Physics 7311 — Professor Scalise
Homework Assignment #8 — Due: 30 October 2002

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21. In lecture, we claimed that the error

$$E_N(c_1, \ldots, c_N) \equiv \int_a^b d\xi \; |f(\xi) - f_N(\xi)|^2 = \int_a^b d\xi \; [f(\xi) - f_N(\xi)]^*[f(\xi) - f_N(\xi)]$$

made in the approximation

$$f_N(\xi) = \sum_{n=1}^{N} c_n u_n(\xi)$$

to the function $f(\xi)$ was minimized if the expansion coefficients were chosen as

$$c_n = \int_a^b d\xi \; u_n^*(\xi) f(\xi) \quad .$$

Prove this assertion. Hint: Write $c_n = a_n + ib_n$ where a_n and b_n are real constants, and $\int_a^b d\xi \; u_n^*(\xi) f(\xi) = A_n + iB_n$ where A_n and B_n are real constants. Then require that

$$\frac{\partial E_N}{\partial a_k} = 0 = \frac{\partial E_N}{\partial b_k} \quad .$$

22. Consider the first full period of the sine function:$\sin(x)$, $0 < x < 2\pi$.

(a) Expand this in a Fourier **cosine** series and list the first four Fourier coefficients. (This is not a trick question. You can expand any function outside its given range as either an even or an odd function.)

(b) Plot the original function and your four-term approximation using a computer for the range $0 < x < 2\pi$.

(c) Plot the original function and your four-term approximation using a computer for the range $-2\pi < x < 0$. Comment.