Lecture 3

lecture slides are at: ***http://www.physics.smu.edu/ryszard/5380fa17/***

Proton mass $m_p = 938.28\ MeV/c^2$
Electron mass $m_e = 0.511\ MeV/c^2$
Neutron mass $m_n = 939.56\ MeV/c^2$

Helium nucleus α: 2 protons+2 neutrons
mass $m_\alpha = 3727\ MeV/c^2$

$2 \times 939.28 + 2 \times 939.56 = 3755.68$

- *What keeps the nucleus together?*
- *Why the mass of helium nucleus is smaller then the sum of masses of protons and neutrons ?*

But first, unfinished subject of the last lecture
– special relativity and the mass of the photon

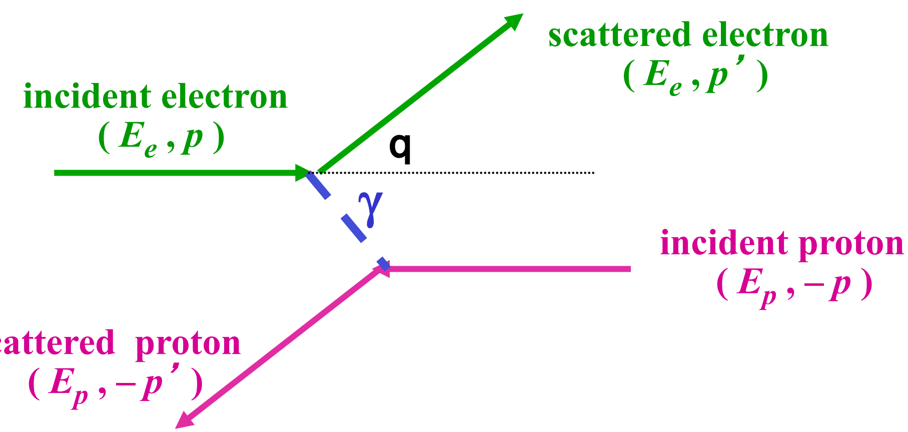
Virtual photon

In Relativistic Quantum Mechanics static fields of forces <u>DO NOT EXIST</u> ;
The interaction between two particles is "transmitted" by intermediate particles acting as "interaction carriers"

Example: electron – proton scattering (an effect of the electromagnetic interactions) is described as a two-step process : 1) **incident electron -> scattered electron + photon**
2) **photon + incident proton -> scattered proton**

The photon (γ) is the carrier of the electromagnetic interaction

In the electron – proton centre-of-mass system



Energy – momentum conservation:

$$E_\gamma = 0$$
$$\vec{p}_\gamma = \vec{p} - \vec{p}\,' \qquad (|\vec{p}\,| = |\vec{p}\,'|)$$

"Mass" of the intermediate photon: $Q^2 = E_\gamma^2 - p_\gamma^2 c^2 = -2\,p^2 c^2\,(1 - \cos\theta)$

The photon is in a VIRTUAL state because for real photons $E_\gamma^2 - p_\gamma^2 c^2 = 0$
(the mass of real photons is ZERO) – **virtual photons can only exist for a very short time interval thanks to the "Uncertainty Principle"**

First ideas of a strong force

Hideki Yukawa 1934

•Nucleus is held together by a force stronger the electromagnetic repulsion between proton – strong force.
•We do not see this force outside the nucleus – therefore it must have a very small range ~ the size of the nucleus, i.e., ~ 1fm
*•If the field of strong force is quantized the carrier of the force must have a mass of about 200 MeV/$c^{2,}$ i.e., in between the electron and proton masses – **meson***

1937-1946 – study of cosmic rays led to discovery of two new light particles: muon (μ) and pion (π).
Pion is consistent with a Yukawa particle. It is a common product of high energy collisions and it decays to muons ("heavy leptons").

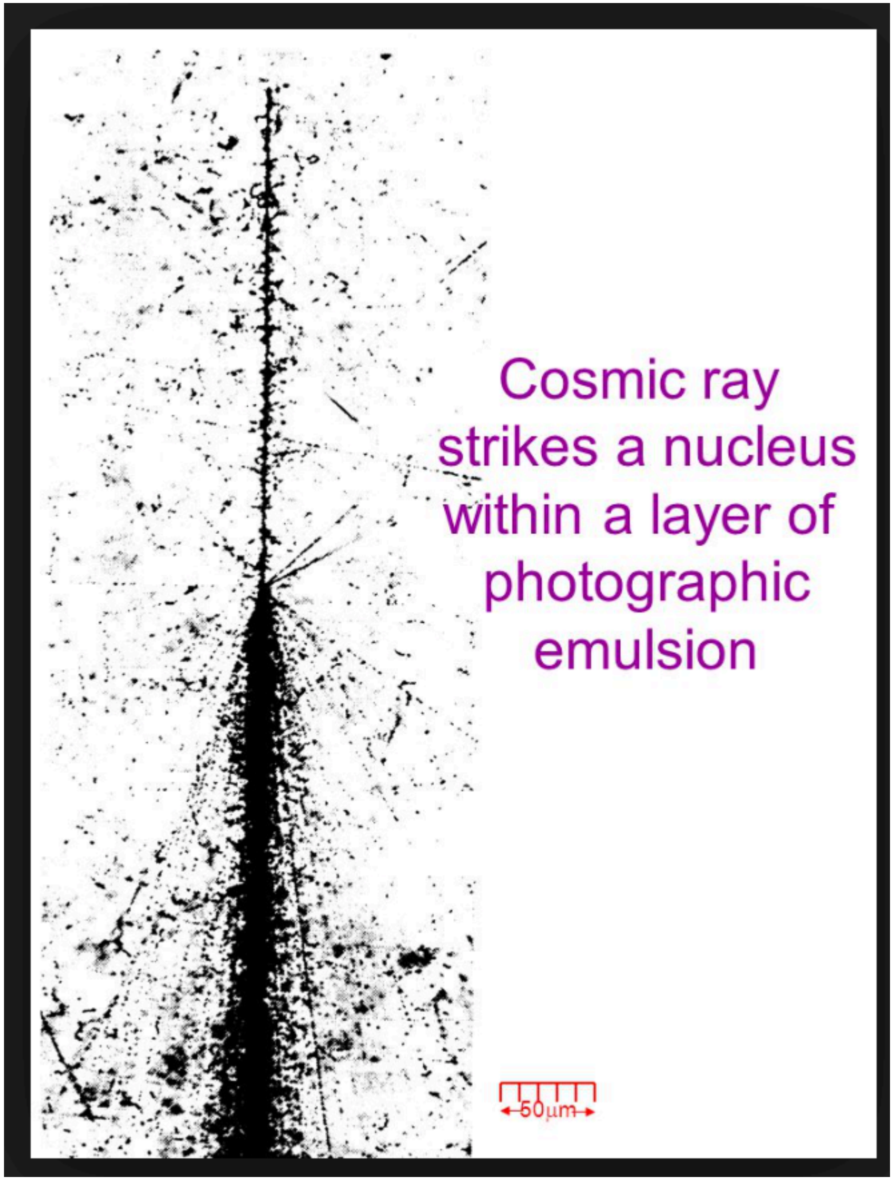
Vertical stack of photographic emulsion (silver bromide AgBr) exposed to cosmic rays in a balloon flight. Light decomposes the compound and the processing removes bromine leaving grains of silver distributed in a low mass transparent gel.

Initial interpretation − fragments of the nucleus

^{108}Ag(silver) − 47 protons
61 neutrons

^{80}Br(bromine) -35 protons
45 neutrons

To many tracks -> new particles are produced



Free neutron has a finite lifetime
Neutron's mean lifetime $\tau = 881.5$ *s*
Lifetime of neutron bound in the stable nucleus $= \infty$
Quantum tunneling is responsible for many decays of radioactive nuclei

Angular Momentum and Spin

- *Objects in motion may have two types of angular momentum*
  - *orbital, e.g., orbital movement of Earth around the Sun*
  - *spin, e.g., earth rotation around its axis*

- *Quantum mechanics –the same angular momentum arguments apply to particles* ***but*** *values of orbital angular momentum are quantized*

  $l(l+1)\, h^2 \quad l = 0,1,2,3,\ldots.$

- *Spin – intrinsic angular momentum is also quantized but may have also half-integer values*

  $s(s+1)h^2 \quad s = 0,\ 1/2,\ 1,\ 3/2,\ 2,\ 5/2,\ \ldots..$

Pauli's exclusion principle

In Quantum Mechanics the electron orbits around the nucleus are "quantized":
only some specific orbits (characterized by integer quantum numbers) are possible.

Example: allowed orbit radii and energies for the Hydrogen atom

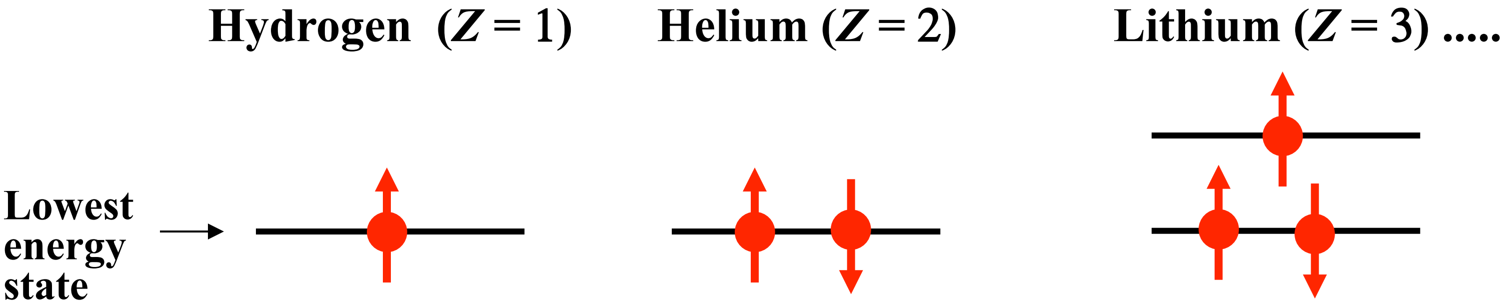
$$R_n = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{me^2} \approx 0.53\times10^{-10} n^2 \text{ [m]}$$

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2 \hbar^2 n^2} \approx -\frac{13.6}{n^2} \text{ [eV]}$$

$\left(\begin{array}{l} m = m_e m_p/(m_e + m_p) \\ n = 1, 2, \ldots\ldots \end{array} \right)$

In atoms with $Z \geq 2$ only two electrons are found in the innermost orbit – WHY?

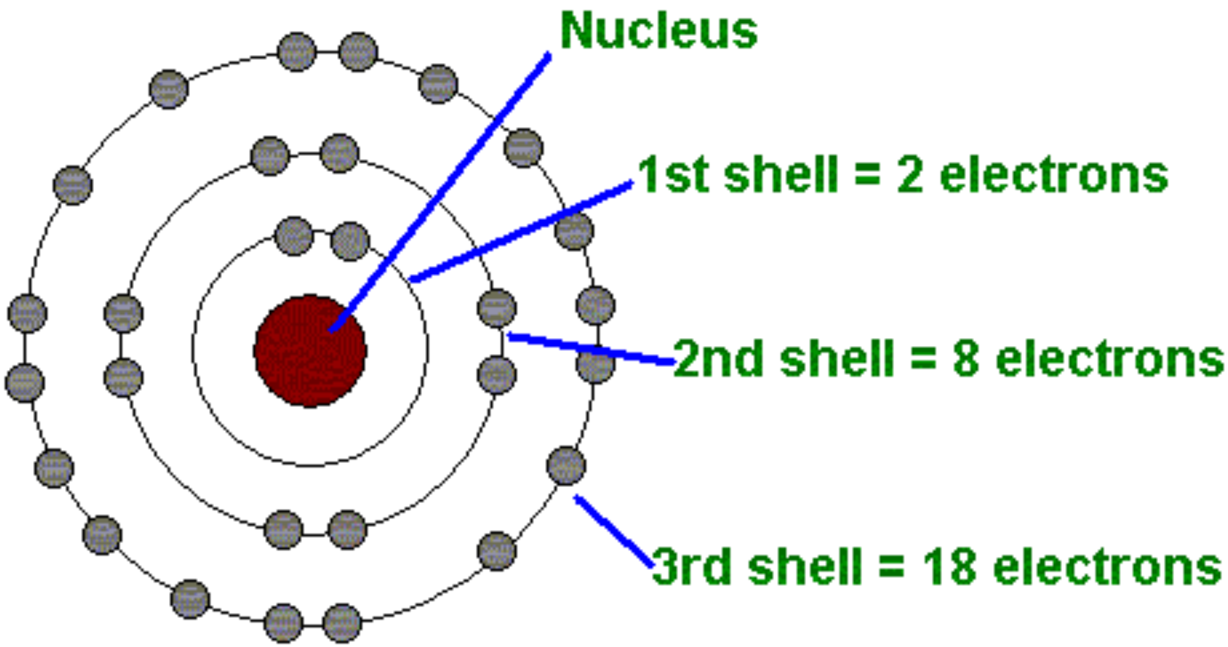
ANSWER (Pauli, 1925): two electrons (spin = ½) can never be in the same physical state



Wolfgang Pauli

Pauli's exclusion principle applies to all particles with half-integer spin (collectively named Fermions)

Atomic shell model



Maximum number of electrons per shell $2n^2$

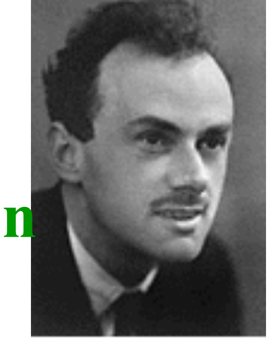
<u>Note</u>

Electron has no dimension, it is point-like

but it has a spin = ½

ANTIMATTER

Discovered "theoretically" by P.A.M. Dirac (1928)

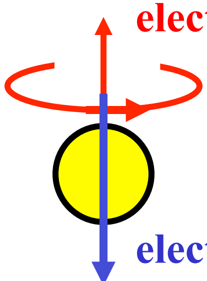


P.A.M. Dirac

Dirac's equation: a relativistic wave equation for the electron

Two surprising results:

- **Motion of an electron in an electromagnetic field:**
  presence of a term describing (for slow electrons) the potential energy of a magnetic dipole moment in a magnetic field
  -> existence of an intrinsic electron magnetic dipole moment opposite to spin



$$\mu_e = \frac{e\hbar}{2m_e} \approx 5.79 \times 10^{-5}\ [\mathrm{eV/T}]$$

- **For each solution of Dirac's equation with electron energy $E > 0$ there is another solution with $E < 0$**

What is the physical meaning of these "negative energy" solutions ?

Generic solutions of Dirac's equation: complex wave functions $\psi(\vec{r}, t)$

In the presence of an electromagnetic field, for each negative-energy solution the complex conjugate wave function ψ^* is a positive-energy solution of Dirac's equation for an electron with opposite electric charge (+*e*)

Dirac's assumptions:

- **nearly all electron negative-energy states are occupied and are not observable.**
- **electron transitions from a positive-energy to an occupied negative-energy state are forbidden by Pauli's exclusion principle.**
- **electron transitions from a positive-energy state to an empty negative-energy state are allowed -> electron disappearance. To conserve electric charge, a positive electron (positron) must disappear -> e^+e^- annihilation.**
- **electron transitions from a negative-energy state to an empty positive-energy state are also allowed -> electron appearance. To conserve electric charge, a positron must appear -> creation of an e^+e^- pair.**

→ empty electron negative–energy states describe positive energy states of the positron

Dirac's perfect vacuum: a region where all positive-energy states are empty and all negative-energy states are full.

Positron magnetic dipole moment = μ_e but oriented parallel to positron spin

Experimental confirmation of antimatter

(C.D. Anderson, 1932)

Carl D. Anderson

Detector: a Wilson cloud – chamber (visual detector based on a gas volume containing vapor close to saturation) in a magnetic field, exposed to cosmic rays

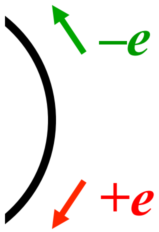
Measure particle momentum and sign of electric charge from magnetic curvature

Lorentz force $\vec{f} = e\vec{v} \times \vec{B}$ → **projection of the particle trajectory in a plane perpendicular to $\vec{B}$ is a circle**

Circle radius for electric charge |*e*|: $$R\,[\mathrm{m}] = \frac{10\,p_\perp[\mathrm{GeV/c}]}{3B\,[\mathrm{T}]}$$

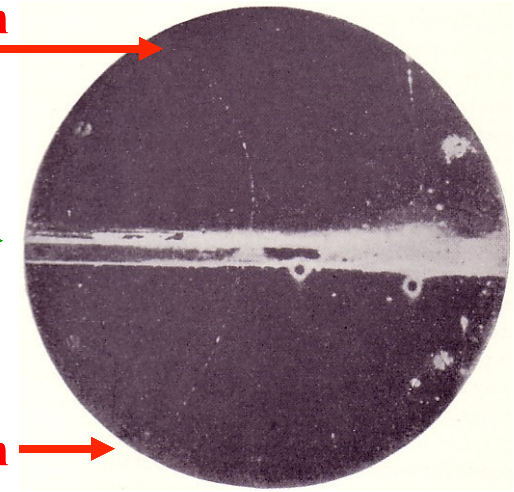
$p_\perp$: momentum component perpendicular to magnetic field direction

NOTE: impossible to distinguish between positively and negatively charged particles going in opposite directio

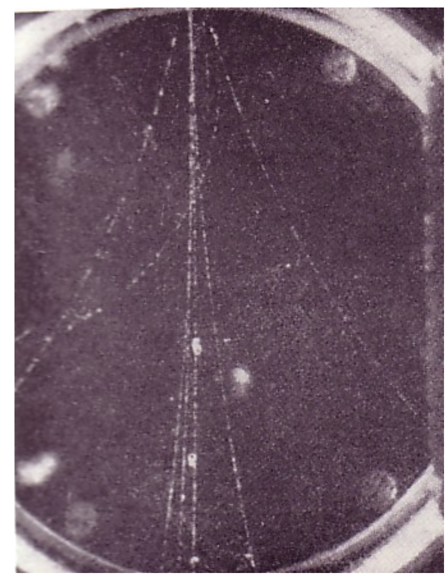


→ **need an independent determination of the particle direction of motion**

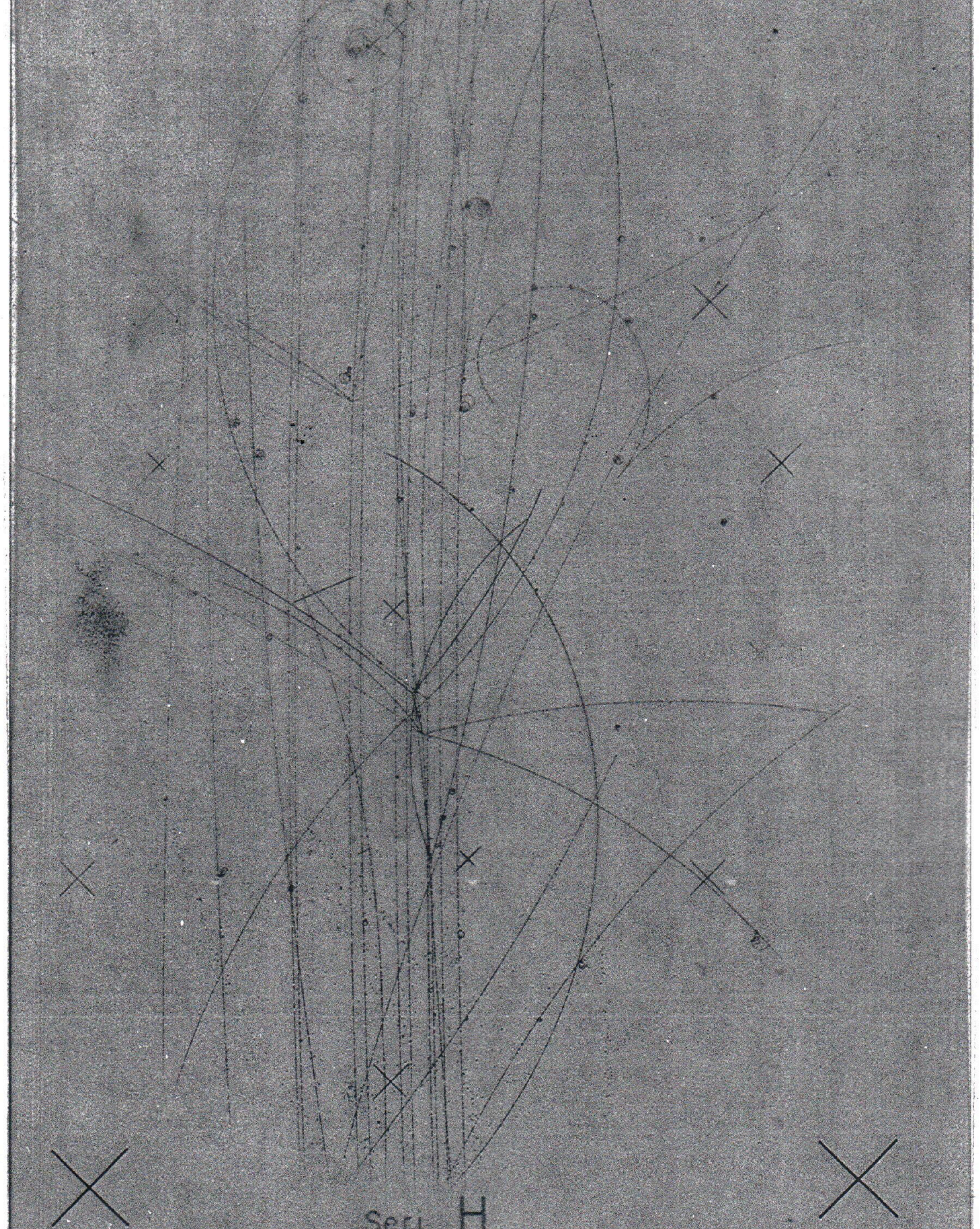
First experimental observation of a positron



Production of an electron-positron pair by a high-energy photon in a Pb plate



Cosmic-ray "shower" containing several $e^+ e^-$ pairs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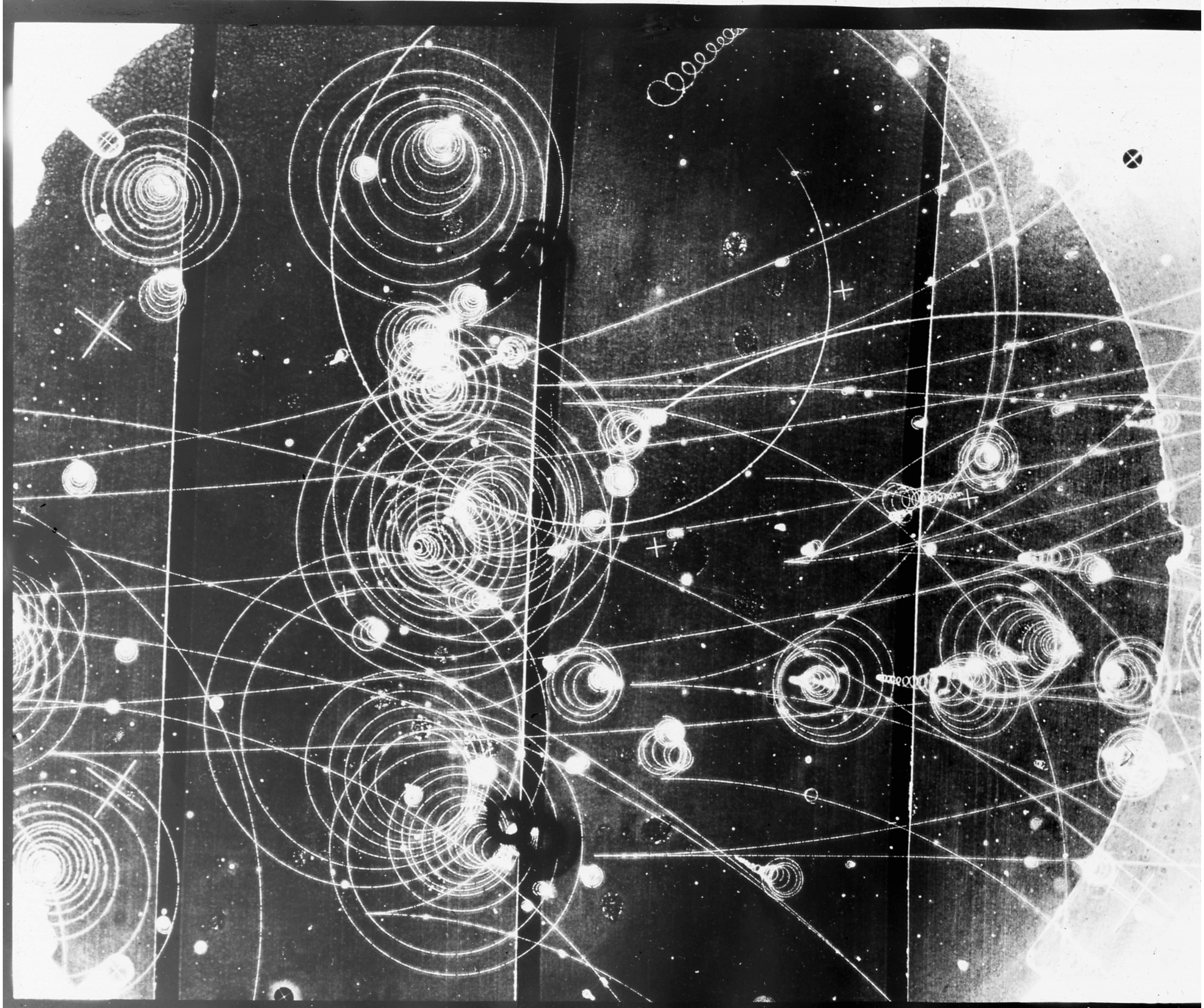
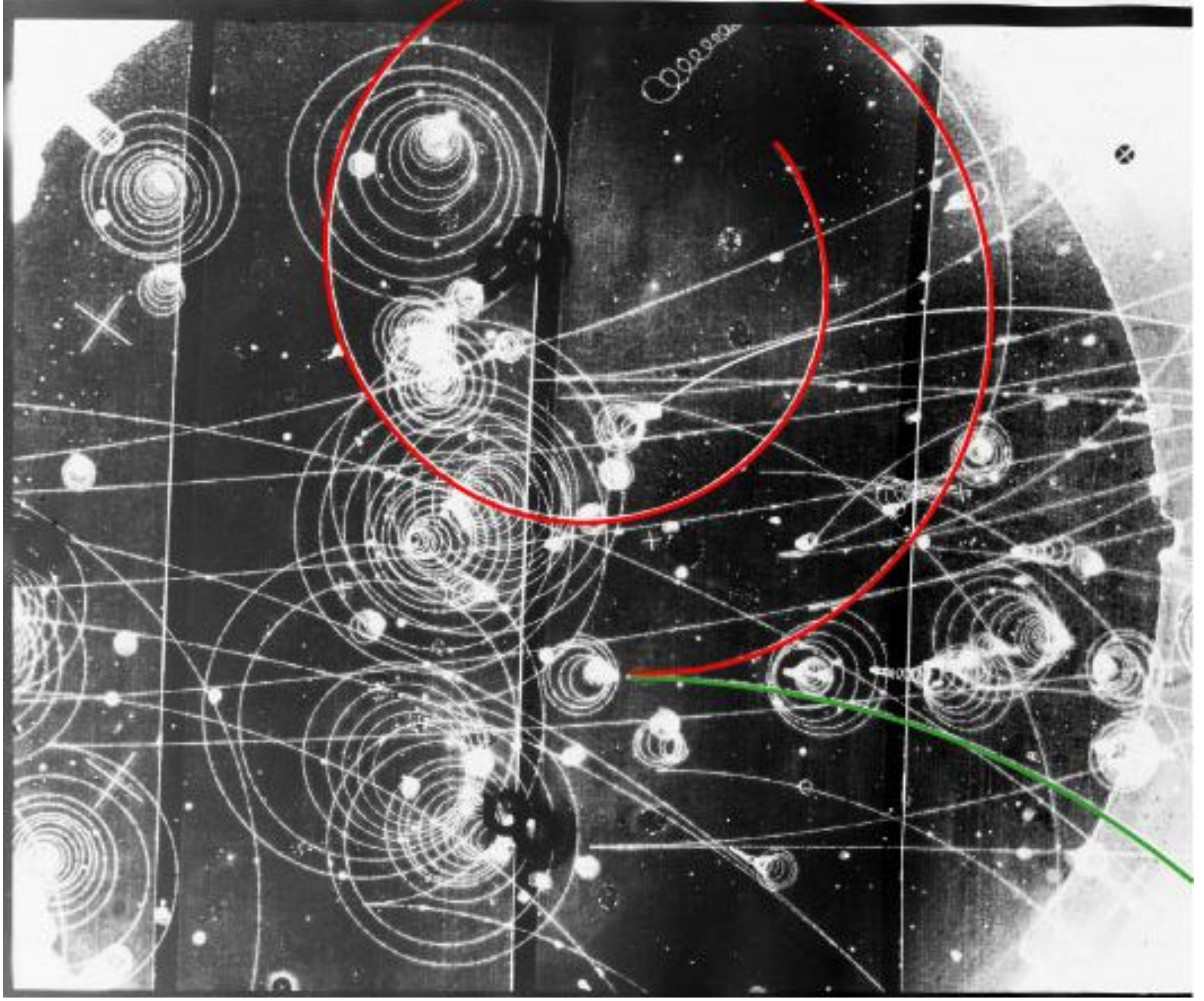
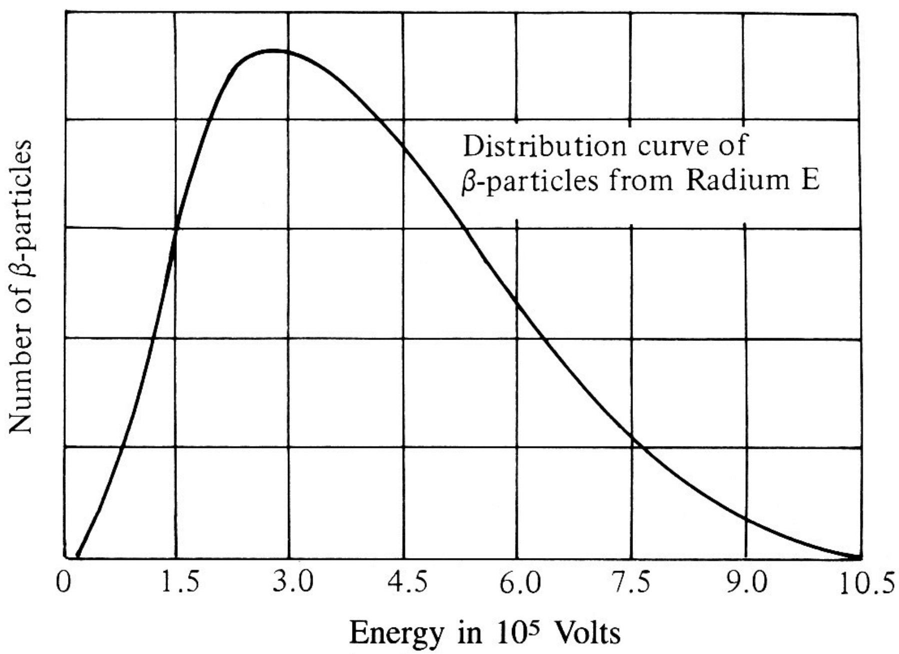




Neutrinos

A puzzle in β – decay: the continuous electron energy spectrum

First measurement by Chadwick (1914)



Radium E: $^{210}Bi_{83}$
(a radioactive isotope produced in the decay chain of ^{238}U)

If b – decay is $(A, Z) \rightarrow (A, Z+1) + e^-$, then the emitted electron is mono-energetic:

electron total energy $E = [M(A, Z) - M(A, Z+1)]c^2$

(neglecting the kinetic energy of the recoil nucleus $\frac{1}{2}p^2/M(A,Z+1) << E$)

Several solutions to the puzzle proposed before the 1930's (all wrong), including violation of energy conservation in b – decay

December 1930: public letter sent by W. Pauli to a physics meeting in Tübingen

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

...because of the "wrong" statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin ½ and obey the exclusion principle ..... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous b-spectrum would then become understandable by the assumption that in b-decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant.

.... For the moment, however, I do not dare to publish anything on this idea ......

So, dear Radioactives, examine and judge it. Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December. ....

W. Pauli

NOTES

- Pauli's neutron is a light particle -> not the neutron that will be discovered by Chadwick one year later
- As everybody else at that time, Pauli believed that if radioactive nuclei emit particles, these particles must exist in the nuclei before emission

Theory of β-decay (Enrico Fermi, 1932-33)

Enrico Fermi

β⁻ decay: $n \rightarrow p + e^- + \bar{\nu}$

β⁺ decay: $p \rightarrow n + e^+ + \nu$ (e.g., ${}^{14}O_8 \rightarrow {}^{14}N_7 + e^+ + \nu$)

ν: the particle proposed by Pauli
(named "neutrino" by Fermi)

$\bar{\nu}$: its antiparticle (antineutrino)

Fermi's theory: point interaction among four spin ½ particles, using the mathematical formalism of creation and annihilation operators invented by Jordan
-> particles emitted in β – decay need not exist before emission they are "created" at the instant of decay

Prediction of β – decay rates and electron energy spectra as a function of only one parameter: Fermi coupling constant G_F (determined from experiments)

Energy spectrum dependence on neutrino mass m
(from Fermi's original article, published in German on Zeitschrift für Physik, following rejection of the English version by Nature)

Measurable distortions for m >0 near the end-point (E_0: max. allowed electron energy)