Physics 7311 — Professor Scalise

Homework Assignment #9 — Due: 6 November 2002

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23. Find the potential $\Phi(x, y, z)$ inside a parallelopiped $(0 < x < a,\ 0 < y < b,\ 0 < z < c)$ for which the x-component of the electric field is specified on the face at $x = 0$, and the others faces are grounded.

$$\begin{array}{l} E_x(0, y, z) = f(y, z) \\ \Phi(a, y, z) = 0 = \Phi(x, 0, z) = \Phi(x, b, z) = \Phi(x, y, 0) = \Phi(x, y, c) \end{array}$$

24. Mathematica^{TM} Problem: Find the potential $\Phi(x, y)$ which satisfies Laplace's equation subject to the following boundary conditions:

$$\begin{array}{l} y = 0, 0 < x < a : \Phi(x, 0) = \Phi_1(x) = V_0 \frac{x}{a} \\ x = 0, 0 < y < b : \Phi(0, y) = \Phi_2(y) = V_0 \frac{y}{b} \\ y = b, 0 < x < a : \Phi(x, b) = \Phi_3(x) = V_0(1 - \frac{x}{a}) \\ x = a, 0 < y < b : \Phi(a, y) = \Phi_4(y) = V_0(1 - \frac{y}{b}) \end{array}$$

Use the principle of superposition: Solve for the potential $\Phi_1(x, y)$ which satisfies Laplace's equation and has

$$\begin{array}{l} \Phi_1(x) = V_0 \frac{x}{a} \\ \Phi_2(y) = 0 = \Phi_3(x) = \Phi_4(y) \quad , \end{array}$$

and similarly for $\Phi_2(x, y)$, $\Phi_3(x, y)$, and $\Phi_4(x, y)$. Then the solution to the original problem is

$$\Phi(x, y) = \Phi_1(x, y) + \Phi_2(x, y) + \Phi_3(x, y) + \Phi_4(x, y) \quad .$$

You can use the symmetry in the problem to obtain one solution from another; for example

$$\Phi_1(x, y; a, b) = \Phi_2(y, x; b, a) \qquad\qquad \Phi_3(x, y; a, b) = \Phi_4(y, x; b, a) \quad .$$

Use Mathematica^{TM} with

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(continued)

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Use Mathematica™ with

$$V_0 = 3$$
$$a = 4$$
$$b = 5$$

to see if the potential looks like the minimal-energy surface that a soap film would form if stretched on the wire frame below. Try one term in the sum, then five terms, then thirty terms to see the convergence. You can also look at the partial solutions $\Phi_i(x, y)$ for $i = 1, \ldots, 4$ separately for your own amusement.

The folowing commands might be useful.

NN=30
V0=3
a=4
b=5
Phi[1][x_,y_] = Sum[??? , {n,1,NN}]
Plot3D[Phi[1][x,y], {x,0,a}, {y,0,b}, PlotRange->All]