Lecture 2: Quantum Mechanics and Relativity

Atom

Atomic number	A
Number of protons	Z
Number of neutrons	$A-Z$
Number of electrons	Z

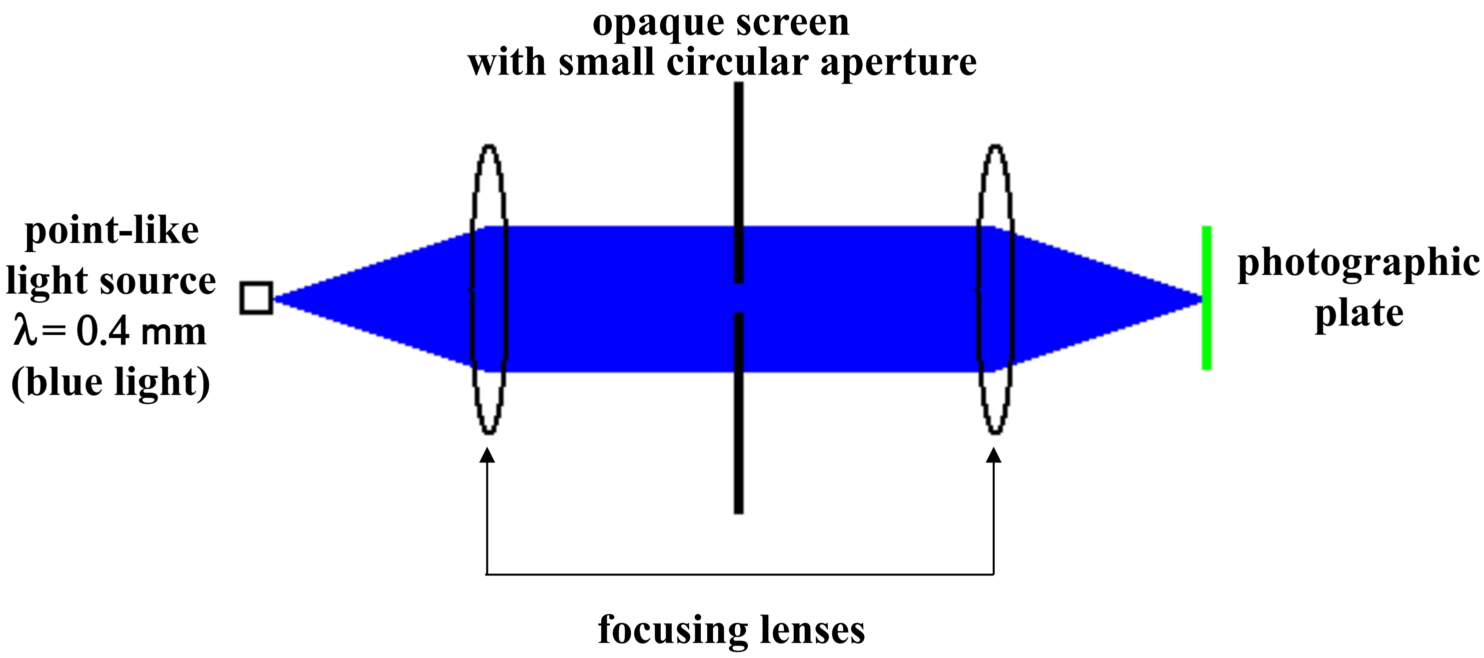
Charge of electron = charge of proton $\sim 1.6\times 10^{-19}\,C$

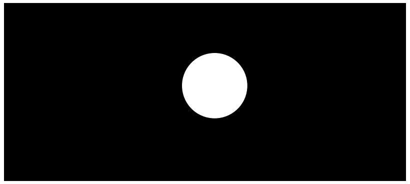
Size of the atom	$\sim 10^{-10}\,m$
Size of the nucleus	$\sim 10^{-15}\,m$

Two questions:

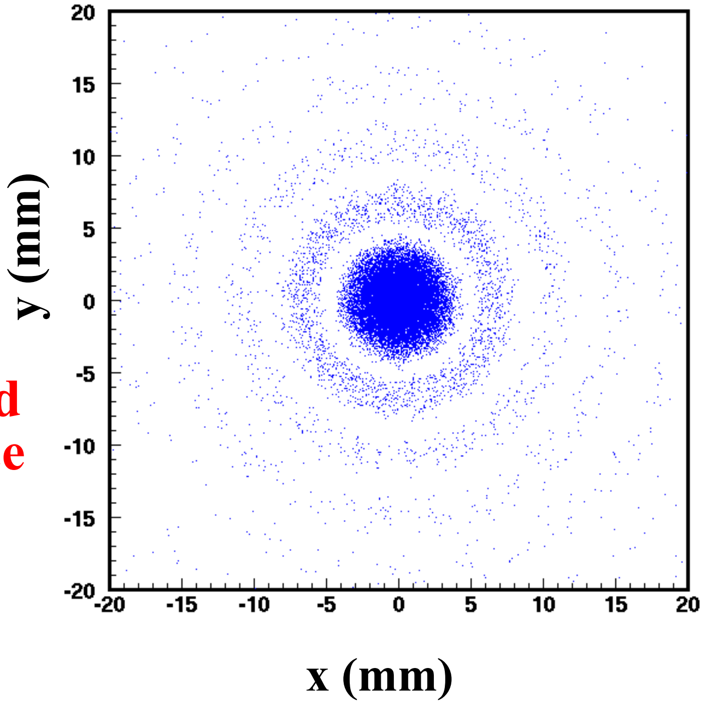
- Why did Rutherford need α – particles to discover the atomic nucleus?
- Why do we need huge accelerators to study particle physics today?

Observation of very small objects using visible light





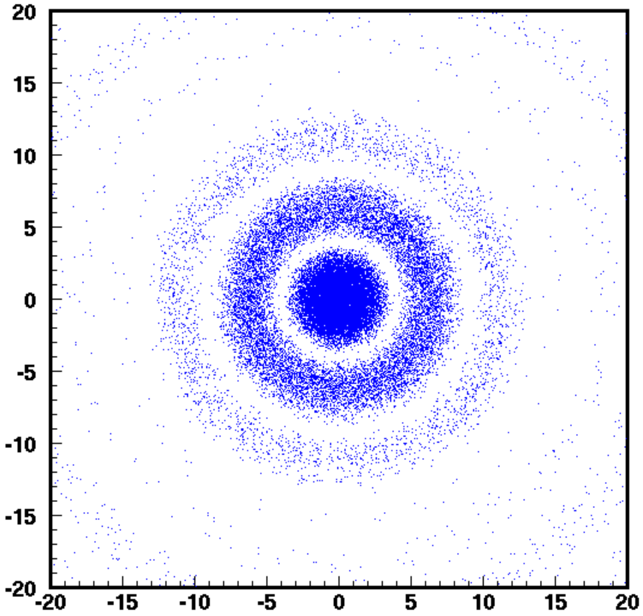
Aperture diameter: *D* = 20 µm
Focal length: 20 cm

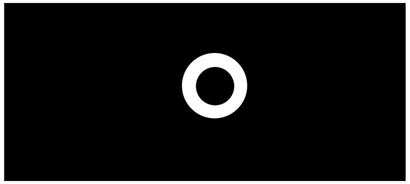


Observation of light diffraction, interpreted as evidence that light consists of waves since the end of the 17th century

Angular aperture of the first circle (before focusing):

$$\alpha = 1.22\lambda / D$$



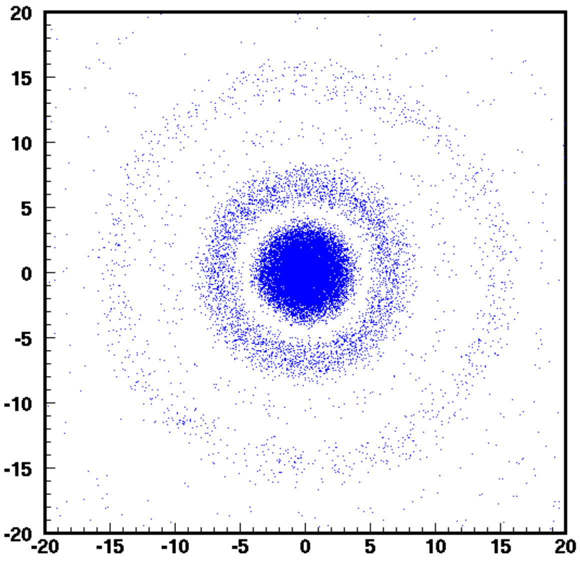


Opaque disk, diam. 10 µm
in the center

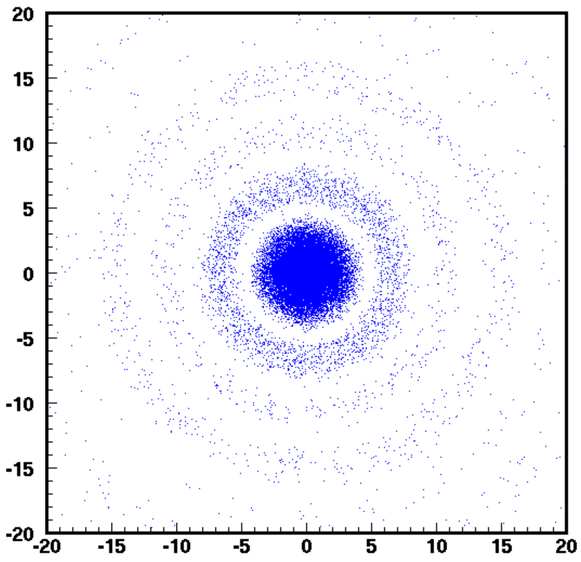
Presence of opaque disk is detectable

Opaque disk of variable diameter

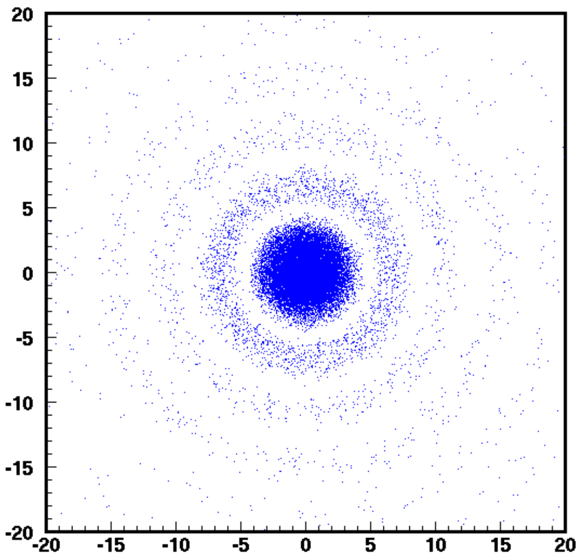
diameter = .4 µm



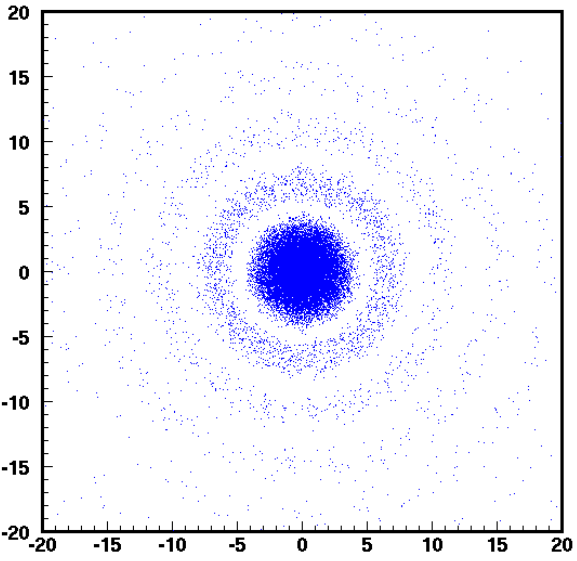
diameter = .2 µm



diameter = .1 µm



no opaque disk

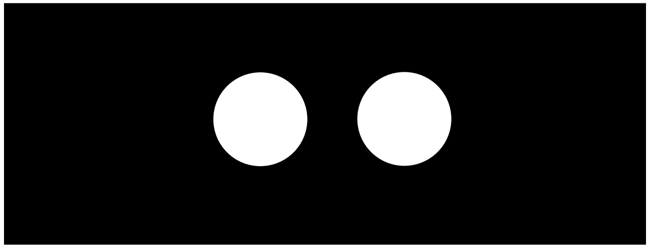


The presence of the opaque disk in the center is detectable if its diameter is larger than the wavelength λ of the light

The <u>RESOLVING POWER</u> of the observation depends on the wavelength λ

Visible light: not enough resolution to see objects smaller than 0.2 – 0.3 µm

Opaque screen with two circular apertures



aperture diameter: 10 μm
distance between centers: 15 μm

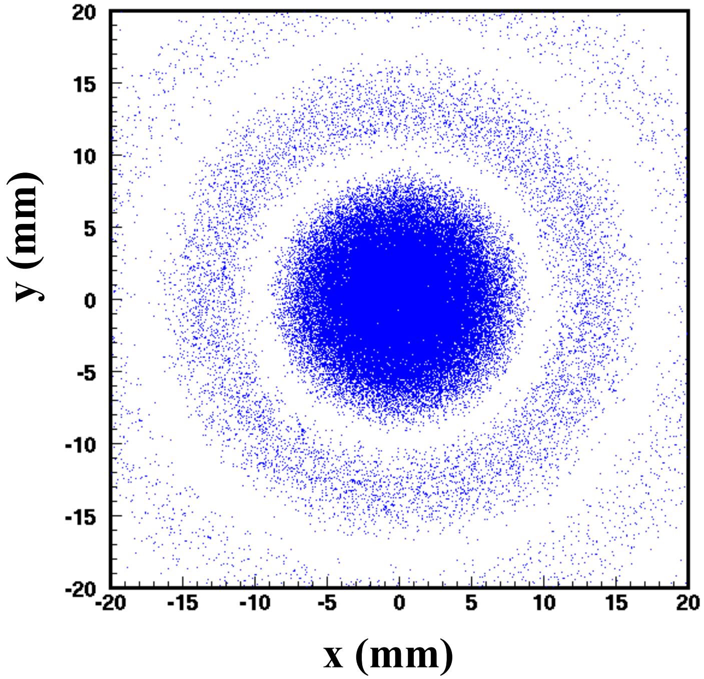
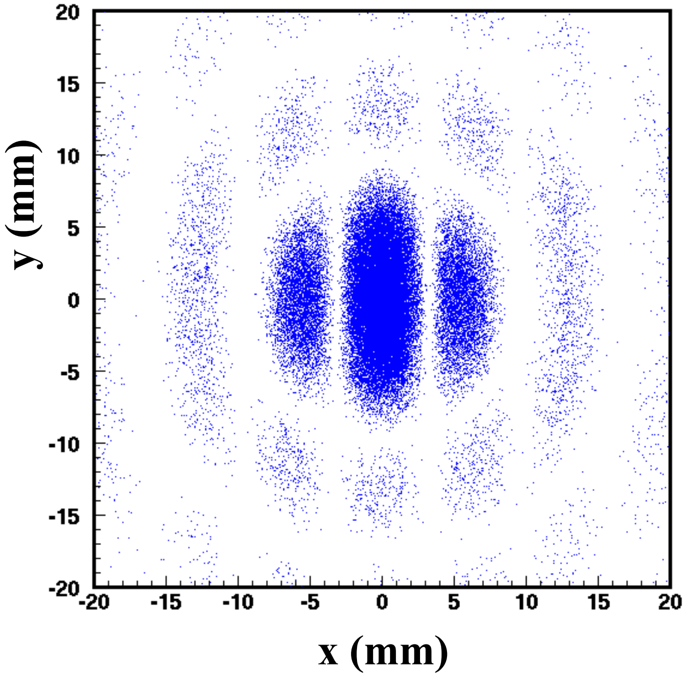


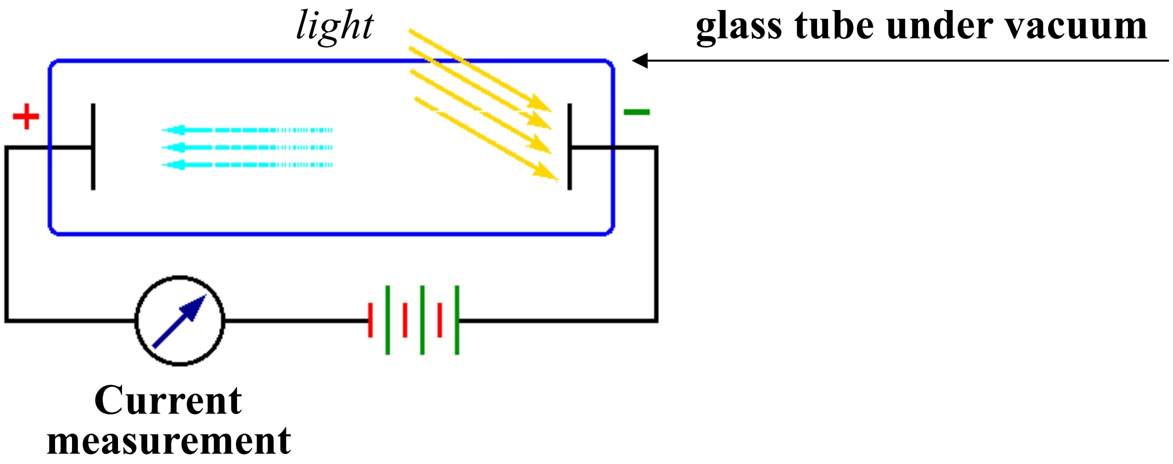
Image obtained by shutting one aperture alternatively for 50% of the exposure time

Image obtained with both apertures open simultaneously

Light is a wave and can interfere !



Photoelectric effect: evidence that light consists of particles



Observation of a threshold effect as a function of the frequency (wavelength) of the light impinging onto the electrode at negative voltage (cathode):

Frequency $\nu < \nu_0$: electric current = zero, independent of luminous flux;

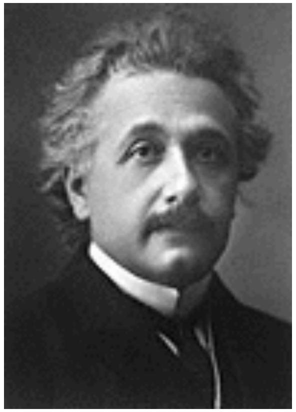
Frequency $\nu > \nu_0$: current > 0, proportional to luminous flux

INTERPRETATION (A. Einstein):

- Light consists of particles ("photons") !!!!
- Photon energy proportional to frequency:

$$\boxed{E = h\nu}$$ (Planck constant $h = 6.626 \times 10^{-34}$ J s)

- Threshold energy $E_0 = h\nu_0$: the energy needed to extract an electron from an atom (depends on the cathode material)



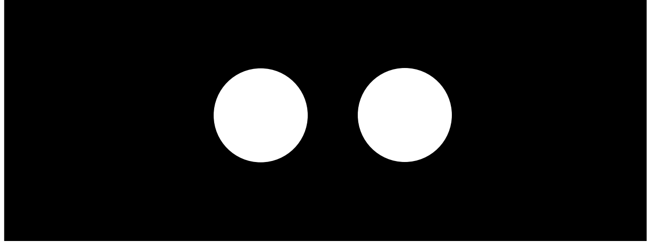
Albert Einstein

Repeat the experiment with two circular apertures using a very weak light source

Luminous flux = 1 photon /second

(detectable using modern, commercially available photomultiplier tubes)

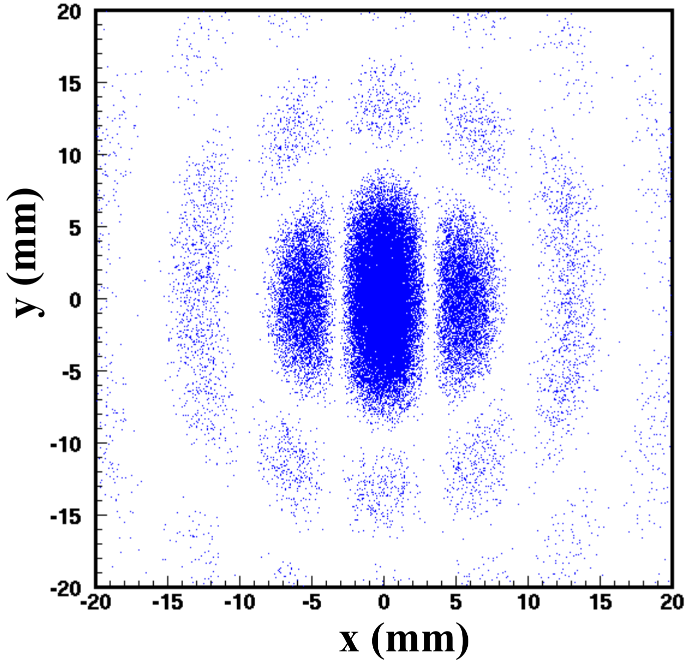
Need very long exposure time



aperture diameter: 10 µm
distance between centres: 15 µm

Question: which aperture will photons choose?

Answer: diffraction pattern corresponds to both apertures simultaneously open, independent of luminous flux



Interference pattern

Photons have both particle and wave properties simultaneously

It is impossible to know which aperture the photon traversed

The photon can be described as a coherent superposition of two states

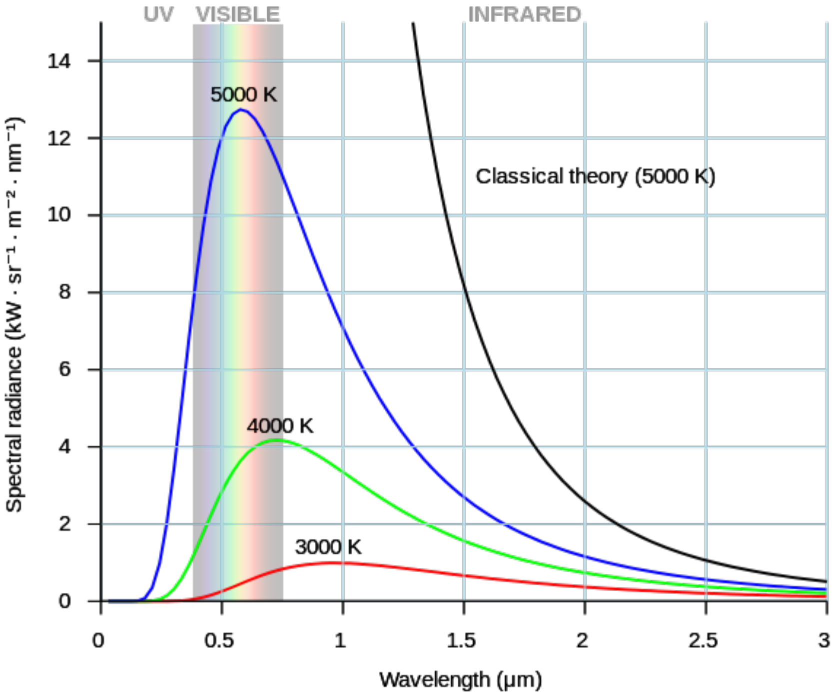
Black body radiation

Electromagnetic radiation emitted by hot object.
Statistical mechanics lead to “ultraviolet catastrophe”
i.e., amount of energy emitted at short wavelength became infinite.

1900 Planck: One can avoid the UV catastrophe and describe the experimentally measured spectrum IF electromagnetic radiation is Quantized, i.e. comes in little packages of energy

$$E = h\nu$$

Planck’s constant
$h = 6.626 \times 10^{-27}$ ergs
ν - frequency
λ - wavelength $\sim 1/\nu$



1924: De Broglie's principle

Louis de Broglie

Not only light, but also matter particles possess both the properties of waves and particles

Relation between wavelength and momentum:

$$\boxed{\lambda = \frac{h}{p}}$$

h: Planck constant
$p = m\,\mathrm{v}$: particle momentum

Hypothesis soon confirmed by the observation of diffraction pattern in the scattering of electrons from crystals, confirming the wave behaviour of electrons (Davisson and Germer, 1927)

Wavelength of the α – particles used by Rutherford in the discovery of the atomic nucleus:

$$\lambda = \frac{h}{m_\alpha \mathrm{v}} \approx \frac{6.626\times10^{-34}\ \mathrm{J\,s}}{(\underset{\substack{\uparrow\\ \alpha\text{ particle mass}}}{6.6\times10^{-27}\,\mathrm{kg}})\times(\underset{\substack{\uparrow\\ 0.05\,c}}{1.5\times10^{7}\,\mathrm{m\,s^{-1}}})} \approx 6.7\times10^{-15}\ \mathrm{m} = \underset{\substack{\uparrow\\ \sim\text{ resolving power of Rutherford's experiment}}}{6.7\times10^{-13}\ \mathrm{cm}}$$

Typical tools to study objects of very small dimensions

		Resolving power
Optical microscopes	Visible light	$\sim 10^{-6}$ m
Electron microscopes	Low energy electrons	$\sim 10^{-9}$ m
Radioactive sources	α−particles	$\sim 10^{-14}$ m
Accelerators	High energy electrons, protons	$\sim 10^{-18}$ m

Units in Particle Physics

Fundamental Units in Physics: mass, length, time (m, kg, s) are not very useful in the world of particle physics.

Typical dimensions are:

Size of the nucleus ~1 fermi = 10^{-15} m

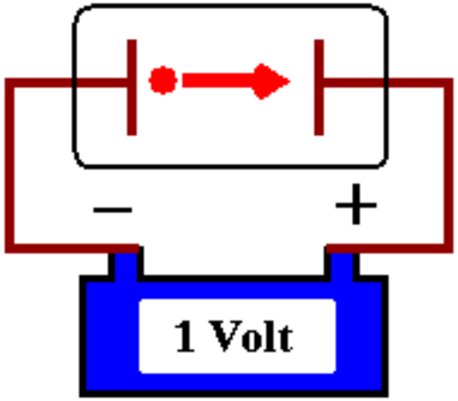
Mass of the proton $m_p = 1.672\ 10^{-27}$ kg

Within an object with mass 80 kg there are

$\sim 5\ 10^{28}$ protons + neutrons

Energy

1 electron-Volt (eV):



the energy of a particle with electric charge = $|e|$, initially at rest, after acceleration by a difference of electrostatic potential = 1 Volt

($e = 1.60 \times 10^{-19}$ C)

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$$

$$\text{J} = \text{kg m/s}^2$$

Multiples:

deca	da	10	deci	d	10^{-1}
hecto	h	10^{2}	centi	c	10^{-2}
kilo	k	10^{3}	mili	m	10^{-3}
mega	M	10^{6}	micro	µ	10^{-6}
giga	G	10^{9}	nano	n	10^{-9}
tera	T	10^{12}	pico	p	10^{-12}
peta	P	10^{15}	femto	f	10^{-15}

Energy:
1 keV = 10^3 eV ; 1 MeV = 10^6 eV
1 GeV = 10^9 eV; 1 TeV = 10^{12} eV

Energy of a proton in the LHC: 7 TeV = 1.12×10^{-6} J
This energy is equal to a body of mass = 1 mg moving at speed = 1.5 m /s
(~120 bees)
The conversion constant between MKS and particle physics units is
$\hbar c = 197.327$ MeV fm

My rest mass (weight =80 kg) is:
M = 80 kg·$(3\cdot10^8$ m/s$)^2$ / $1.6\cdot10^{-19}$ J/eV = $45\cdot10^{14}$ TeV/c^2

Energy and momentum for relativistic particles

Speed of light in vacuum $c = 2.99792 \times 10^8$ m / s (in the absence of magnetic field)

Total energy:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-(\mathrm{v}/c)^2}}$$

***m*:** relativistic mass
***m*$_0$:** rest mass

Expansion in powers of (v/*c*):
(only valid for v/c small)

$$E = \underbrace{m_0 c^2}_{\text{energy associated with rest mass}} + \underbrace{\frac{1}{2} m_0 \mathrm{v}^2}_{\text{“classical” kinetic energy}} + \dots$$

Momentum:

$$p = m\mathrm{v} = \frac{m_0 \mathrm{v}}{\sqrt{1-(\mathrm{v}/c)^2}}$$

$$\frac{pc}{E} = \frac{\mathrm{v}}{c} \equiv \beta$$

$E^2 - p^2c^2 = (m_0c^2)^2$ "relativistic invariant" (effective mass)

(same value in all reference frames)

Special case: the photon (v = *c* in vacuum)

Einstein $E = h\nu$

de Brogile $\lambda = h / p$

$\Longrightarrow$

$E / p = \nu\lambda = c$ **(in vacuum)**

$E^2 - p^2c^2 = 0$

photon rest mass $m_g = 0$

Momentum units: eV/*c* (or MeV/*c*, GeV/*c*, ...)

Mass units: eV/*c*^{2} (or MeV/*c*^{2}, GeV/*c*^{2}, ...)

Numerical example: electron with v = 0. 99 *c*

Rest mass: $m_e = 0.511$ MeV/c^2

$$\gamma \equiv \frac{1}{\sqrt{1-(\mathrm{v}/c)^2}} = 7.089$$ (often called "Lorentz factor")

Total energy: $E = \gamma\, m_e\, c^2 = 7.089 \times 0.511 = 3.62$ MeV

Momentum: $p = (\mathrm{v} / c) \times (E / c) = 0.99 \times 3.62 = 3.58$ MeV/c

"Natural System of Units" **$c = 1$**

$$E^2 = p^2 + m^2$$

- **All quantities in multiples of eV**
- **Unit of length** **1 fm =1/197.3 MeV**
- **Unit of time** **1 s = 1/6.58 10^{-22} MeV**