Classical Mechanics **Professor Scalise**
Spring 2000 **Final Examination**

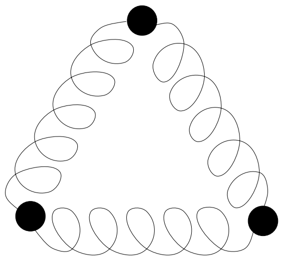
Name: ________________ ID number:________________

All students at SMU are under the jurisdiction of the Honor Code, which you have already signed a pledge to uphold upon entering the University. For this particular exam, you may not give help to nor receive help from any other student. This includes not only students currently in the examination room, but also students in other sections of this course. You may not discuss this final practicum with anyone until Tuesday at 11:00am. You are bound by the Honor Code to report any infractions to me or to the Honor Council.

Issues of academic integrity aside, there is a practical reason to avoid cheating. Let me remind you that since this course is curved, helping students in another section will lower your own grade.

Show your work on the following pages to receive partial credit. Box final answers.

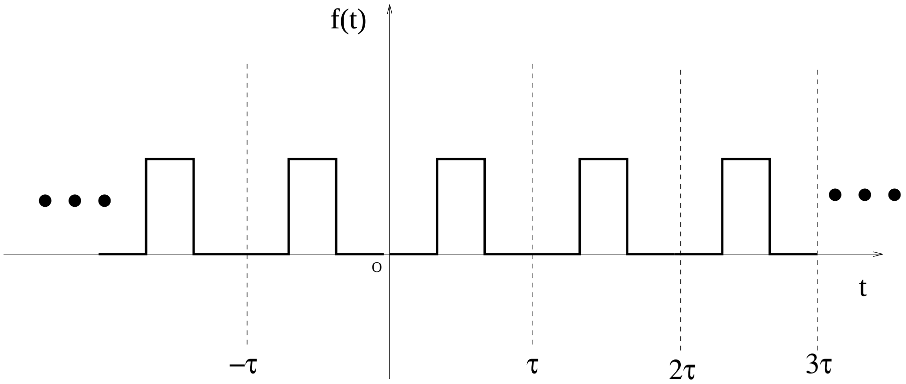
1. Three masses, each of mass m, are attached to the corners of an equilateral triangle. The sides of the triangle are formed by three identical springs, each with force constant k and natural length ℓ. Each mass is displaced the same distance directly away from the center of the triangle (so that a new, larger equilateral triangle is formed) then released. Find the natural frequency of the subsequent oscillation.



2. Dallas is at latitude +32° 47' 09". A projectile is accelerated along the barrel of a gun and leaves the barrel with some muzzle velocity. In which direction must a projectile be fired so that the initial Coriolis force acting on it just after it leaves the barrel is zero. Be numerically specific.

3. Calculate the Fourier coefficients for general summation index for the following signal, and list the first six numerically.

$$f(t) = \begin{cases} 0, & n\tau < t < (n+\frac{1}{3})\tau \\ 1, & (n+\frac{1}{3})\tau < t < (n+\frac{2}{3})\tau \\ 0, & (n+\frac{2}{3})\tau < t < (n+1)\tau \end{cases}$$



4. The suspension point of a simple pendulum of length 1 meter and mass 1 kg is driven back and forth by a motor which provides a force $3\text{mN}\sin(2\pi t)$. When you check on the system an hour after starting the motor, what is

   (a) the frequency of the oscillation?

   (b) the largest angle away from vertical to which the pendulum rises?

5. A mass m attached to a spring of force constant k is constrained to move along a frictionless (no damping) one-dimensional horizontal track. For $t < 0$, the mass is at rest and the spring is unstressed. At $t = 0$, the free end of the spring suddenly acquires a speed v_0 along the track which remains constant thereafter. What is the displacement $x(t)$?

6. Graph your effective weight in a rocket from one minute before launch to one minute after the fuel burns out. Assume a constant burn rate.

7. For each of the following differential equations, state: its order; whether it is linear or non-linear; and if linear, whether homogeneous or non-homogeneous.

   (a) $3\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 7 = 0$

   (b) $2\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 6t = 0$

   (c) $4\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 6x\sin(t) = 0$

   (d) $\dfrac{d^3x}{dt^3} = 9x$

   (e) $\dfrac{d^2y}{dt^2} + 2\beta\dfrac{dy}{dt} + \omega_0^2 y = \sin(y)$ (β and ω_0 are constants)

   (f) $7\dfrac{d^4y}{dx^4} + 5\dfrac{d^3y}{dx^3} + y^2 = 0$

   (g) $x^2 + 3\dfrac{dx}{dt} = 13$

8. A mass m is attached to a spring of force constant k and natural length ℓ. It is spun in a horizontal circle on a frictionless table with angular speed $\Omega < \sqrt{\frac{k}{m}}$.

   (a) Draw a free-body diagram in the non-inertial frame, labeling and explaining all forces clearly.

   (b) What is the equilibrium length of the spring?

   (c) What is the natural angular frequency of oscillations?

9. A simple pendulum oscillating back and forth in a plane has a period that is independent of the amplitude of oscillation for small angles. Does the period of a conical simple pendulum depend on the angle that the pendulum makes with the vertical?

10. Where on Earth (at what latitude) must you be to see a Foucault pendulum precess twice in one day?

11. Graph $\displaystyle\int_{x'=-\infty}^{x}\left[\int_{x''=-\infty}^{x'}\delta(x'')dx''\right]dx'$ vs. x.

12. The trace of a square matrix is defined to be the sum of its diagonal elements. If A is a matrix with 2 rows and 4 columns with elements a_{11}, a_{12}, etc.; and B is a matrix with 4 rows and 2 columns with elements b_{11}, b_{12}, etc.; write the trace of AB in index notation. Make sure that all sums have explicit limits.