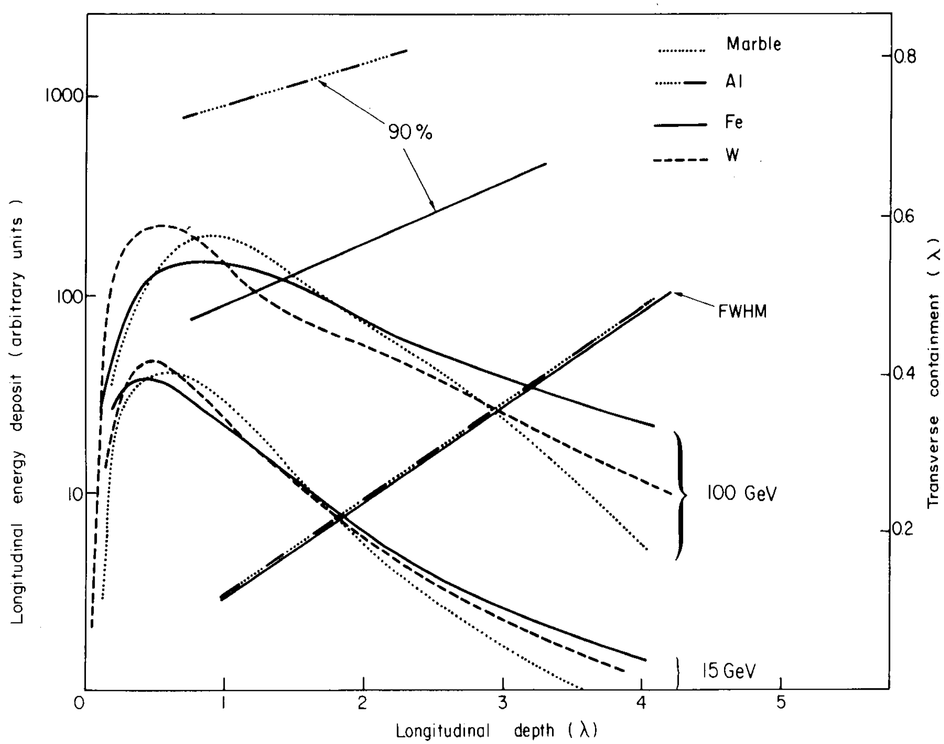
for 100 GeV pp collision: $n(\pi^0) \approx 18$

Shower development

Longitudinal shower shape

$$t_{\max}(\lambda_I) \approx 0.2 \ln E[GeV] + 0.7$$
$$t_{95\%}(cm) \approx a \ln E + b$$

For Iron: a = 9.4, b=39 λ_a=16.7 cm
E =100 GeV → $t_{95\%}$ ≈ 80 cm



Lateral shower shape

The shower consists of core + halo. 95% containment in a cylinder of radius λ.
Hadronic showers are much longer and broader than electromagnetic ones

Types of calorimeters

Homogeneous calorimeters: (e.g., crystals)

- ⇨ Detector = absorber
- ⇨ good energy resolution
- ⇨ limited spatial resolution (particularly in longitudinal direction)
- ⇨ only used for electromagnetic calorimetry

Sampling calorimeters:

- ⇨ Detectors and absorber separated → only part of the energy is sampled
- ⇨ limited energy resolution
- ⇨ good spatial resolution
- ⇨ used both for electromagnetic and hadron calorimetry

Homogeneous calorimeters

Signal = photons (scintillation or Cherenkov radiation).
Readout via photomultiplier, -diode/triode….

- **Scintillators (crystals)**

Scintillator	Density $[g/cm^3]$	X_0 [cm]	Light Yield γ/MeV (rel. yield)	τ_1 [ns]	λ_1 [nm]	Rad. Dam. [Gy]	Comments
NaI (Tl)	3.67	2.59	4×10^4	230	415	≥10	hydroscopic, fragile
CsI (Tl)	4.51	1.86	5×10^4 (0.49)	1005	565	≥10	Slightly hygroscopic
CSI pure	4.51	1.86	4×10^4 (0.04)	10 36	310 310	10^3	Slightly hygroscopic
BaF_2	4.87	2.03	10^4 (0.13)	0.6 620	220 310	10^5	
BGO	7.13	1.13	8×10^3	300	480	10	
$PbW0_4$	8.28	0.89	≈100	10 10	≈440 ≈530	10^4	light yield =f(T)

Light yield relative to NaI(Tl) readout with PM (bialkali photocathode)

Homogeneous calorimeters

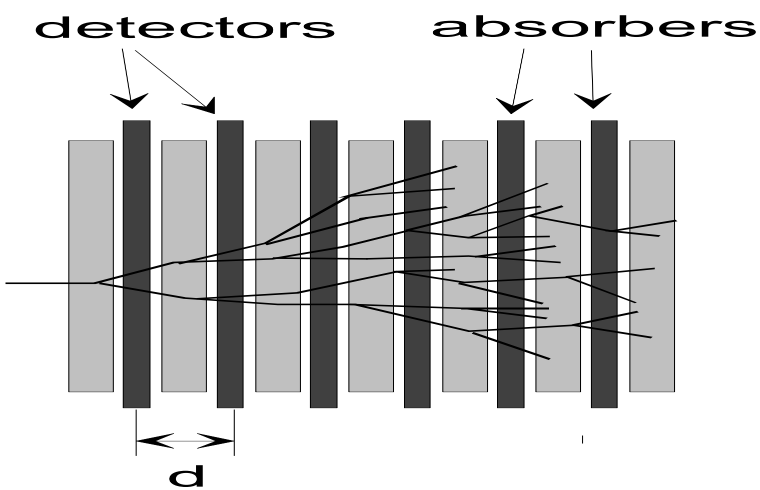
Cherenkov radiators

Material	Density [g/cm^3]	X_0 [cm]	n	Light yield [p.e./GeV] (rel. p.e.)	λ_{cut} [nm]	Rad. Dam. [Gy]	Comments
SF-5 Lead glass	4.08	2.54	1.67	600 (1.5×10^{-4})	350	10^2	
SF-6 Lead glass	5.20	1.69	1.81	900 (2.3×10^{-4})	350	10^2	
PbF_2	7.66	0.95	1.82	2000 (5×10^{-4})		10^3	Not available in quantity

Light yield relative to NaI(Tl) readout with PM (bialkali photocathode)

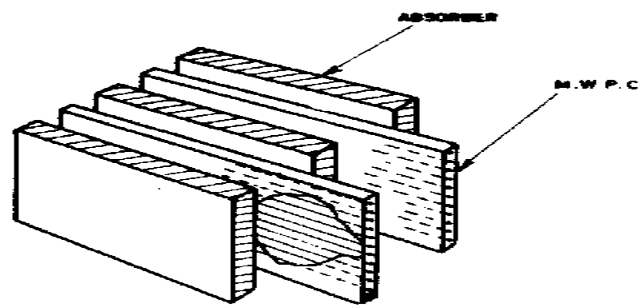
Sampling calorimeters

Absorber + detector separated → sampling fluctuations

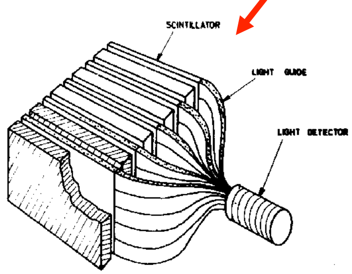


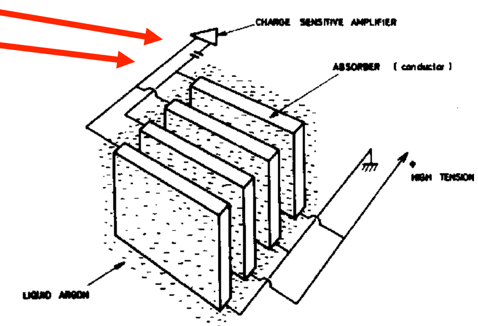
Detectable track segments

$$N = \frac{T_{\text{det}}}{d}$$
$$= F(\xi)\frac{E}{E_c}\, X_0\, \frac{1}{d}$$
$$\frac{\sigma(E)}{E} \propto \frac{\sqrt{N}}{N} \propto \sqrt{\frac{1}{E}} \cdot \sqrt{\frac{d}{X_0}}$$



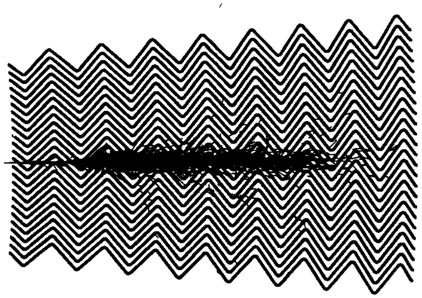
MWPC, streamer tubes
warm liquids:
 TMP = tetramethylpentane,
 TMS = tetramethylsilane
cryogenic noble gases:
 mainly LAr (LXe, LKr)
scintillators, scintillation fibres, silicon detectors



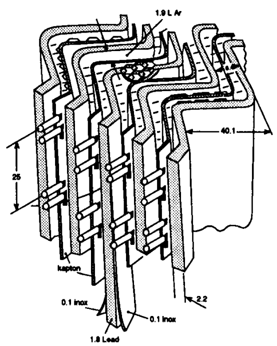


ATLAS LAr Calorimeter

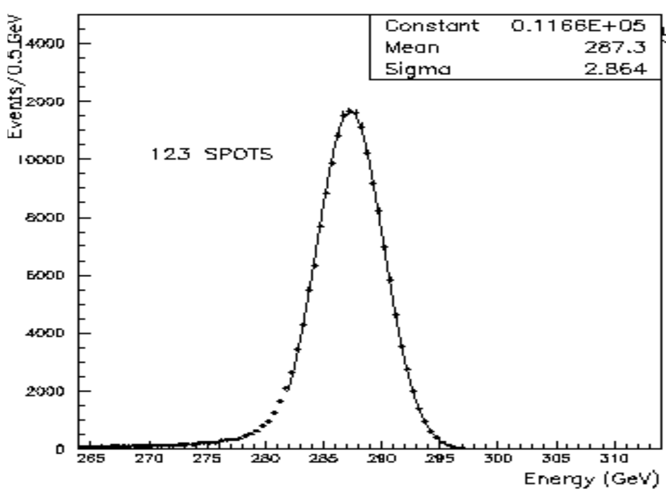
Accordion geometry absorbers immersed in Liquid Argon

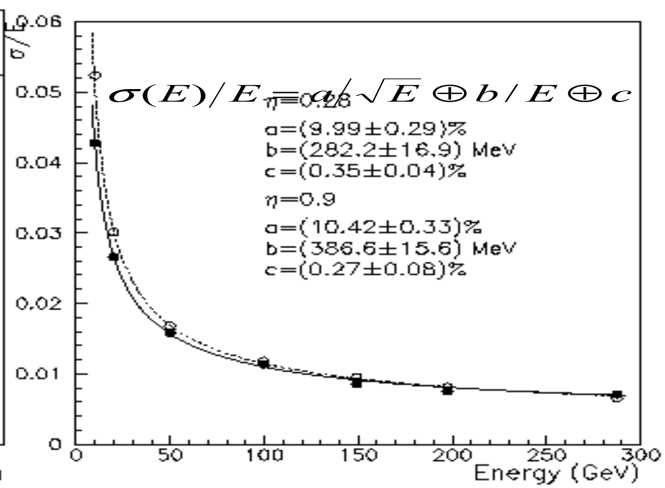


Liquid Argon (90K)
+ lead-steal absorbers (1-2 mm)
+ multilayer copper-polyimide readout boards
1 GeV E-deposit → 5 × 10^6 e^-



Accordion geometry minimizes dead zones.
Liquid Ar is intrinsically radiation hard.
Readout board allows fine segmentation (azimuth, pseudo-rapidity and longitudinal)





Spatial and angular uniformity ≈ 0.5%

Spatial resolution ≈ 5mm / $E^{1/2}$

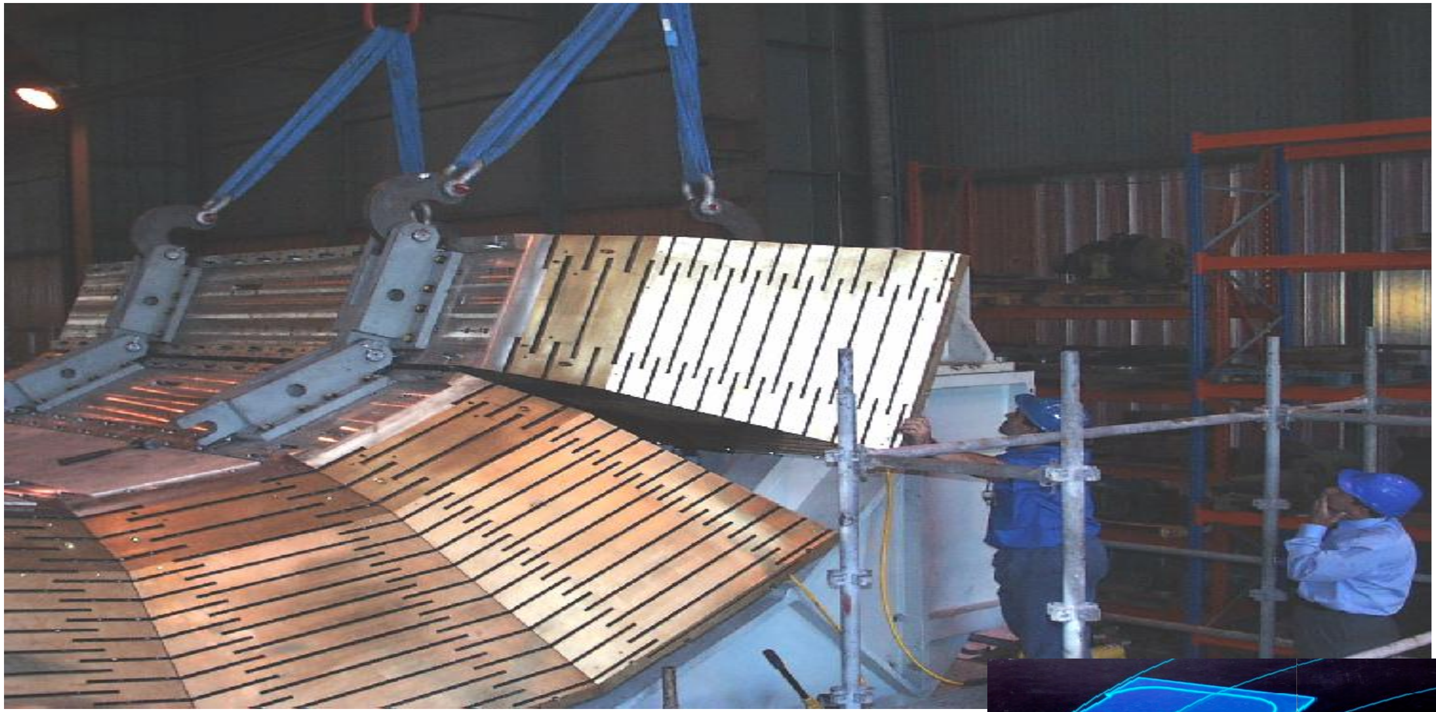
Pointing

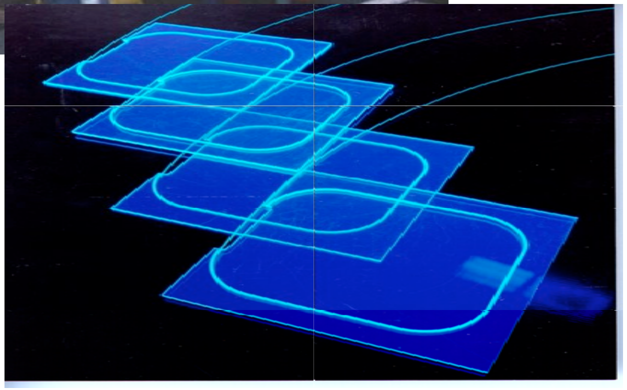
CMS hadron calorimeter

Cu absorber + scintillator

2 × 18 wedges (barrel)

+ 2 × 18 wedges (endcap) ≈ 1500 T absorber





Scintillators fill slots and are read out via fibres by HPDs (hybrid photodiodes)

Test beam resolution for single hadrons

$$\frac{\sigma_E}{E} = \frac{65\%}{\sqrt{E}} \oplus 5\%$$

Energy Measurement

How we determine the energy of a particle from the shower?

– Detector response must have linearity i.e., signal proportional to the energy deposit

- The average calorimeter signal vs. the energy of the particle is different for homogenous and sampling calorimeters
- Hadronic showers may include electromagnetic component from π^0' s
- Detector resolution is controlled by fluctuations, i.e., event to event variations of the signal.

In general EM calorimeters have linear responses while hadronic calorimeters do not.

Sources of non-linearity:

- saturation of the medium (gas, crystal, scintillator)
- non-linearity of detectors (PMT, Photodiodes, electronics)
- leakage of the signal outside the detector

Homogeneous calorimeters - crystals or liquid Xe

Scintillation proportional to the total electron energy

Advantages:

- excellent energy resolution -> best statistical precision
  for mean energy W required to produce a signal
  eg., visible photon in a crystal

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}}$$

- uniform response -> good linearity

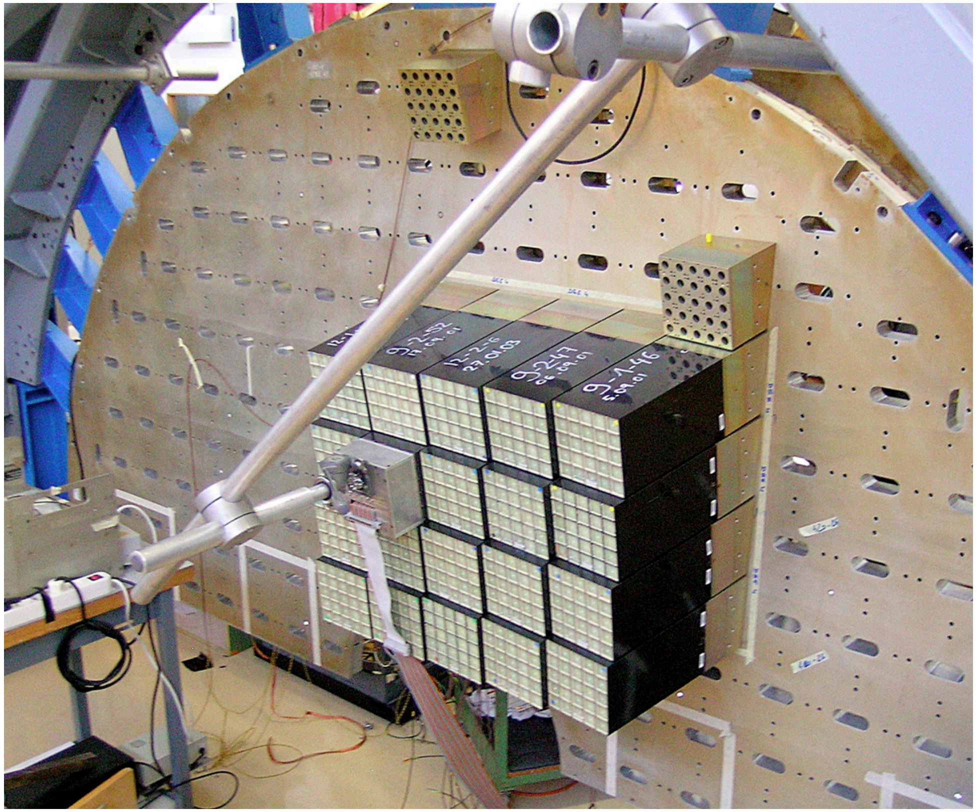
Disadvantages:

- limited segmentation
- high cost

CMS EM Calorimeter

barrel + endcaps 77,000 $PbWO_4$ crystals

Energy resolution – 1% at 30 GeV

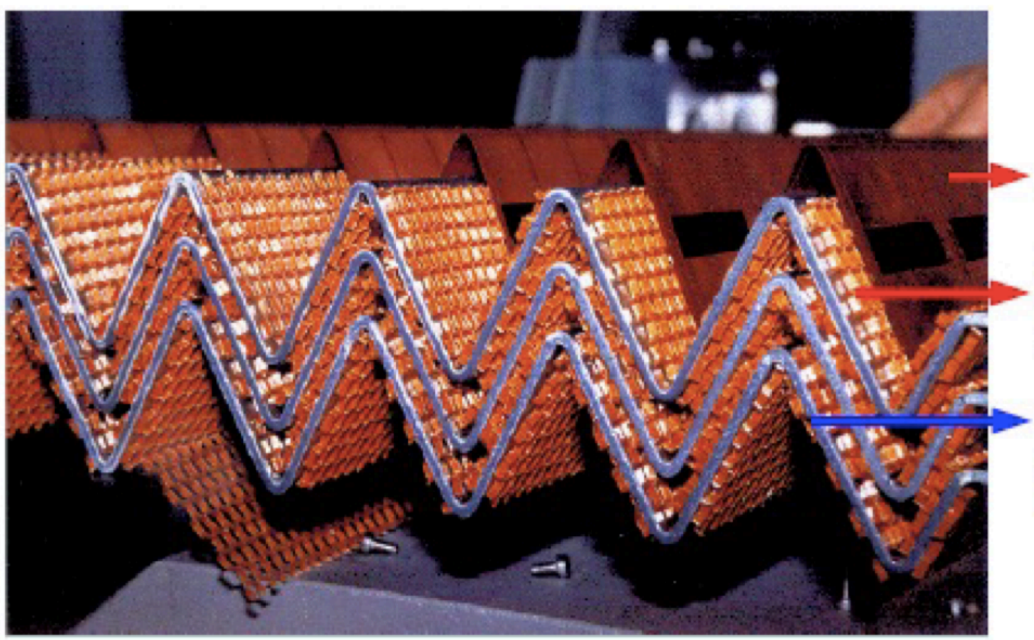


Sampling calorimeters

Sandwich of dense material to induce showering interspaced with a detector (scintillator counting tracks, LAr counting ionization,..)
Advantage – good spatial segmentation, both lateral and in depth
Disadvantage – only see part of the shower

$$f_{sampling} = \frac{E_{visible}}{E_{deposited}}$$

ATLAS
LAr



Energy resolution

$$\sigma_E = a\sqrt{E} \oplus bE \oplus c$$

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

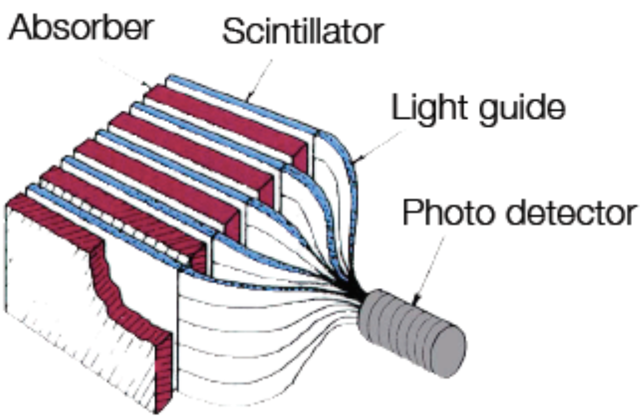
a – stochastic term due to
- intrinsic statistical shower fluctuations
- sampling fluctuations
- signal quantum fluctuations (e.g., photo-electron statistics)

b – constant term due to
- inhomogeneities and imperfections
- non-linearity of electronics
- fluctuation of the energy lost in the absorber

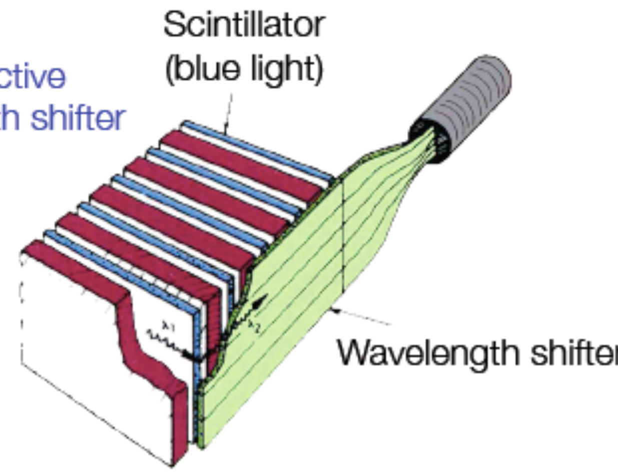
c – noise term due to electronics noise, natural radioactivity, pile-up

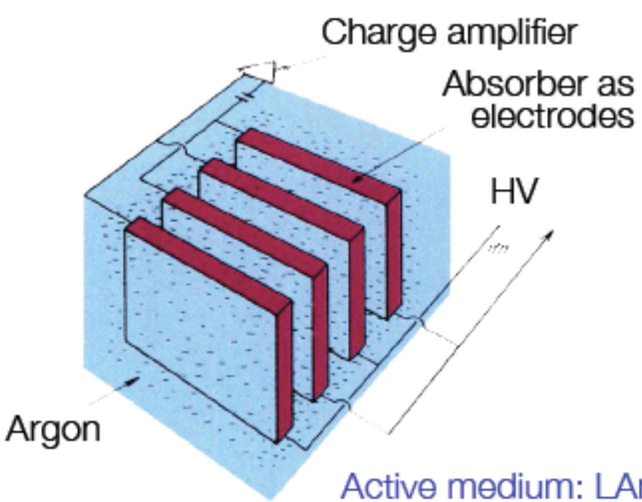
Possible setups

Scintillators as active layer;
signal readout via photo multipliers



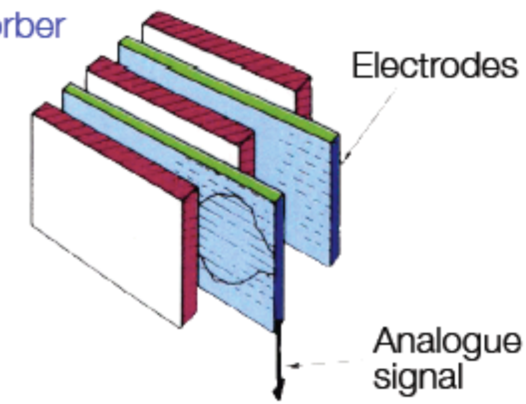
Scintillators as active
layer; wave length shifter
to convert light





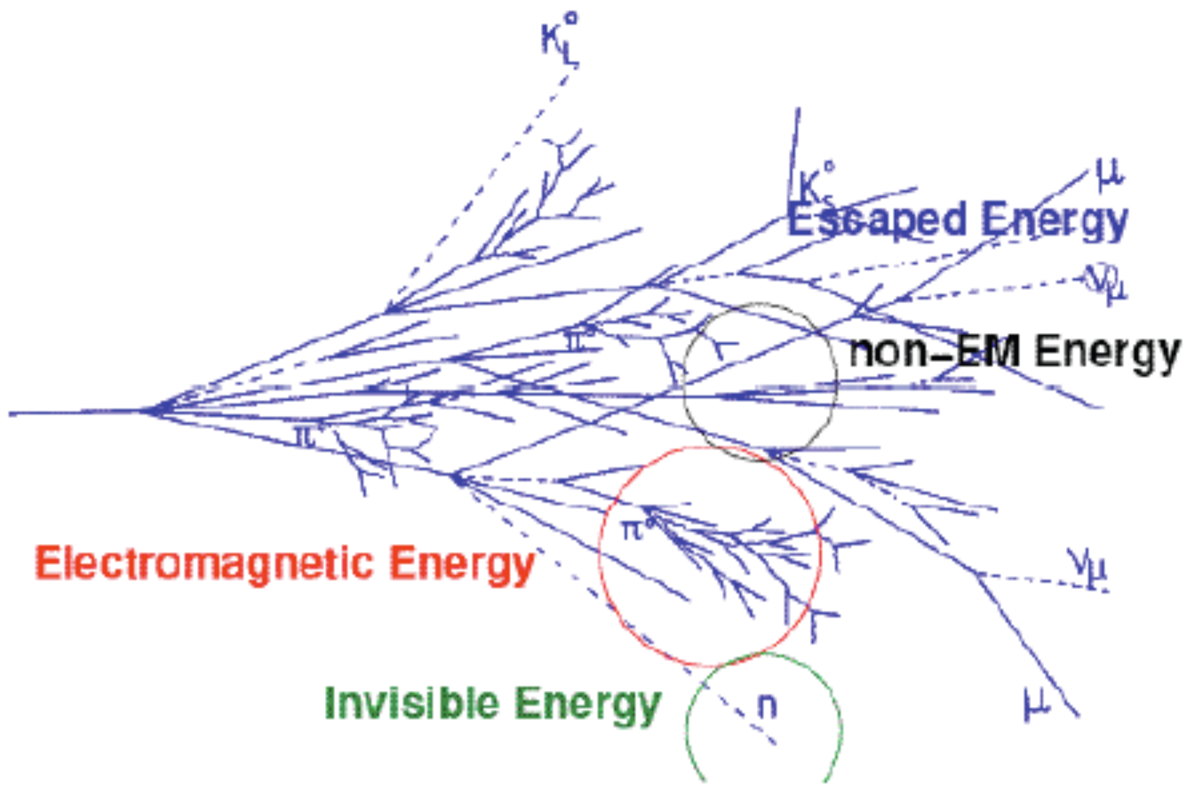
Active medium: LAr; absorber
embedded in liquid serve as electrods

Ionization chambers
between absorber
plates



Hadron showers

Initiated by strong interactions
Characterized by hadronic interaction length
Contain electromagnetic components
Large complexity – requires simulation tools



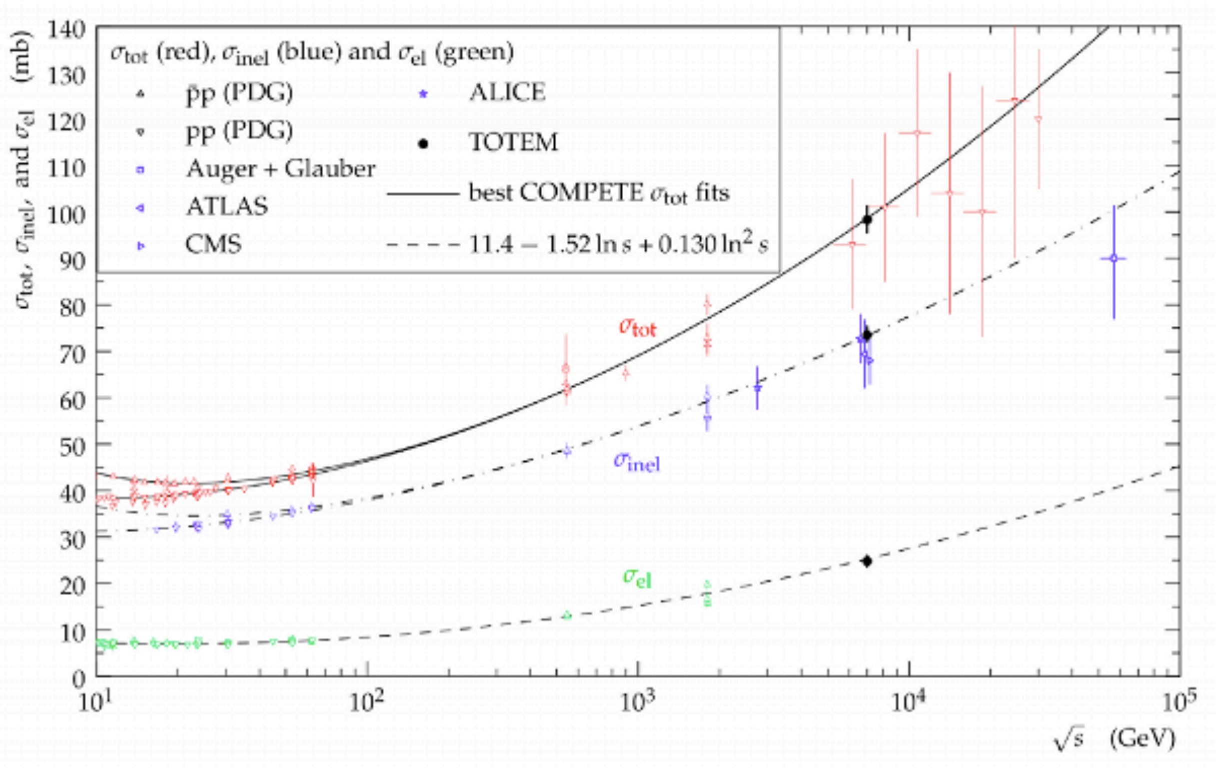
Hadronic interactions cross sections

$\sigma_{tot} = \sigma_{el} + \sigma_{inel}$

$\sigma_{el} \approx 10mb$

$\sigma_{inel} \approx A^{2/3}$

$\sigma_{tot}(nucl) = \sigma_{tot}(pp)A^{2/3}$ σ *tot (pp) increases with s*



Energy resolution of hadron showers

Hadronic energy resolution of non-compensating calorimeters does not scale with $1/\sqrt{E}$ but as

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b\left(\frac{E}{E_0}\right) \approx \frac{a}{\sqrt{E}} \oplus b$$