Physics 7311 — Professor Scalise
Homework Assignment #7 — Due: 23 October 2002

18. From the expression
$$\vec{F}_{\text{sphere on q}} = -\frac{q^2 a \vec{r}\,'}{(r'^2 - a^2)^2}$$
for the force on a point charge q at position $\vec{r}\,'$ outside a grounded conducting sphere of radius a, compute directly the work done by an external agent in bringing the point charge q in from infinity without acceleration. Compare this expression for the system energy with that obtained from the general expression
$$U = \frac{1}{2}\int dV\ \rho(\vec{r})\Phi(\vec{r}) \ .$$
In the present case, what is $\rho(\vec{r})$ and what is $\Phi(\vec{r})$? Recall that you do not want to include the infinite self-energy of the point charge.

19. The potential due to a point charge q at position $\vec{r}\,'$ outside a conducting sphere of radius a held at potential Φ_0 was found in class by the method of images to be
$$\Phi(\vec{r}) = \Phi_0 \frac{a}{r} + \frac{q}{|\vec{r} - \vec{r}\,'|} - \frac{a}{r'}\frac{q}{|\vec{r} - \frac{a^2}{r'^2}\vec{r}\,'|}$$
Find this potential by using the Green function method. [Hint: Use polar coordinates to do the surface integral in the Green function formula for the potential with $\vec{r}\,'$ (the source location) on the polar axis.]

20. Suppose that the problem of determining the potential outside a closed conductor of arbitrary shape is solved in terms of an image charge density $\rho_{\text{image}}(\vec{r})$, which is non-zero only in the interior region of the conductor. Prove that the net charge induced on the outer surface of the conductor is
$$Q = \int dV\ \rho_{\text{image}}(\vec{r}) \ .$$