Physics 1303(001),1403(801) **Professor Scalise**
Spring 1996 **Midterm #3**

Name: ________________ **ID number:**________________

This exam is worth 100 points. It consists of 13 multiple-choice questions worth 6 points each and a partial credit section worth a maximum of 22 points.

Don't get hung up on the questions. They should only take a few minutes each. If you find yourself spending more than a few minutes on a question you are probably looking at it the wrong way. You should skip it temporarily and return to it later.

NOTE - The equation sheets are on the last pages. If you think that it makes referring to them easier, you can remove them from the rest of the exam.

<u>GOOD LUCK</u>

1. Block A, with a mass of 4 kg, is stationary while block B, with a mass of 8 kg, is moving at 3 m/s. The center of mass of the two block system has a speed in m/s of:
   - (a) 0
   - (b) 1.5
   - (c) 2
   - (d) 3
   - (e) 12

2. Blocks A and B are moving toward each other. A has a mass of 2.0 kg and a velocity of 50 m/s, while B has a mass of 4.0 kg and a velocity of -25 m/s. They suffer a completely inelastic collision. The kinetic energy dissipated during the collision is:
   - (a) 0
   - (b) 1250 J
   - (c) 2500 J
   - (d) 3750 J
   - (e) 5000 J

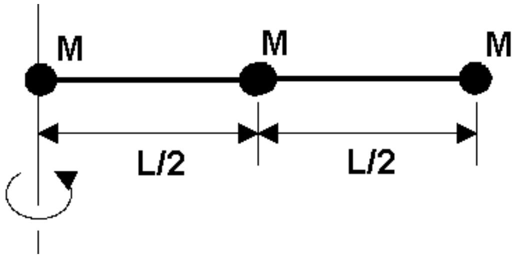
3. The angular speed in rad/s of the minute hand of a watch is:

   (a) $60/\pi$
   (b) $1800/\pi$
   (c) π
   (d) $\pi/1800$
   (e) $\pi/60$

4. A wheel starts from rest and has a constant angular acceleration of 4.0 rad/s^2. The time it takes to make 10 revolutions is:

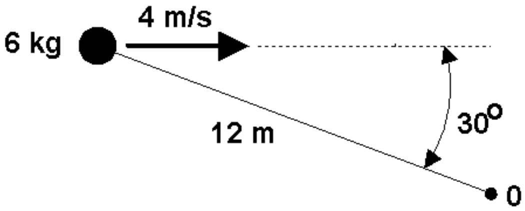
   (a) 0.50 s
   (b) 0.71 s
   (c) 5.6 s
   (d) 1.8 s
   (e) 3.1 s

5. Three identical objects of mass M are fastened to a massless rod of length L as shown. The rotational inertia about one end of the rod of this array is:



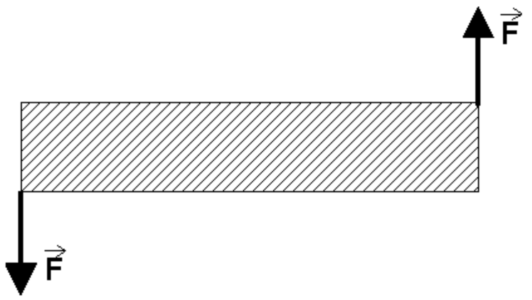
(a) $ML^2/2$
(b) ML^2
(c) $3ML^2/2$
(d) $5ML^2/4$
(e) $3ML^2$

6. A 6-kg particle moves to the right at 4 m/s as shown. Its angular momentum in kg·m^2/s about the point O is:



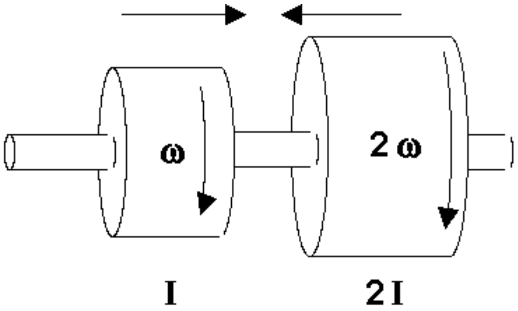
(a) zero
(b) 288
(c) 144
(d) 24
(e) 249

7. A rod rests on frictionless ice. While forces that are equal in magnitude and opposite in direction are simultaneously applied to its ends as shown, the quantity that vanishes is its:



(a) angular momentum about its center of mass
(b) angular acceleration about its center of mass
(c) linear momentum
(d) kinetic energy
(e) rotational inertia about its center of mass

8. Two disks are mounted on low friction bearings on a common shaft. The first disc has rotational inertia I and is spinning with angular velocity ω. The second disc has rotational inertia 2I and is spinning (in the same direction as the first disc) with angular velocity 2ω as shown. The two disks are slowly forced toward each other along the shaft until they couple and have a final common angular velocity of:

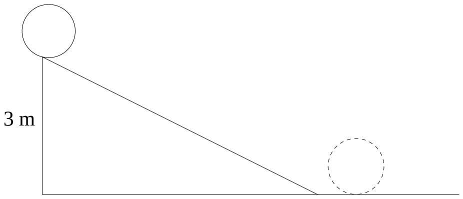


(a) $5\omega/3$
(b) $\omega\sqrt{3}$
(c) $\omega\sqrt{7/3}$
(d) ω
(e) 3ω

9. A disk with a rotational inertia of 5.0 kg·m^2 and a radius of 0.25 m rotates on a frictionless fixed axis perpendicular to the disk and through its center. A force of 2.0 N is applied tangentially to the rim. The angular acceleration of the disk is:
   (a) 0.40 rad/s^2
   (b) 0.4 rad/s^2
   (c) 1.0 rad/s^2
   (d) 2.5 rad/s^2
   (e) 0.1 rad/s^2

10. To increase the rotational inertia of a solid disk about its axis without changing its mass:
   (a) drill holes near the rim and put the material near the axis
   (b) drill holes near the axis and put the material near the rim
   (c) drill holes at points on a circle near the rim and put the material at points between the holes
   (d) drill holes at points on a circle near the axis and put the material at points between the holes
   (e) do none of the above (the rotational inertia cannot be changed without changing the mass)

11. A solid sphere of mass 2 kg starts from rest and rolls down a ramp inclined 20° from the horizontal from a height of 3 m. What is the speed of the center of mass of the sphere when it reaches the bottom?

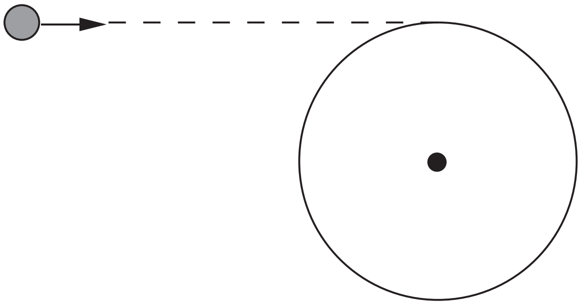


(a) 5.42 m/s
(b) 7.67 m/s
(c) 6.48 m/s
(d) 2.71 m/s
(e) need to know the radius of the disk

12. Almost the same problem as above, but now the mass of the solid sphere is 8 kg. What is the speed of the center of mass of the sphere at the bottom of the ramp?

(a) same as in the previous problem
(b) 8 times as large as before
(c) $\frac{1}{8}$ as large as before
(d) $\sqrt{8}$ times as large as before
(e) still need to know the radius of the disk

13. A particle of mass m = 0.1 kg and speed v_0 = 5 m/s collides and sticks to the end of a uniform solid cylinder of mass M = 1 kg and radius R = 0.2 m. If the cylinder is initially at rest and is pivoted about a frictionless axle through its center, what is the final angular velocity (in rad/s) of the system after the collision?



(a) 8

(b) 2

(c) 6

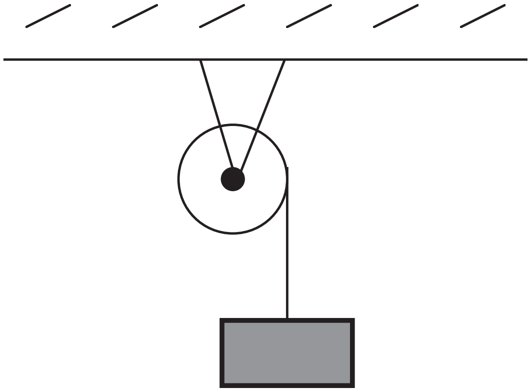
(d) 4

(e) 10

Partial Credit Section (22 points)

It would be prudent to use variables as much as possible and only substitute numbers in at the very end of each part. SHOW YOUR WORK!

A pulley consisting of a hollow cylinder (hoop) of mass 3 kg and radius 0.5 m is mounted on the ceiling. A rectangular mass 2 kg is suspended by a light cord wrapped several times around the pulley.



What is the moment of inertia of the pulley about an axis through its center of mass?

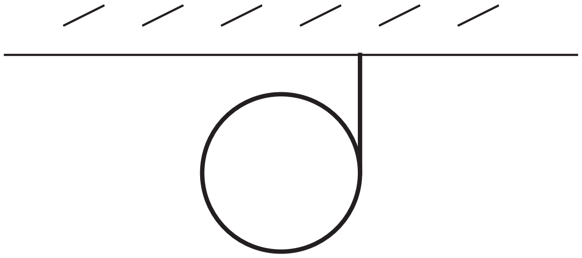
What is the acceleration of the rectangular mass?

What is the angular acceleration of the pulley about its center of mass?

What is the tension in the string?

What is the torque acting on the pulley about its center of mass?

One end of a light cord is rigidly fixed to the ceiling. The rest of the cord is wrapped several times around a hollow cylinder (hoop) of mass 10 kg and radius 10 cm which is released from rest.



What is the moment of inertia of the hoop about an axis through its center of mass?

What is the moment of inertia of the hoop about an axis through a point on its rim (where the string is attached)?

What is the speed of the center of mass of the hoop after it has fallen 1 m?

What is the speed of a point mass that falls freely from rest through 1 m?

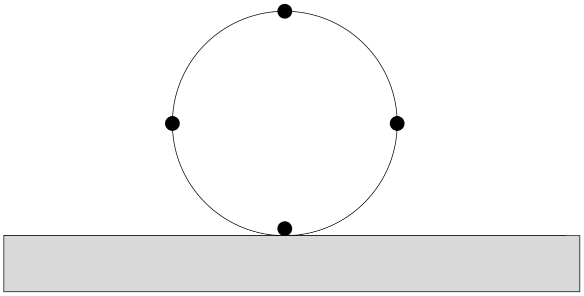
Which object's center of mass is moving faster after falling 1 m? Explain why this should be so.

Short Answer Section

If you are riding a bicycle forward, what is the direction of the angular momentum of the front wheel about its axle?

(left, right, up, down, forward, backward)

Draw velocity vector arrows representing velocity WITH RESPECT TO THE GROUND for the four points indicated on the bicycle wheel.



USEFUL FORMULÆ AND CONSTANTS

Average velocity and acceleration

$$\vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} \qquad \vec{\bar{a}} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous velocity and acceleration

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Equations for motion with a constant acceleration

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$

$$\vec{r}(t) - \vec{r}_0 = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{r}(t) - \vec{r}_0 = \tfrac{1}{2}[\vec{v}_0 + \vec{v}(t)]t$$

$$[\vec{v}(t)]^2 = \vec{v}_0^{\,2} + 2\vec{a}\cdot[\vec{r}(t) - \vec{r}_0]$$

Relative Velocity

$$\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc} \qquad \vec{v}_{ab} = -\vec{v}_{ba}$$

Radial Acceleration

$$a_r = \frac{v^2}{r}$$

Tangential Acceleration

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}(\text{speed})$$

Newton's Second Law $\vec{F} = \frac{d\vec{p}}{dt}$ **(**$= m\vec{a}$ **if the mass is <u>constant</u>)**

Derivatives and integrals of power functions

$$\frac{d}{dt}(At^n) = nAt^{n-1} \qquad \int Bt^n\,dt = \frac{B}{n+1}t^{n+1} + constant$$

Quadratic equation

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Weight $W = mg$

Friction $f_k = \mu_k N$ $f_s \leq \mu_s N$

Newton's Third Law $\vec{F}_{12} = -\vec{F}_{21}$

Centripetal Force $\sum F_r = ma_r = m\frac{v^2}{r}$

The Work-Energy Theorem

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot \vec{ds} \qquad\qquad K = \frac{1}{2}m(\vec{v}\,)^2$$

$$W_{\text{net}} = K_f - K_i = \Delta K$$

Potential Energy (for Conservative Forces)

$$\Delta U = U_f - U_i = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = -W_{\text{cons}}$$

$$F_x = -\frac{dU}{dx}$$

Some Common Forces and Their Potential Energies

Gravity	$F = -mg$	$U = mgh$
Spring	$F = -kx$	$U = \frac{1}{2}kx^2$

Conservation of Energy

$$E = K + U$$

$\Delta E = E_f - E_i = W_{\text{nc}} =$ **Work done by non-conservative forces**

$$W_{\text{nc}} = \Delta K + \Delta U$$

Dot Product

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}|\cos\theta$$

Instantaneous Power

$$P = \vec{F} \cdot \vec{v}$$

Average Power

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Momentum $\vec{p} = m\vec{v}$

Impulse $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int \vec{F}(t)\, dt$

1-D Collisions between a particle of mass $\mathbf{m}_1$ with initial velocity $\mathbf{v}_{1i}$ and a particle of mass $\mathbf{m}_2$ with initial velocity $\mathbf{v}_{2i}$:

Totally Elastic

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Totally Inelastic

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Center of Mass

$$\vec{r}_{cm} = \frac{\sum\limits_i m_i \vec{r}_i}{\sum\limits_i m_i} \qquad \vec{v}_{cm} = \frac{\sum\limits_i m_i \vec{v}_i}{\sum\limits_i m_i} \qquad \vec{a}_{cm} = \frac{\sum\limits_i m_i \vec{a}_i}{\sum\limits_i m_i}$$

Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{\boldsymbol{\imath}} + (A_z B_x - A_x B_z)\hat{\boldsymbol{\jmath}} + (A_x B_y - A_y B_x)\hat{\boldsymbol{k}}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta = |\vec{A}| B_\perp = A_\perp |\vec{B}|$$

Circular Motion:

Relation of Linear and Angular Variables

$$s = r\theta \qquad v = r\omega \qquad a_t = r\alpha$$

Rotational Kinematics – Make the following replacements in linear equations:

$$\begin{array}{ll} x \to \theta & m \to I \\ v \to \omega & F \to \tau \\ a \to \alpha & p \to L \end{array}$$

Rotational Kinetic Energy

$$K_{\mathrm{ROT}} = \tfrac{1}{2} I \omega^2$$

Work

$$W = \int_{\theta_i}^{\theta_f} \tau \; d\theta$$

Acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r \qquad a_t = \frac{d|\vec{v}|}{dt} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

Rotational Dynamics

Scalar	**Vector**
$\tau = I\alpha$	$\vec{\tau} = I\vec{\alpha}$
$\tau = Fr\sin\theta$	$\vec{\tau} = \vec{r} \times \vec{F}$
$\tau = \dfrac{dL}{dt}$	$\vec{\tau} = \dfrac{d\vec{L}}{dt}$
$L = I\omega$	$\vec{L} = I\vec{\omega}$
$L = rp\sin\theta$	$\vec{L} = \vec{r} \times \vec{p}$

Moment of Inertia

$$I = \sum_i m_i r_i^2 = \int r^2 \; dm$$

Parallel-Axis Theorem

$$I = I_{\mathrm{cm}} + MD^2$$

Physical Constants

Acceleration due to gravity (g)	$9.80\ \mathrm{m/s^2} = 32\ \mathrm{ft/s^2}$
Average earth-moon distance	3.84×10^{8} m
Average earth-sun distance	1.49×10^{11} m
Average radius of the earth	6.37×10^{6} m
Mass of the earth	5.98×10^{24} kg
Mass of the moon	7.36×10^{22} kg
Mass of the sun	1.99×10^{30} kg
Gravitational constant (G)	$6.672 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

Conversion Constants

Length

1 in. = 2.54 cm
1 m = 39.37 in. = 3.281 ft
1 ft = 0.3048 m
12 in. = 1 ft
3 ft = 1 yd
1 yd = 0.9144 m
1 km = 0.621 mi
1 mi = 1.609 km
1 Å= 10^{-10} m
1 mm = 10^{-3} m
1 μm = 10^{-6} m = 10^{4} Å
1 lightyear = 9.461×10^{15} m

Mass

1000 kg = 1 t (metric ton)
1000 g = 1 kg
1 slug = 14.59 kg
1 u = 1.66×10^{-27} kg

Energy

1 J = 0.738 ft·lb = 10^{7} erg
1 cal = 4.186 J
1 BTU = 252 cal = 1.054×10^{3} J
1 eV = 1.6×10^{-19} J
931.5 MeV = 1 u
1 kW·h = 3.6×10^{6} J

Force

1 N = 10^{5} dyne = 0.2248 lb
1 lb = 4.448 N
1 dyne = 10^{-5} N = 2.248×10^{-6} lb

Velocity

1 mi/h = 1.47 ft/s = 0.447 m/s
1 mi/h = 1.61 km/h
1 m/s = 100 cm/s = 3.281 ft/s
1 mi/min = 60 mi/h = 88 ft/s

Acceleration

1 $\mathrm{m/s^2}$ = 3.28 $\mathrm{ft/s^2}$ = 100 $\mathrm{cm/s^2}$
1 $\mathrm{ft/s^2}$ = 0.3048 $\mathrm{m/s^2}$ = 30.48 $\mathrm{cm/s^2}$

Power

1 hp = 550 ft·lb/s = 0.746kW
1 W = 1 J/s = 0.738 ft·lb/s
1 BTU/h = 0.293 W

Angle

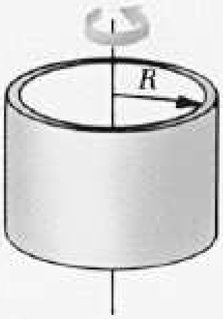
1 radian = 57.29578°
1° = 0.01745 rad

Improper Conversions

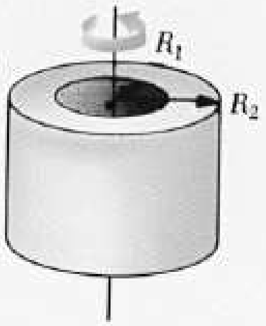
1 lb (weight) = 0.454 kg (mass) at the surface of the earth

TABLE 10.2 Moments of Inertia of Homogeneous Rigid Bodies with Different Geometries

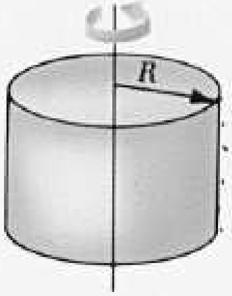
Hoop or
cylindrical shell
$I_c = MR^2$



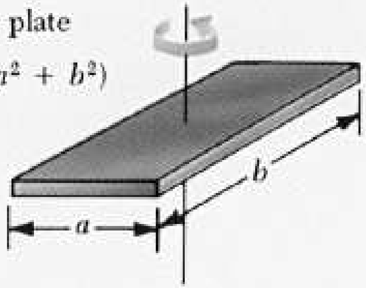
Hollow cylinder
$I_c = \frac{1}{2}M(R_1^2 + R_2^2)$



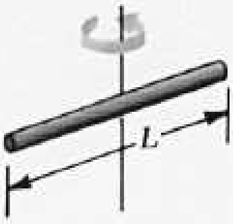
Solid cylinder
or disk
$I_c = \frac{1}{2}MR^2$



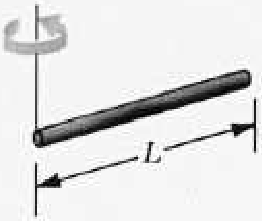
Rectangular plate
$I_c = \frac{1}{12}M(a^2 + b^2)$



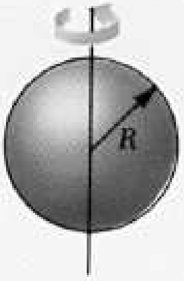
Long thin rod
$I_c = \frac{1}{12}ML^2$



Long thin rod
$I = \frac{1}{3}ML^2$



Solid sphere
$I_c = \frac{2}{5}MR^2$



Thin spherical
shell
$I_c = \frac{2}{3}MR^2$