

# Lecture 21

Muon Spectrometer

Muon

Neutrino

Hadronic Calorimeter

Proton

Neutron

The dashed tracks are invisible to the detector

Electromagnetic Calorimeter

Electron

Photon

Solenoid magnet

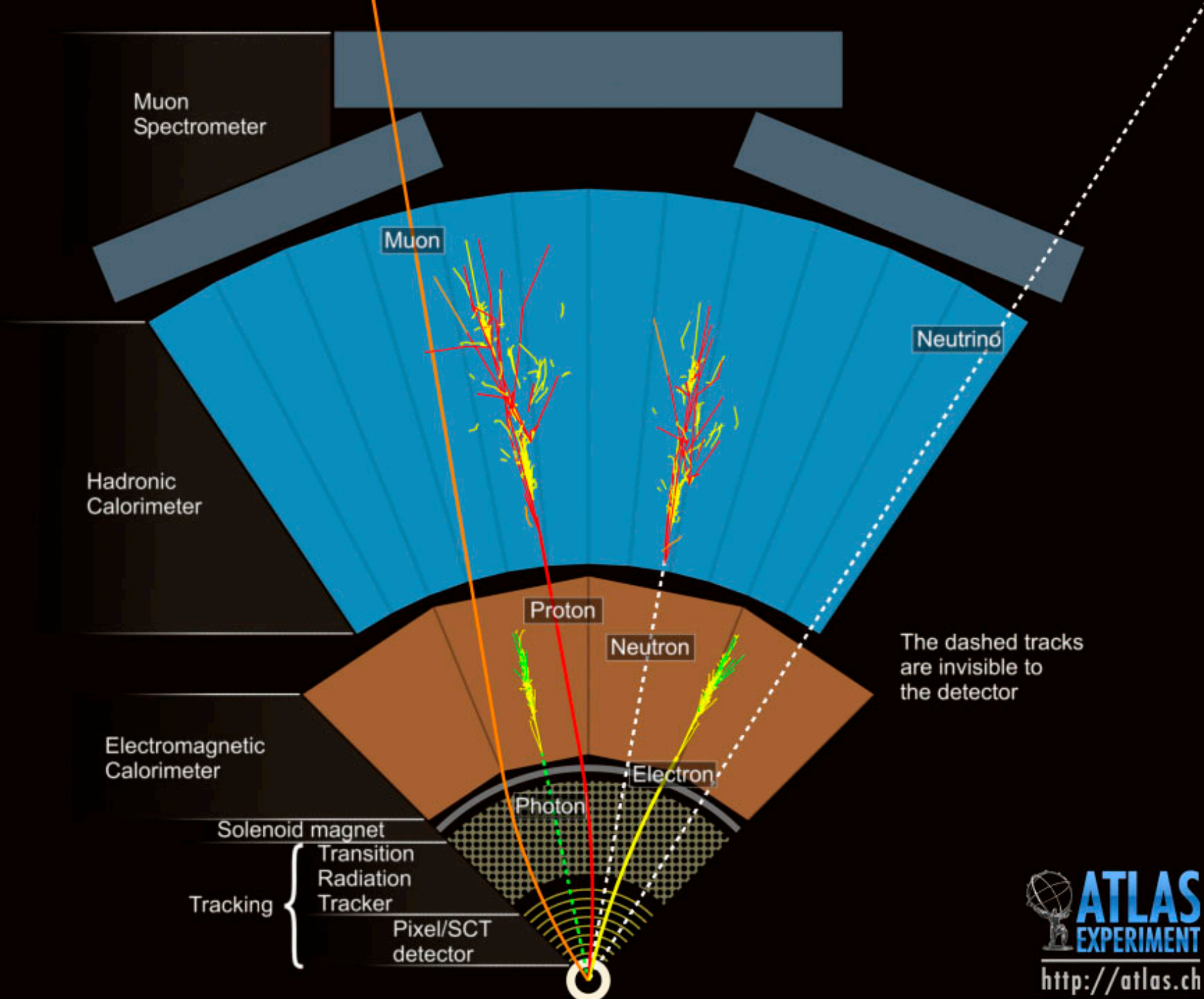
Tracking

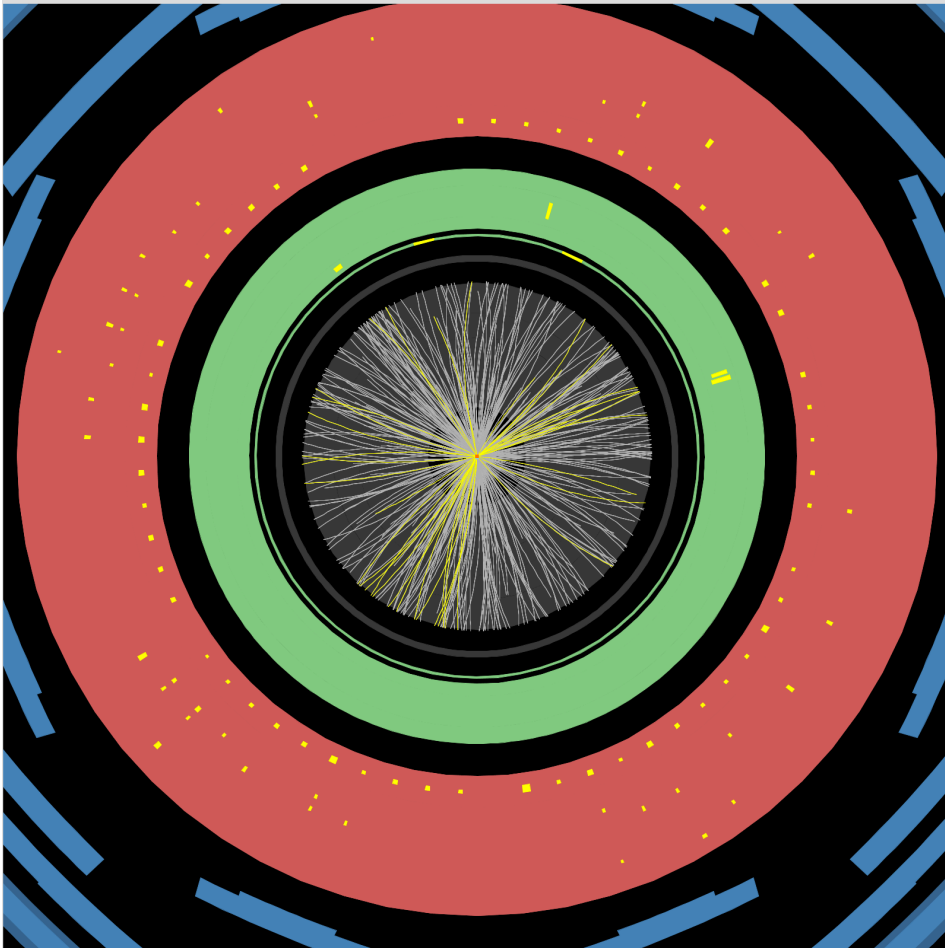
Transition Radiation Tracker

Pixel/SCT detector



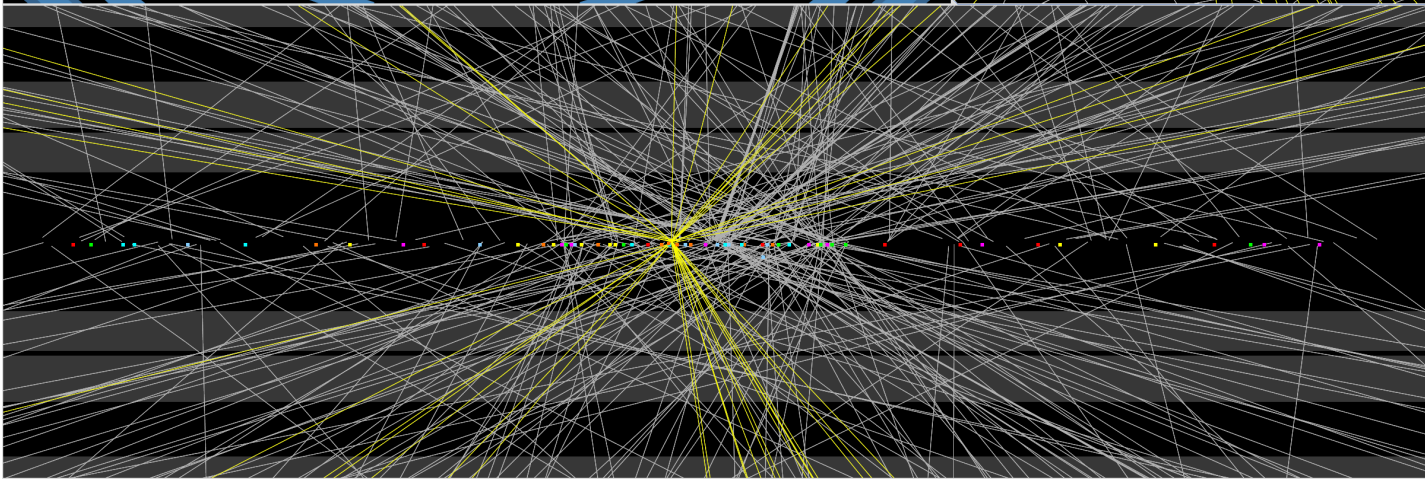
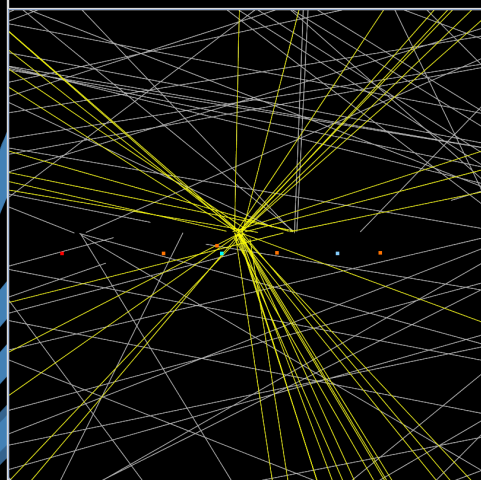
<http://atlas.ch>





Run Number: 330160, Event Number: 391419735

Date: 2017-07-20 02:18:47 CEST



# Tracking

Momentum measurement

Multiple scattering

Bethe-Bloch formula  
/ Landau tails

Ionization of gases

Wire chambers

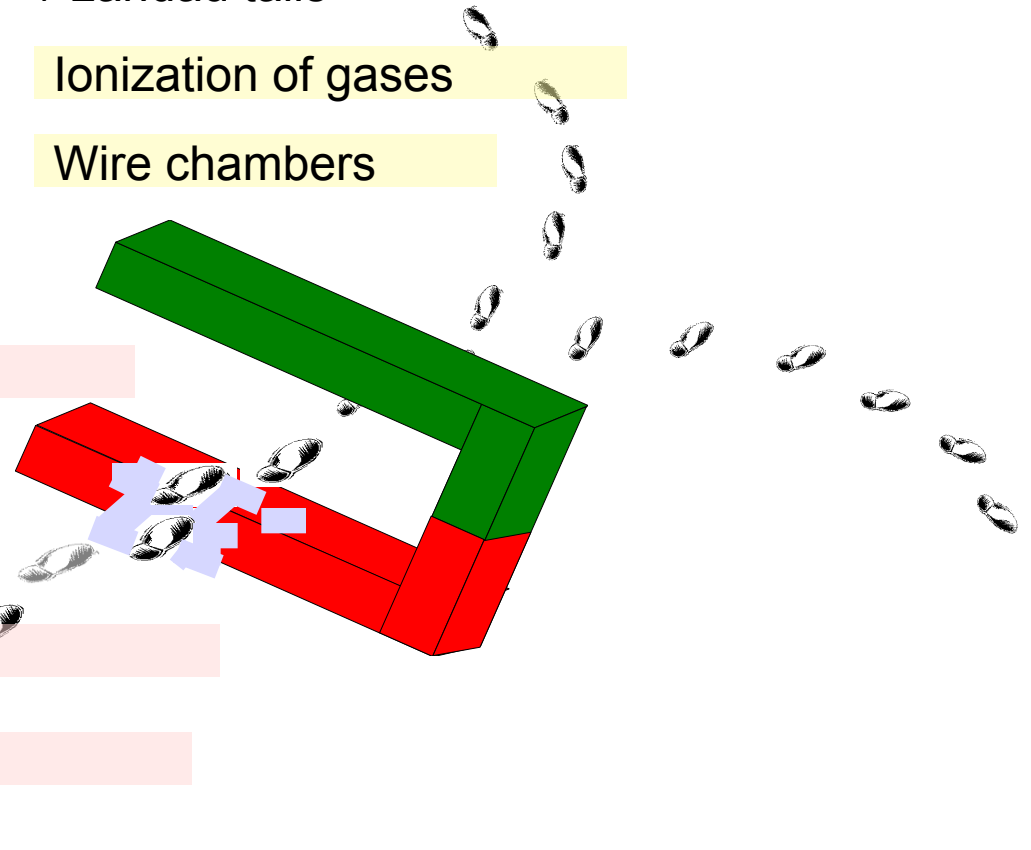
Drift and diffusion in gases

Drift chambers

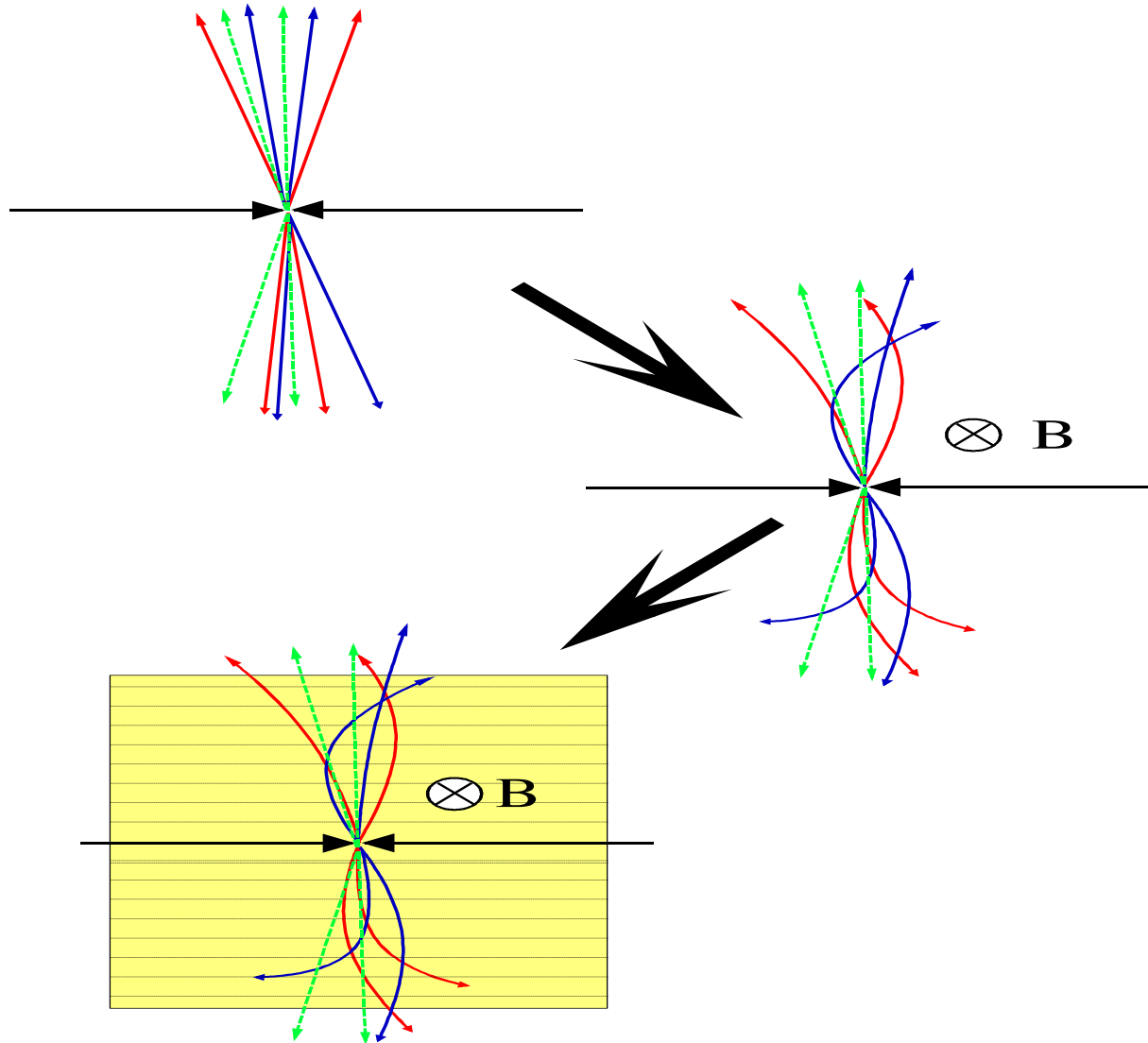
Micro gas detectors

Silicon as a detection medium

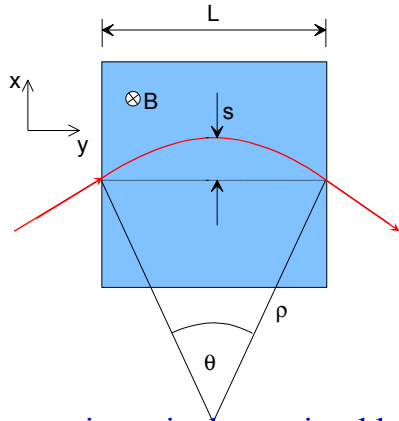
Silicon detectors strips/pixels



# Momentum measurement



# Momentum measurement



$$\frac{mv^2}{\rho} = q(v \times B) \rightarrow p_T = qB\rho$$

$$p_T \text{ (GeV/c)} = 0.3B\rho \text{ (T} \cdot \text{m)}$$

$$\frac{L}{2\rho} = \sin \theta/2 \approx \theta/2 \rightarrow \theta \approx \frac{0.3L \cdot B}{p_T}$$

**sagitta**  $s = \rho(1 - \cos \theta/2) \approx \rho \frac{\theta^2}{8} \approx \frac{0.3}{8} \frac{L^2 B}{p_T}$

The sagitta  $s$  is determined by 3 measurements with error  $s(x)$ :

**Error on the sagitta**  $s = x_2 - \frac{1}{2}(x_1 + x_3)$

$$\left. \frac{\sigma(p_T)}{p_T} \right|^{meas.} = \frac{\sigma(s)}{s} = \frac{\sqrt{\frac{3}{2}}\sigma(x)}{s} = \frac{\sqrt{\frac{3}{2}}\sigma(x) \cdot 8p_T}{0.3 \cdot BL^2}$$

For  $N$  equidistant measurements (R.L. Gluckstern, NIM 24 (1963) 381)

$$\left. \frac{\sigma(p_T)}{p_T} \right|^{meas.} = \frac{\sigma(x) \cdot p_T}{0.3 \cdot BL^2} \sqrt{720/(N+4)} \quad (\text{for } N \geq 10)$$

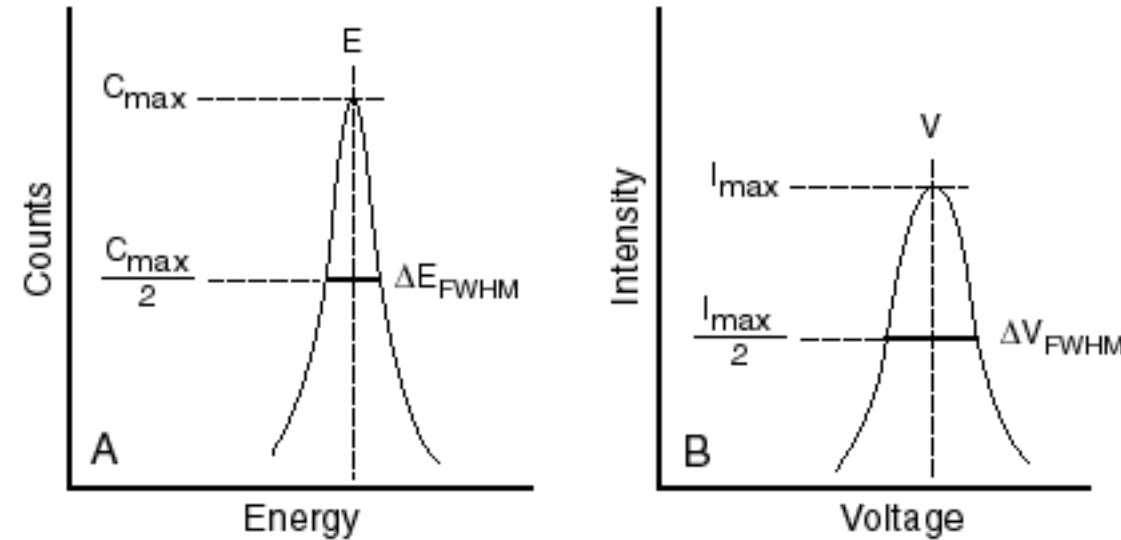
**Example:**  $p_T = 1 \text{ GeV/c}$ ,  $L = 1 \text{ m}$ ,  $B = 1 \text{ T}$ ,  $s(x) = 200 \text{ mm}$ ,  $N = 10$

$$\left. \frac{\sigma(p_T)}{p_T} \right|^{meas.} \approx 0.5\% \quad (s \approx 3.75 \text{ cm})$$

## Momentum resolution

Resolution is generally defined as 1 standard deviation -  $1\sigma$  – for a gaussian distribution dominated by Poisson fluctuations:

$$\sigma = \text{“FWHM-full width at half-maximum”} / 2.36$$



$$E \text{ resolution} = \sigma/E$$

For box distribution (uniform within an interval  $p$ ):  $\sigma = p/\sqrt{12}$

$$\sigma^2 = \frac{\int_{-\frac{p}{2}}^{\frac{p}{2}} (x_r - x_m)^2 D(x_r) dx_r}{\int_{-\frac{p}{2}}^{\frac{p}{2}} D(x_r) dx_r} = \frac{p^2}{12}$$

$D(x) = 1$  uniform distribution of tracks  
 $X_m = 0$  pixel centre

Can we distinguish curved track from the straight line ?

$$s = \rho - \sqrt{\rho^2 - L^2/4}$$

CLEO (electron-positron collider):

Maximum momentum  $p = 5 \text{ GeV}/c$ , B field = 1.5 T

$$\rightarrow \rho = p/(0.3 \text{ B}) = 11.11 \text{ m}$$

Track radius = 1.0 m

$$\rightarrow s = 0.011 \text{ m (1.1 mm)}$$

**EASY !!!**

ATLAS

Tracking length  $L = 1.15 \text{ m}$

B field = 2 T

$$p = 50 \text{ GeV}/c \quad \rho = 144.9 \text{ m} \quad s = 0.0011 \text{ m (1.1 mm)}$$

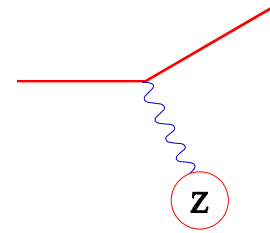
$$p = 1000 \text{ GeV}/c \quad \rho = 1666.7 \text{ m} \quad s = 0.0001 \text{ m (0.1 mm) !!!!}$$



**must consider measurement errors**



# Scattering



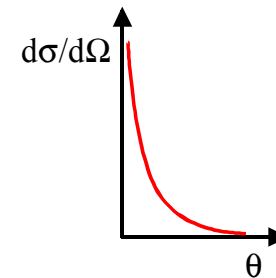
**An incoming particle with charge  $z$  interacts with a target of nuclear charge  $Z$  via exchange of the virtual photon. The cross-section for this e.m. process is**

$$\frac{d\sigma}{d\Omega} = 4zZr_e^2 \left( \frac{m_e c}{\beta p} \right)^2 \frac{1}{\sin^4 \theta/2}$$

**Rutherford formula**

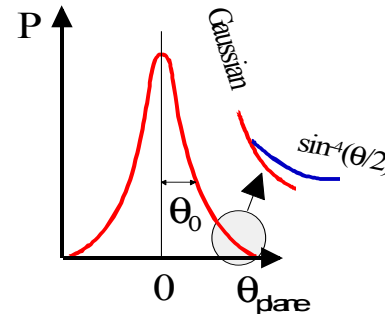
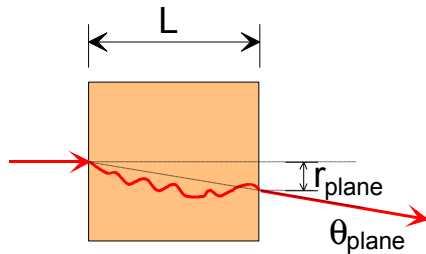
**Depends on charge of the passing particle  
and on the charge and density of the material**

- **Average scattering angle  $\langle \theta \rangle = 0$**
- **Cross-section for  $\theta \rightarrow 0$  is infinite !**



# Multiple Scattering

In sufficiently thick material layer the particle will undergo multiple scattering. Each individual scattering is independent of the previous one → statistical process in 3 dimensions



$$\theta_0 = \theta_{plane}^{RMS} = \sqrt{\langle \theta_{plane}^2 \rangle} = \frac{1}{\sqrt{2}} \theta_{space}^{RMS}$$

Approximation:  $\theta_0 \propto \frac{1}{p} \sqrt{\frac{L}{X_0}}$

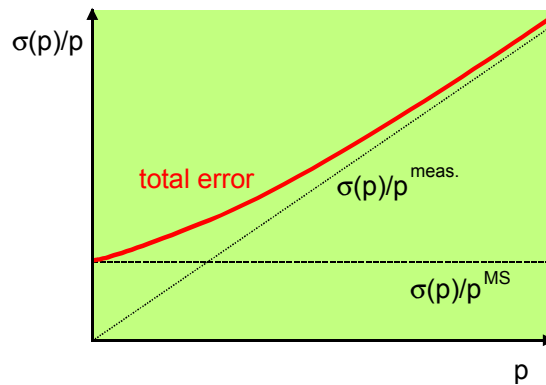
$X_0$  is radiation length of the medium

# Contribution of multiple scattering to $\frac{\sigma(p)}{p_T}$

$$\left. \begin{aligned} \frac{\sigma(p)}{p_T} &\propto \sigma(x) \cdot p_T \\ \sigma(x)|^{MS} &\propto \theta_0 \propto \frac{1}{p} \end{aligned} \right\} \frac{\sigma(p)}{p_T} \Big|^{MS} = \text{constant} \\ \text{independent of } p !$$

More precisely:

$$\frac{\sigma(p)}{p_T} \Big|^{MS} = 0.045 \frac{1}{B\sqrt{LX_0}}$$



- example: Argon gas ( $X_0=110\text{m}$ ),  $L=1\text{m}$ ,  $B=1\text{T}$

$$\frac{\sigma(p)}{p_T} \Big|^{MS} \approx 0.5\%$$

# The Helix Equation

The helix is described in parametric form

$$x(s) = x_o + R \left[ \cos \left( \Phi_o + \frac{hs \cos \lambda}{R} \right) - \cos \Phi_o \right]$$

$$y(s) = y_o + R \left[ \sin \left( \Phi_o + \frac{hs \cos \lambda}{R} \right) - \sin \Phi_o \right]$$

$$z(s) = z_o + s \sin \lambda$$

$\lambda$  is the dip angle

$h = \pm 1$  is the sense of rotation on the helix

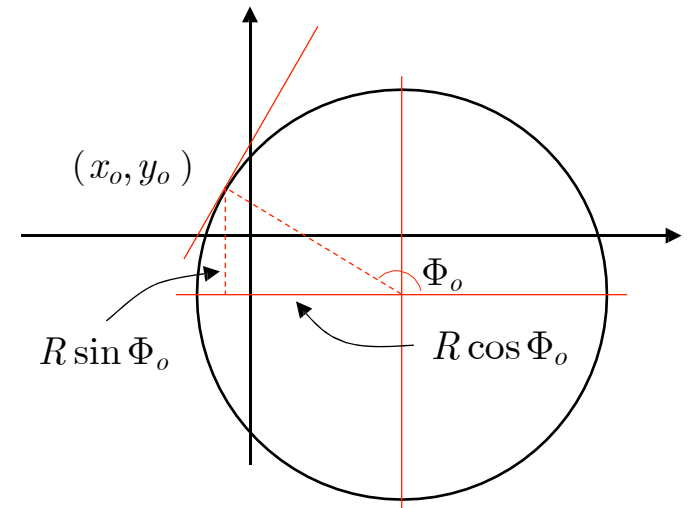
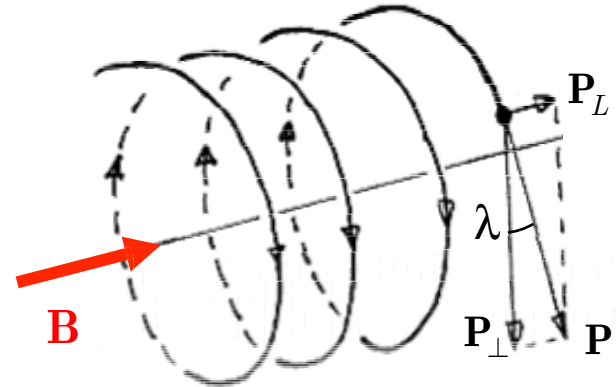
The projection on the  $x$ - $y$  plane is a circle

$$(x - x_o + R \cos \Phi_o)^2 + (y - y_o + R \sin \Phi_o)^2 = R^2$$

$x_o$  and  $y_o$  the coordinates at  $s = 0$

$\Phi_o$  is also related to the slope of the tangent to the circle at  $s = 0$

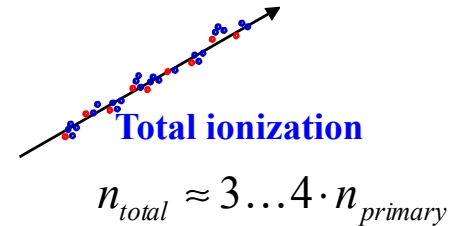
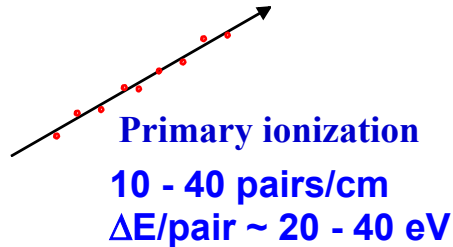
$$R(m) = \frac{p_{\perp} (GeV)}{0.3B(T)}$$



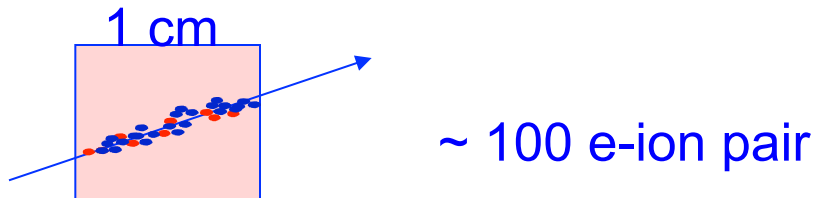
right-handed system

# Gas detectors

Fast charged particles ionize the atoms of a gas. Often the resulting primary electron will have enough kinetic energy to ionize other atoms.



Assume detector, 1 cm thick, filled with Ar gas:



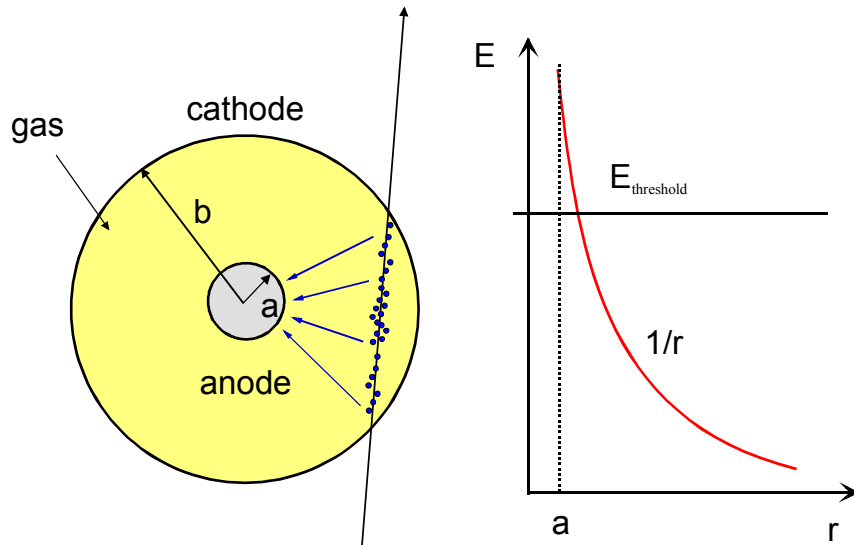
→ 100 electron-ion pairs are not easy to detect! (Noise of amplifier  $\sim 1000 \text{ e}^-$ )

We need to increase the number of e-ion pairs.

# Gas amplification

Simplest case - cylindrical field geometry

## Proportional Counter



$$E(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \frac{1}{r}$$

$$V(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \ln \frac{r}{a}$$

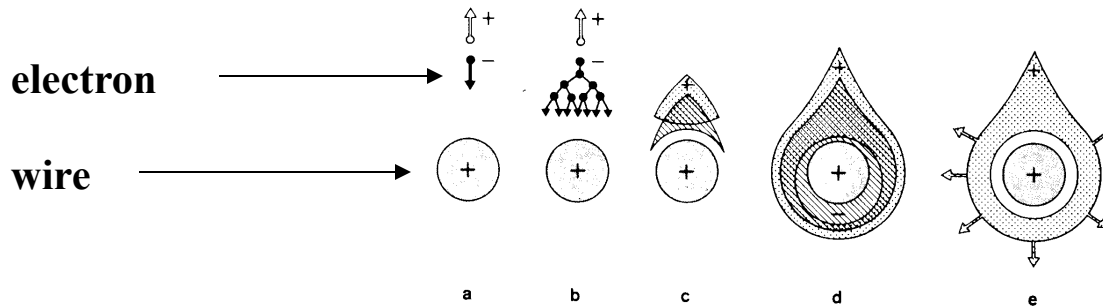
$C$  = capacitance / unit length

## MECHANISM OF SIGNAL AMPLIFICATION

- ◆ Electrons drift towards the anode wire.
- ◆ Close to the anode wire the field is **high** ( ~kV/cm)
- ◆ Electrons gain enough energy for further ionization → exponential increase of number of e<sup>-</sup>-ion pairs.

# Signal Formation

(F. Sauli, CERN 77-09)

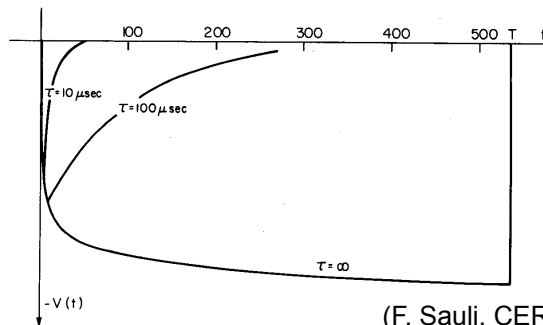


**Avalanche formation within a few wire radii and within  $t < 1$  ns!**

Signal is induced by the moving charges both on anode and on cathode (electrons and ions).

$$dv = \frac{Q}{lCV_0} \frac{dV}{dr} dr$$

**Electrons collected by anode wire, i.e.,  $dr$  is small (few  $\mu\text{m}$ )**



(F. Sauli, CERN 77-09)

**Ions have to drift back to cathode,  $dr$  is big. Signal duration limited by total ion drift time !**

**Need electronic signal differentiation to limit dead time.**

# Cross section for ionization by collisions of electrons with atoms of noble gases

$$n = n_0 e^{\alpha(E)x} \quad \text{or} \quad n = n_0 e^{\alpha(r)x}$$

$$\alpha = \frac{1}{\lambda}$$

$$M = \frac{n}{n_0} = \exp \left[ \int_a^{r_c} \alpha(r) dr \right]$$

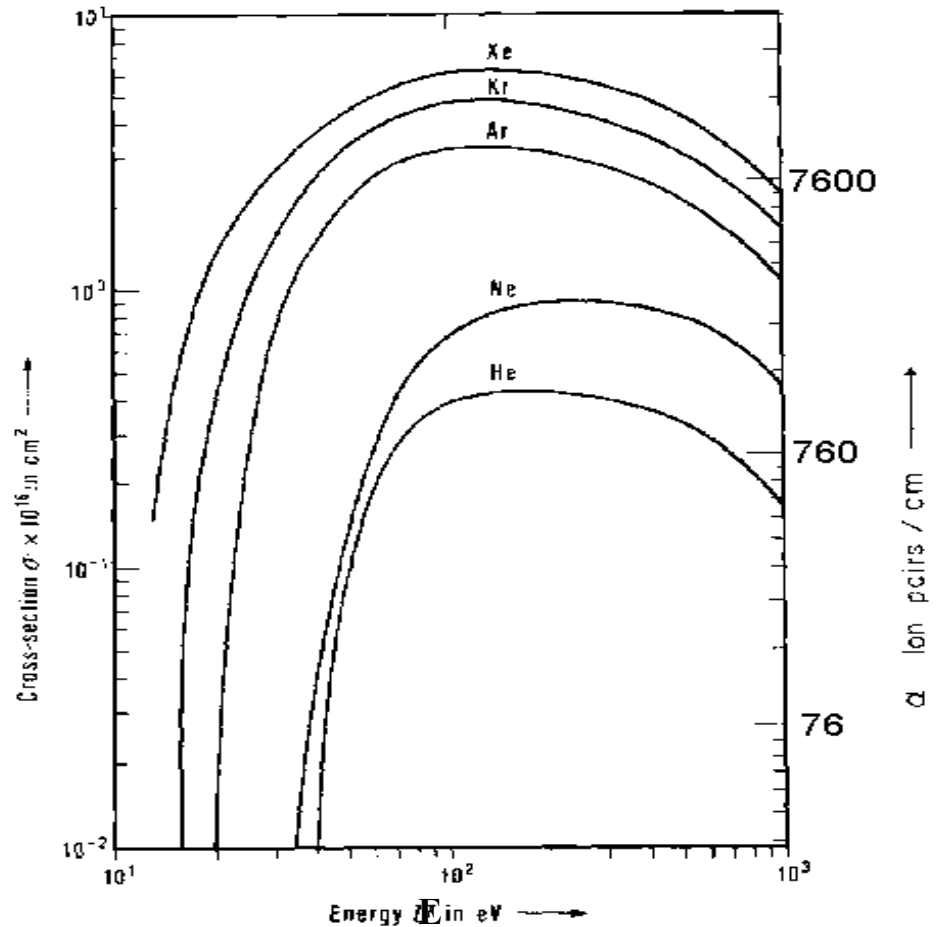
**Gain**  $M \approx ke^{CV_0}$

$\alpha$ : First Townsend coefficient (e<sup>-</sup>-ion pairs/cm)

$\lambda$ : mean free path

Total signal (number of collected electrons) is proportional to the total ionization generated by the passage of charged particle.

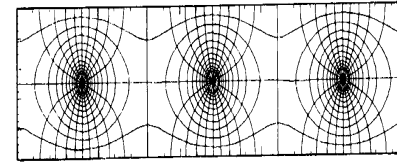
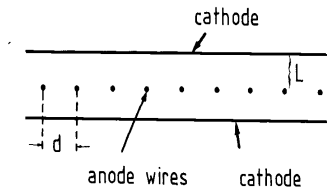
-> proportional chambers





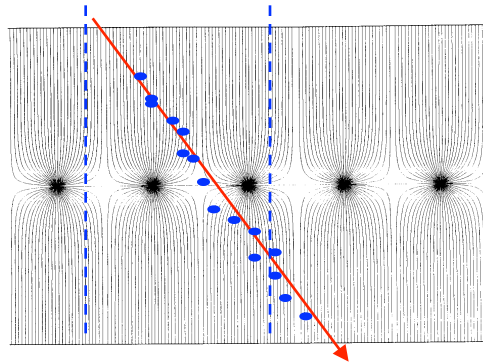
# Multi wire proportional chambers

G. Charpak et al. 1968, Nobel prize 1992



field lines and equipotentials around anode wires

Negative signals on all wires.



Typical parameters:

$L=5\text{mm}$ ,  $d=1\text{mm}$ ,

$a_{\text{wire}}=20\mu\text{m}$ .

Digital readout:  
spatial resolution limited to

$$\sigma_x \approx \frac{d}{\sqrt{12}}$$

(  $d=1\text{mm}$ ,  $s_x=300\mu\text{m}$  )