Large Hadron Collider

Dipoles:

Field	8.3T
Length	15 m
Number	1232

Quadrupoles:

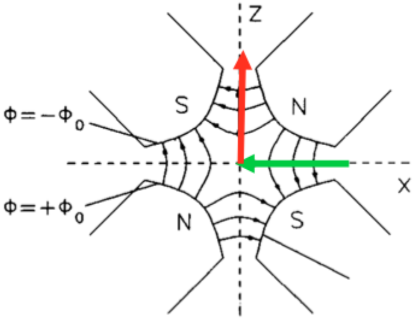
Length	3.2 m
Gradient	223 T/m

$$g = \frac{2\mu_0 n I}{r^2}\left[\frac{T}{m}\right]$$

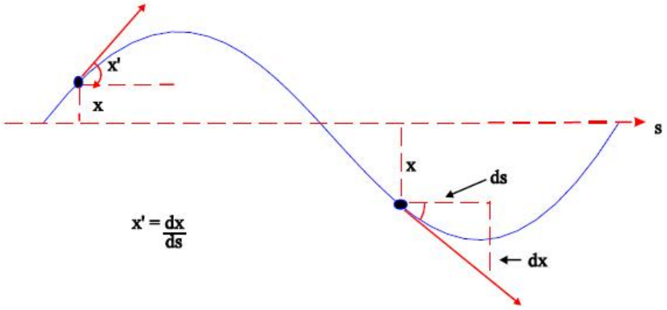
gradient

$$k = \frac{g}{p/e}\left[m^{-2}\right]$$

focusing strength



Particle motion is described by 6 coordinates wrt to ideal trajectory
deviations from ideal trajectory in x and y, velocities in x and y, position along the ideal trajectory and phase:
x, dx/dt, y, dy/dt, s-βct, δ



-> oscillatory motion around ideal trajectory

Equation of Motion

$$F = m\,a = e\,B\,v$$

Keep only terms linear in x or y for magnetic field

Quadrupole field changes sign between x and y

Solution in one coordinate: $\boxed{x(s) = a_1 \cos(\sqrt{K}\,s) + a_2 \sin(\sqrt{K}\,s)}$

$$y'' + ky = 0$$
$$x'' + x\left(\frac{1}{\rho^2} - k\right) = 0$$

Transfer matrices $\begin{pmatrix} x \\ x' \end{pmatrix} = M_{foc} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$

Focusing quadrupole K 0

$$\boxed{M_{foc} = \begin{pmatrix} \cos(\sqrt{K}s) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}s) \\ -\sqrt{K}\sin(\sqrt{K}s) & \cos(\sqrt{K}s) \end{pmatrix}}$$

Defocusing quadrupole K < 0

$$\boxed{M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}s) \\ \sqrt{|K|}\sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix}}$$

Drift space

$$\boxed{M_{drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}}$$

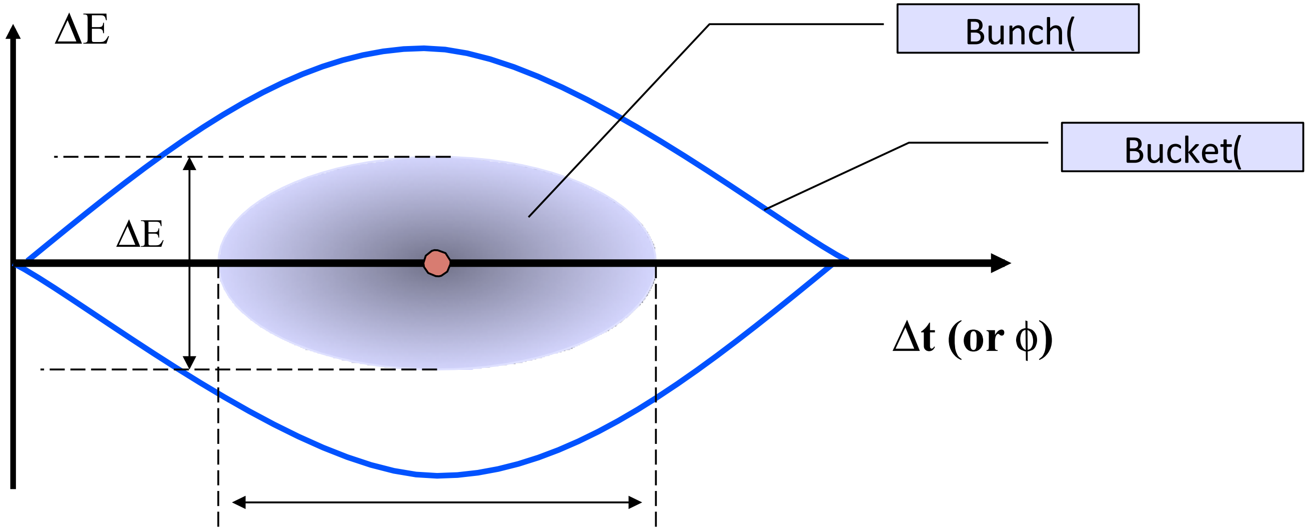
$$M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots$$

- Particles (protons) follow an trajectory that oscillates around the ideal circular path.
- The oscillation is independent for vertical and radial components.
- The oscillation has same properties but each particle has an independent trajectory govern by its initial vector and the derivatives.
- The bunch is defined by an envelope of all trajectories.
- At any point along the ring, the distribution of the density of the trajectories is gaussian and we assume that the beam pipe must be large enough to allow for 10σ of this density distribution to pass.

- In order to increase the probability of particle collision we want to maximize the particle density at the beam intersection point. This is done by “low beta inserts” – a special set of magnets with strong focusing that narrows the beam envelope.
- Close to the interaction point the beam must get close to each other into one beam pipe.
- We do not want the interactions to take place away from the center of the detector so beams are blown up when travelling along the same path and brought to high density (small transverse cross section) at the interaction point.

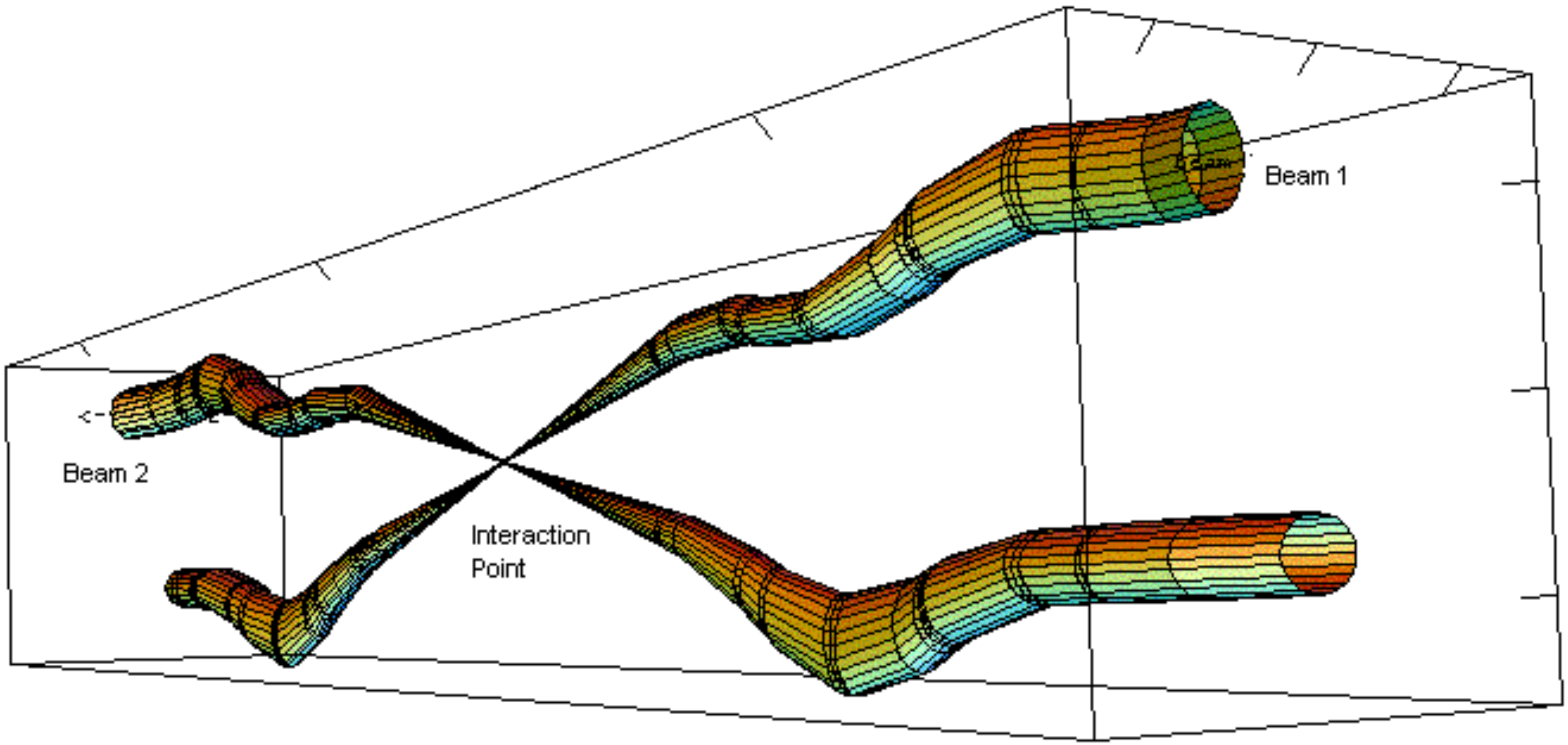
Bucket & Bunch

The bunches of the beam fill usually a part of the bucket area.



LHC – bucket spacing 25 ns -> 40 MHz of bucket-bucket crossing
Not all buckets are filled up with particles
Not all bunches have the same number of particles

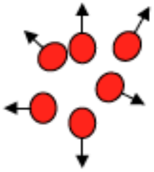
Mini-beta inserts: relative size of the beam near ATLAS interaction point



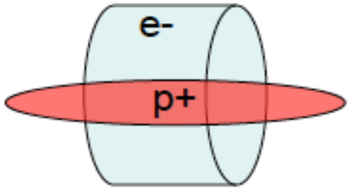
Collective Effects

Three categories: can cause beam instabilities, emittance blow-up, beam loss....!

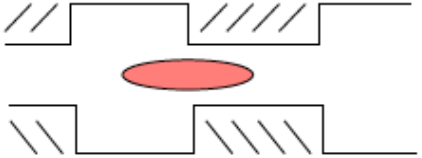
Beam-self: beam interacts with itself through space charge.



Beam-beam: colliding beams in colliders or ambient electron clouds (e-p instability).



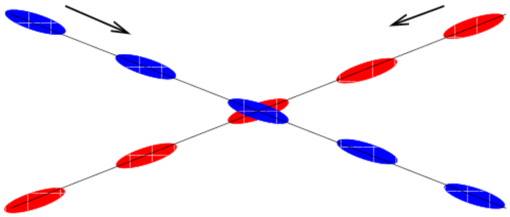
Beam-environment: beam interacts with machine (impedance-related instabilities).



Tune spread $\propto \frac{1}{\beta^2\gamma^3}$

THE limitation in low energy machines

Colliding beams. Tune spread/shift due to head-on collisions and long range collisions.



Beam induces field in accelerator environment. Wake fields. Wake fields can act back on trailing beam.

Fourier transform of Wake field is impedance.

Can lead to component heating and/or instability.

Beam crossing

For continuous beam – intersection of two ribbons

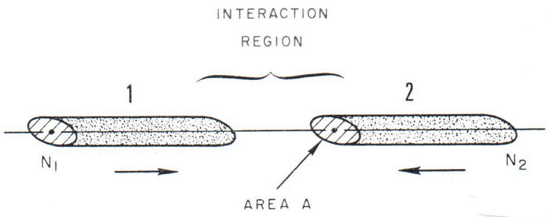
Bunches – limited in space – allow for grouping of of collisions in time

Crossing at an angle minimize number of multiple interactions away from the central collision point.

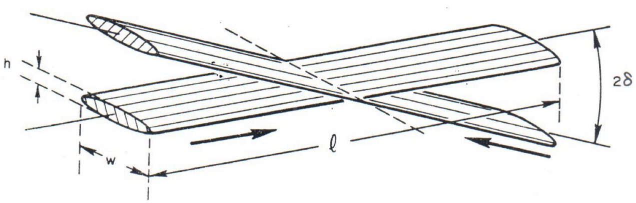
Bunch separation at LHC is 25 ns, i.e.,

40 MHz collision frequency

→ impact on detector design



Head-on collision of two bunches.



Bunches colliding with a vertical crossing angle.

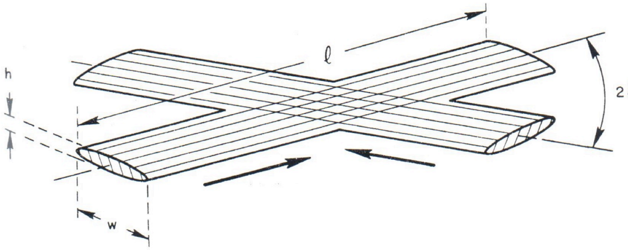
Bunch crossing

Bunches with gaussian density distributions in all 3 dimensions

Ideal case:

N_1 = number of particles in bunch 1

N_2 = number of particles in bunch 2



Bunches colliding with a horizontal crossing angle.

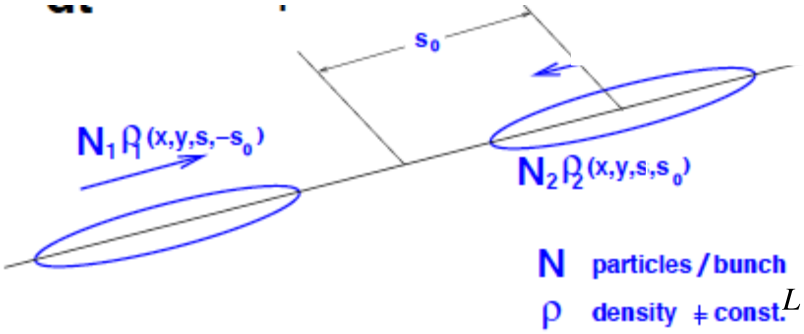
rate of collisions of collisions ~ particle density × σ

$$dR/dt \sim L \times \sigma$$

LUMINOSITY

Luminosity is the proportionality factor between cross section and number of interactions per second (instantaneous luminosity)

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$



s – position along the trajectory

Two colliding bunches

$$L_{inst} = f\,N_1 N_2 K \int \rho_1(x,y,z,t)\rho_2(x,y,z,t)\,dx\,dy\,dz\,dt$$

Effect of crossing angle $K = \sqrt{(v_1 - v_2)^2 - \frac{(v_1 \times v_2)^2}{c^2}}$

v – velocity of particles in the bunch

For $v_1 = v_2 = c$ and crossing angle in the plane of the orbit with half-angle α

$$L = f\,N_1 N_2\, 2\, c\, \cos\alpha \int \rho_1(x,y,z,t)\rho_2(x,y,z,t)\,dx\,dy\,dz\,dt$$

Gaussian particle distributions

$$\rho(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp(-\frac{x^2}{2\sigma_x^2})$$

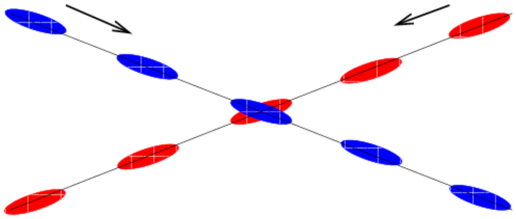
Equal shape of the colliding bunches

$$\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$$

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot S \cdot H$$

Correction factors S and H.

If colliding with many bunches, need collision with crossing angle to avoid unwanted collisions.



The larger the crossing angle, the smaller S

S corrects for the non-uniform bunch density during collision. Important for long bunches.

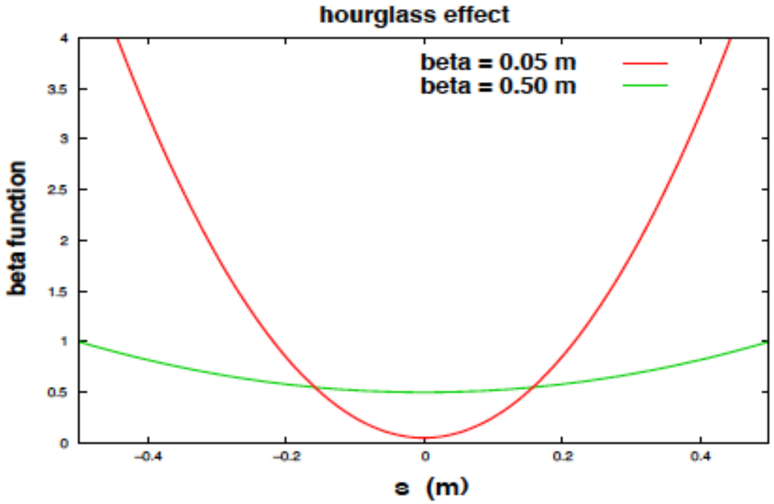
Correction factor H - Hour glass effect

$$\sigma_{x,y} = \sqrt{\beta^*_{x,y} \cdot \varepsilon}$$

Size of the focused bunch β depends on position along the trajectory s

$$\beta(s) = \beta^* \cdot (1 + (\frac{s}{\beta^*})^2)$$

The longer the bunch the larger the effect.



LHC Design Parameters

Design parameters for 7 TeV/c per beam operations

$N_1 = N_2 = 1.15 \times 10^{11}$ protons per bunch

bunch spacing = 25 ns -> $n_b = 2808$ bunches per beam

with one bunch in each beam frequency of collisions $f = 11.2455$ kHz

crossing angle $\varphi = 285$ μrad

beta function at the collision point $\beta^*_x = \beta^*_y = 55$ cm

bunch dimension at the collision point

$\sigma^*_x = \sigma^*_y = 16.6$ μm

$\sigma_s = 7.7$ cm

LHC Design Luminosity (round beams)

$$L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} = \frac{N_1 N_2 f n_b}{4\pi\beta^*\varepsilon}$$

Without crossing angle and hourglass effect

$$\mathcal{L} = 1.2 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

With crossing angle

$$\mathcal{L} = 0.973 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

With crossing angle and hourglass effect

$$\mathcal{L} = 0.969 \times 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

Integrated luminosity

$$L_{\mathrm{int}} = \int L dt$$

Number of events: N_{ev} = σ(cross section) × L_{int}

units of cross section: 1 barn = 10^{-24} cm^2
units of L_{int}: 1 fb^{-1} = 10^{39} cm^{-2}

Luminosity during fill

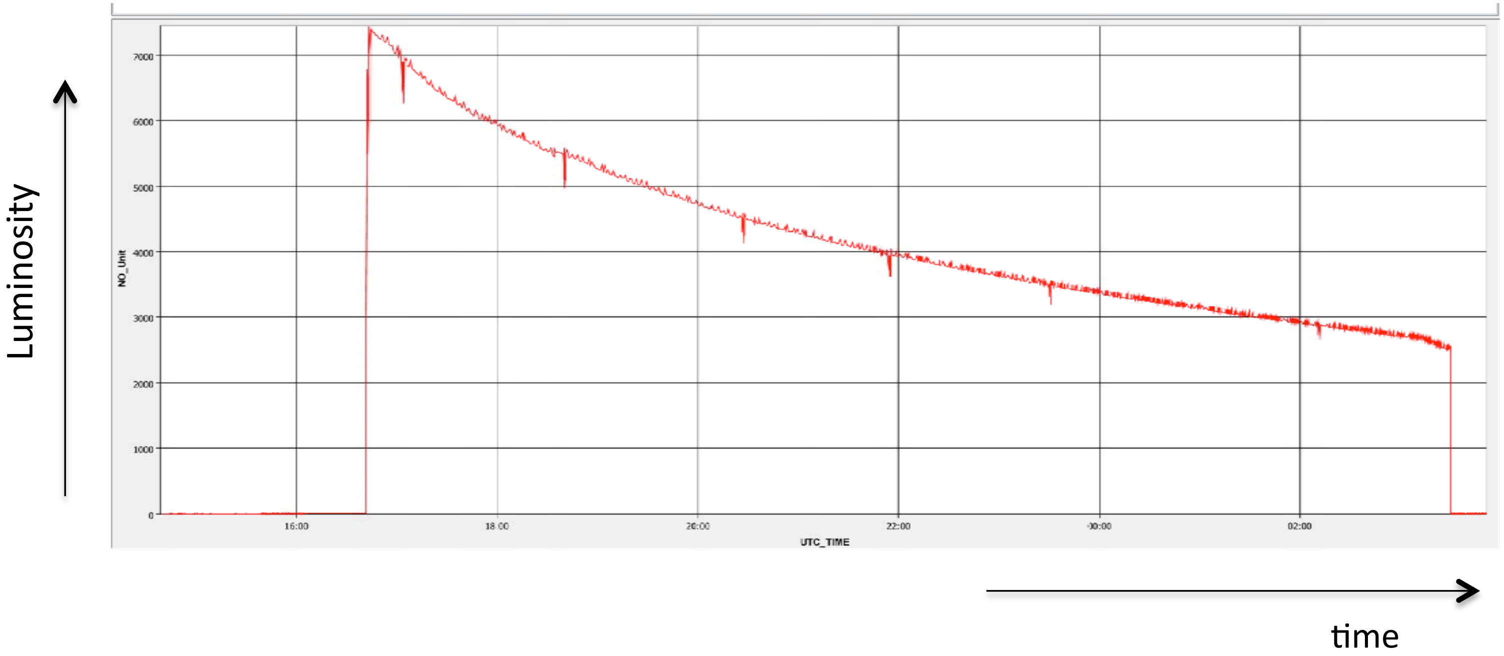
LHC fill = one full operational cycle

injection + energy ramp + data taking + beam abort

Luminosity decays during fil after beams have been put into collision

- Intensity burn-off (about 20 – 30 interactions per crossing, μ =20-30
- Emittance growth (larger dispersion of particle directions leading to increased losses)

$$\mathcal{L}(t) = \mathcal{L}_0 \exp(-\frac{t}{\tau})$$



Questions:

How do we know how many bunches are in each beam?
How do we know the number of protons per bunch?
How do we measure luminosity?
What is the measurement precision required by ATLAS physics analyses?
What is measured and what provides calibration?