Physics 1303(001),1403(801) **Professor Scalise**
Spring 1996 **Final Exam**

Name: ________________ ID number:________________

This exam is worth 100 points. It consists of 20 multiple-choice questions worth 4 points each and a partial credit section worth a maximum of 20 points.

Don't get hung up on the questions. They should only take a few minutes each. If you find yourself spending more than a few minutes on a question you are probably looking at it the wrong way. You should skip it temporarily and return to it later.

NOTE - The equation sheets are on the last pages. If you think that it makes referring to them easier, you can remove them from the rest of the exam.

GOOD LUCK

1. An inelastic collision is one in which:
    (a) momentum is not conserved but kinetic energy is conserved
    (b) neither momentum nor kinetic energy is conserved
    (c) momentum is conserved but kinetic energy is not conserved
    (d) both momentum and kinetic energy are conserved
    (e) kinetic energy is conserved, but potential energy is not

2. Let F_1 be the magnitude of the gravitational force exerted on the sun by the earth and F_2 be the magnitude of the force exerted on the earth by the sun. Then:
    (a) F_1 is much greater than F_2
    (b) F_1 is slightly greater than F_2
    (c) F_1 is equal to F_2
    (d) F_1 is slightly less than F_2
    (e) F_1 is much less than F_2

3. An astronomer on Earth wishes to know the mass of Jupiter. This information can be obtained by:
   (a) measuring the period and orbital radius of Jupiter about the Sun
   (b) measuring the periods of two of Jupiter's moons
   (c) measuring the orbital radius of one of Jupiter's moons and the period of a different moon
   (d) measuring the orbital radii of two of Jupiter's moons
   (e) measuring the period and orbital radius of one of Jupiter's moons

4. In simple harmonic motion, the restoring force must be proportional to the:
   (a) amplitude
   (b) frequency
   (c) velocity
   (d) displacement
   (e) displacement squared

5. A body is in equilibrium under the combined action of several forces. Then:

   (a) all the forces must be applied at the same point
   (b) all of the forces are composed of pairs of equal and opposite forces
   (c) any two of these forces must be balanced by a third force
   (d) the lines of action of all the forces must pass through the center of mass of the body
   (e) none of these

6. A 0.200-kg mass attached to a spring whose spring constant is 500 MKS units executes simple harmonic motion with amplitude 0.100 m. Its maximum speed is:

   (a) 25 m/s
   (b) 5 m/s
   (c) 1 m/s
   (d) 15.8 m/s
   (e) 0.2 m/s

7. A hoop has a mass of 0.2 kg and a radius of 0.25 m. It rolls without slipping along the ground at 5.0 m/s. Its total kinetic energy in MKS units is:

   (a) 2.5
   (b) 5
   (c) 10
   (d) 250
   (e) need to know the angular velocity

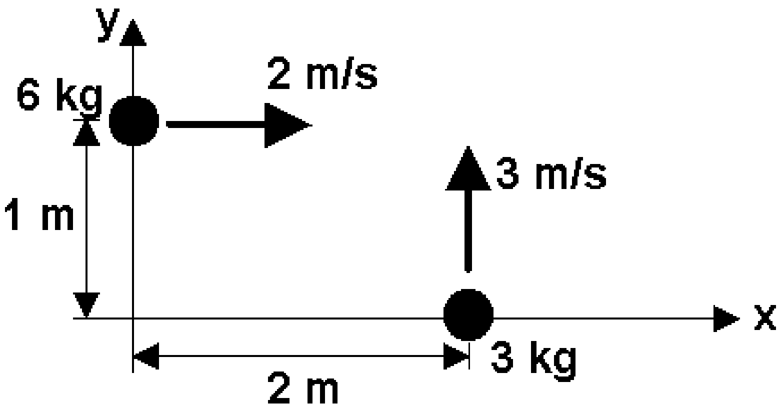
8. We may apply conservation of energy to a cylinder rolling without slipping down an incline without slipping and exclude friction because:

   (a) there is no friction present
   (b) the angular velocity of the center of mass about the point of contact is zero
   (c) the coefficient of kinetic friction is zero
   (d) the linear velocity of the point of contact, relative to the inclined surface, is zero
   (e) the coefficients of static and kinetic friction are equal

9. Two uniform cylinders have different masses and different rotational inertias. They simultaneously start from rest at the top of an inclined plane and roll without slipping down the plane. The cylinder that gets to the bottom first is:
   (a) the one with the larger mass
   (b) the one with the smaller mass
   (c) the one with the larger rotational inertia
   (d) the one with the smaller rotational inertia
   (e) neither (they arrive together)

10. If you look down on the North Pole, the Earth rotates counter-clockwise. The angular momentum vector of the earth, due to its daily rotation, is directed:
   (a) tangent to the equator toward the east
   (b) tangent to the equator toward the west
   (c) due north
   (d) due south
   (e) toward the sun

11. Two objects are moving in the x-y plane as shown. The magnitude of their total angular momentum (about the origin O) in MKS units is



(a) zero

(b) 6

(c) 12

(d) 30

(e) 78

12. A mass slides down a frictionless ramp inclined 35° to the horizontal. The acceleration of the mass is

(a) 5.6 $\mathrm{m/s^2}$

(b) 9.8 $\mathrm{m/s^2}$

(c) 8.0 $\mathrm{m/s^2}$

(d) need to know if the mass is pointlike or an extended body

(e) need to know the mass

13. A flywheel, initially at rest, has a constant angular acceleration. After 9 seconds the flywheel has rotated 450 rad. Its angular acceleration in MKS units is:

    (a) 100
    (b) 1.77
    (c) 50
    (d) 11.1
    (e) 15.9

14. Block A, with a mass of 4 kg, is stationary while block B, with a mass of 8 kg, is moving at 3 m/s. The center of mass of the two block system has a speed in m/s of:

    (a) 0
    (b) 1.5
    (c) 2
    (d) 3
    (e) 12

15. The center of mass of a system of particles obeys an equation similar to Newton's second law $\mathbf{F} = \mathrm{m}\mathbf{a}_{cm}$, where

    (a) **F** is the total internal force and m is the total mass of the system
    (b) **F** is the total internal force and m is the mass acting on the system
    (c) **F** is the total external force and m is the total mass of the system
    (d) **F** is the force of gravity and m is the mass of the earth
    (e) **F** is the force of gravity and m is the total mass of the system

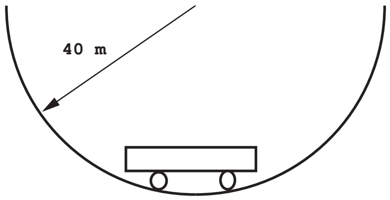
16. If the total momentum of a system is changing:

    (a) particles of the system must be exerting forces on each other
    (b) the system must be under the influence of gravity
    (c) the center of mass must have constant velocity
    (d) a net external force must be acting on the system
    (e) none of the above

17. A forward force of 3 lb is used to pull a 60-lb sled at constant velocity on a frozen pond. The coefficient of friction in MKS units is:

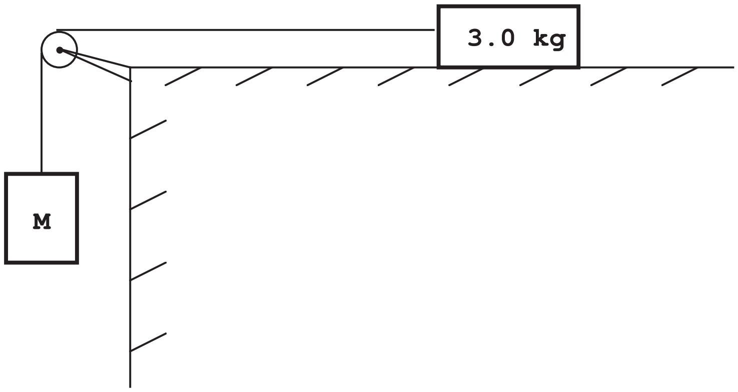
    (a) 0.5
    (b) 0.05
    (c) 2
    (d) 0.2
    (e) 20

18. A roller-coaster car has a mass of 500 kg when fully loaded with passengers. At the bottom of a circular dip of radius 40 m (as shown in the figure) the car has a speed of 16 m/s. What is the magnitude of the force of the track on the car at the bottom of the dip?



    (a) 3.2 kN
    (b) 8.1 kN
    (c) 4.9 kN
    (d) 1.7 kN
    (e) 5.3 kN

19. If M is 1 kg and the horizontal surface is frictionless, what will happen to the system?



(a) Nothing, because M is less massive than the block on the surface.
(b) The system will accelerate at a<g.
(c) The system will accelerate at 9.8m/s^2.
(d) The system will accelerate at a>g.
(e) Nothing, because the system is in stable equilibrium.

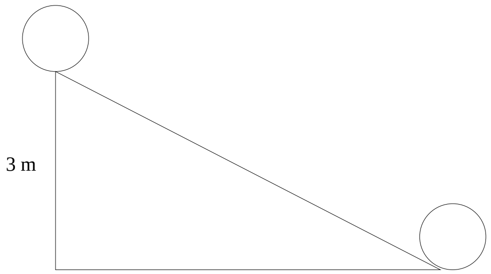
20. A 0.2 kg rubber ball is dropped from the window of a building. It strikes the sidewalk below at 30 m/s and rebounds up at 20 m/s. The magnitude of the impulse due to the collision with the sidewalk in MKS units is:

(a) 10
(b) 6.0
(c) 2.0
(d) 19.6
(e) 9.8

Partial Credit Section (20 points)

It would be prudent to use variables as much as possible and only substitute numbers in at the very end of each part. SHOW YOUR WORK!

Find the velocity of the center of mass of a hoop (hollow cylinder) after it has rolled without slipping down a ramp of height 3 m. (4 points)



A 3 kg mass moving to the right at 2 m/s collides elastically with a 9 kg mass moving to the left at 2 m/s. What are the final velocities (magnitudes **AND** directions) of the masses? (4 points)

3 kg mass: speed=________________________; direction=________________________

9 kg mass: speed=________________________; direction=________________________

An Atwood's machine consists of a 10 kg mass and an 8 kg mass connected by a massless rope over a massless, frictionless pulley. Find the tension in the rope and the acceleration of either mass. (4 points)

A stone is thrown downward at a speed of 10 m/s from the top of a 100 m tower. What is the velocity of the stone when it reaches the ground? (4 points)

Short Answer Section

What are the MKS units for the following quantities? (1 point each)

Angular Acceleration	$\vec{\alpha}$	____________
Angular Frequency	$\vec{\omega}$	____________
Angular Momentum	$\vec{L}$	____________
Coefficient of Kinetic Friction	μ_k	____________
Impulse	$\vec{I}$	____________
Kinetic Energy	K	____________
Linear Frequency	f	____________
Linear Momentum	$\vec{p}$	____________
Moment of Inertia (Rotational Inertia)	I_{cm}	____________
Potential Energy	U	____________
Power	P	____________
Spring Constant	k	____________
Torque	$\vec{\tau}$	____________
Work	W	____________

USEFUL FORMULÆ AND CONSTANTS

Average velocity and acceleration

$$\vec{\bar{v}} = \frac{\Delta \vec{r}}{\Delta t} \qquad \vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous velocity and acceleration

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Equations for motion with a constant acceleration

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$

$$\vec{r}(t) - \vec{r}_0 = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{r}(t) - \vec{r}_0 = \tfrac{1}{2}[\vec{v}_0 + \vec{v}(t)]t$$

$$[\vec{v}(t)]^2 = \vec{v}_0^{\,2} + 2\vec{a} \cdot [\vec{r}(t) - \vec{r}_0]$$

Relative Velocity

$$\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc} \qquad \vec{v}_{ab} = -\vec{v}_{ba}$$

Radial Acceleration

$$a_r = \frac{v^2}{r}$$

Tangential Acceleration

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}(\text{speed})$$

Newton's Second Law $\vec{F} = \frac{d\vec{p}}{dt}$ **(**$= m\vec{a}$ **if the mass is constant)**

Derivatives and integrals of power functions

$$\frac{d}{dt}(At^n) = nAt^{n-1} \qquad \int Bt^n \, dt = \frac{B}{n+1}t^{n+1} + constant$$

Quadratic equation

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Weight $W = mg$

Friction $f_k = \mu_k N \qquad f_s \leq \mu_s N$

Newton's Third Law $\vec{F}_{12} = -\vec{F}_{21}$

Centripetal Force $\sum F_r = ma_r = m\frac{v^2}{r}$

The Work-Energy Theorem

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot \vec{ds} \qquad\qquad K = \frac{1}{2}m(\vec{v}\,)^2$$

$$W_{\text{net}} = K_f - K_i = \Delta K$$

Potential Energy (for Conservative Forces)

$$\Delta U = U_f - U_i = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = -W_{\text{cons}}$$

$$F_x = -\frac{dU}{dx}$$

Some Common Forces and Their Potential Energies

Gravity	$F = -mg$	$U = mgh$
Spring	$F = -kx$	$U = \frac{1}{2}kx^2$

Conservation of Energy

$$E = K + U$$

$$\Delta E = E_f - E_i = W_{\text{nc}} = \textbf{Work done by non-conservative forces}$$

$$W_{\text{nc}} = \Delta K + \Delta U$$

Dot Product

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos\theta$$

Instantaneous Power

$$P = \vec{F} \cdot \vec{v}$$

Average Power

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Momentum $\vec{p} = m\vec{v}$

Impulse $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int \vec{F}(t)\, dt$

1-D Collisions between a particle of mass $\mathbf{m}_1$ with initial velocity $\mathbf{v}_{1i}$ and a particle of mass $\mathbf{m}_2$ with initial velocity $\mathbf{v}_{2i}$:

Totally Elastic

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Totally Inelastic

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Center of Mass

$$\vec{r}_{cm} = \frac{\sum\limits_i m_i \vec{r}_i}{\sum\limits_i m_i} \qquad \vec{v}_{cm} = \frac{\sum\limits_i m_i \vec{v}_i}{\sum\limits_i m_i} \qquad \vec{a}_{cm} = \frac{\sum\limits_i m_i \vec{a}_i}{\sum\limits_i m_i}$$

Cross Product

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{\boldsymbol{\imath}} + (A_z B_x - A_x B_z)\hat{\boldsymbol{\jmath}} + (A_x B_y - A_y B_x)\hat{\boldsymbol{k}}$$

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta = |\vec{A}| B_\perp = A_\perp |\vec{B}|$$

Circular Motion:

Relation of Linear and Angular Variables

$$s = r\theta \qquad v = r\omega \qquad a_t = r\alpha$$

Rotational Kinematics – Make the following replacements in linear equations:

$$\begin{array}{ll} x \to \theta & m \to I \\ v \to \omega & F \to \tau \\ a \to \alpha & p \to L \end{array}$$

Rotational Kinetic Energy

$$K_{\text{ROT}} = \tfrac{1}{2} I \omega^2$$

Work

$$W = \int_{\theta_i}^{\theta_f} \tau \; d\theta$$

Acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r \qquad a_t = \frac{d|\vec{v}|}{dt} = r\alpha = r\frac{d\omega}{dt} = r\frac{d^2\theta}{dt^2}$$

Rotational Dynamics

Scalar	Vector
$\tau = I\alpha$	$\vec{\tau} = I\vec{\alpha}$
$\tau = Fr\sin\theta$	$\vec{\tau} = \vec{r} \times \vec{F}$
$\tau = \frac{dL}{dt}$	$\vec{\tau} = \frac{d\vec{L}}{dt}$
$L = I\omega$	$\vec{L} = I\vec{\omega}$
$L = rp\sin\theta$	$\vec{L} = \vec{r} \times \vec{p}$

Moment of Inertia

$$I = \sum_i m_i r_i^2 = \int r^2 \; dm$$

Parallel-Axis Theorem

$$I = I_{\text{cm}} + MD^2$$

Simple Harmonic Oscillator

$$x(t) = A\cos(\omega t + \delta)$$

$$v(t) = -\omega A \sin(\omega t + \delta)$$

$$a(t) = -\omega^2 A\cos(\omega t + \delta) = -\omega^2 x(t)$$

Period, Frequency, and Angular Frequency

$$\omega_{\text{spring}} = \sqrt{\frac{k}{m}} \qquad \omega_{\substack{\text{simple}\\ \text{pendulum}}} = \sqrt{\frac{g}{L}} \qquad \omega_{\substack{\text{physical}\\ \text{pendulum}}} = \sqrt{\frac{mgd}{I}}$$

$$T = \frac{2\pi}{\omega} \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Gravity $F = G\frac{M_1 M_2}{r^2}$

Kepler's Third Law $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

Physical Constants

Acceleration due to gravity (g)	$9.80\ \mathrm{m/s^2} = 32\ \mathrm{ft/s^2}$
Average earth-moon distance	3.84×10^{8} m
Average earth-sun distance	1.49×10^{11} m
Average radius of the earth	6.37×10^{6} m
Mass of the earth	5.98×10^{24} kg
Mass of the moon	7.36×10^{22} kg
Mass of the sun	1.99×10^{30} kg
Gravitational constant (G)	$6.672 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

Conversion Constants

Length

1 in. = 2.54 cm
1 m = 39.37 in. = 3.281 ft
1 ft = 0.3048 m
12 in. = 1 ft
3 ft = 1 yd
1 yd = 0.9144 m
1 km = 0.621 mi
1 mi = 1.609 km
1 Å= 10^{-10} m
1 mm = 10^{-3} m
1 μm = 10^{-6} m = 10^{4} Å
1 lightyear = 9.461×10^{15} m

Mass

1000 kg = 1 t (metric ton)
1000 g = 1 kg
1 slug = 14.59 kg
1 u = 1.66×10^{-27} kg

Energy

1 J = 0.738 ft·lb = 10^{7} erg
1 cal = 4.186 J
1 BTU = 252 cal = 1.054×10^{3} J
1 eV = 1.6×10^{-19} J
931.5 MeV = 1 u
1 kW·h = 3.6×10^{6} J

Force

1 N = 10^{5} dyne = 0.2248 lb
1 lb = 4.448 N
1 dyne = 10^{-5} N = 2.248×10^{-6} lb

Velocity

1 mi/h = 1.47 ft/s = 0.447 m/s
1 mi/h = 1.61 km/h
1 m/s = 100 cm/s = 3.281 ft/s
1 mi/min = 60 mi/h = 88 ft/s

Acceleration

1 $\mathrm{m/s^2}$ = 3.28 $\mathrm{ft/s^2}$ = 100 $\mathrm{cm/s^2}$
1 $\mathrm{ft/s^2}$ = 0.3048 $\mathrm{m/s^2}$ = 30.48 $\mathrm{cm/s^2}$

Power

1 hp = 550 ft·lb/s = 0.746kW
1 W = 1 J/s = 0.738 ft·lb/s
1 BTU/h = 0.293 W

Angle

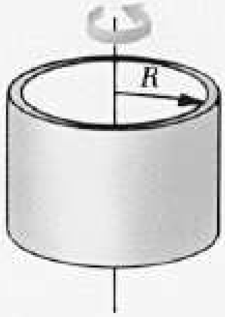
1 radian = 57.29578°
1° = 0.01745 rad

Improper Conversions

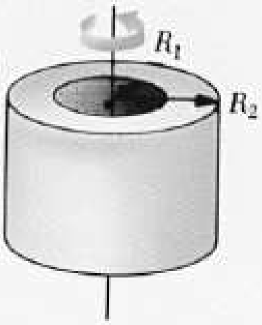
1 lb (weight) = 0.454 kg (mass) at the surface of the earth

TABLE 10.2 Moments of Inertia of Homogeneous Rigid Bodies with Different Geometries

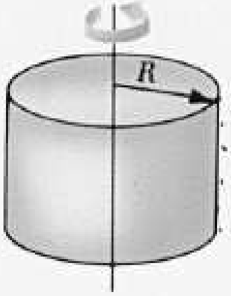
Hoop or
cylindrical shell
$I_c = MR^2$



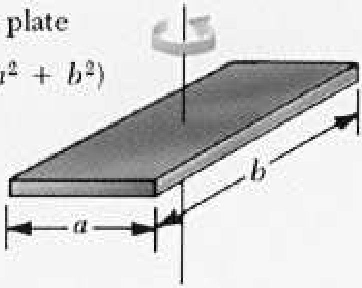
Hollow cylinder
$I_c = \frac{1}{2} M(R_1^2 + R_2^2)$



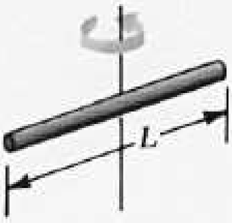
Solid cylinder
or disk
$I_c = \frac{1}{2} MR^2$



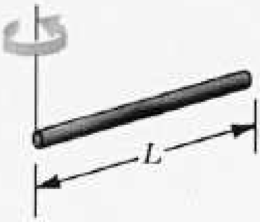
Rectangular plate
$I_c = \frac{1}{12} M(a^2 + b^2)$



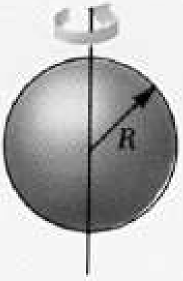
Long thin rod
$I_c = \frac{1}{12} ML^2$



Long thin rod
$I = \frac{1}{3} ML^2$



Solid sphere
$I_c = \frac{2}{5} MR^2$



Thin spherical
shell
$I_c = \frac{2}{3} MR^2$