II. Magnetostatics

A) Basic Aspects of Magnetostatics

The basic magnetic force between two moving electric charges as inferred from experiment is:

$$\vec{F}_{1\,\text{on}\,2} = q_1 q_2 \frac{\vec{v}_2}{c} \times \left[\frac{\vec{v}_1}{c} \times \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} \right]$$

This is the analogue of Coulomb's law for <u>electro</u>-statics.

q_1 and q_2 are the electric charges, $\vec{r}_1$ and $\vec{r}_2$ are their position vectors, and $\vec{v}_1$ and $\vec{v}_2$ are their velocities $\vec{v}_i = \frac{d\vec{r}_i}{dt}$.

Notice that this force law violates Newton's third law: $\vec{F}_{1\,\text{on}\,2} \neq -\vec{F}_{2\,\text{on}\,1}$!

We define the magnetic field $\vec{B}$ by

$$\vec{F}_{1\,\text{on}\,2} = q_2 \frac{\vec{v}_2}{c} \times \vec{B}_1(\vec{r}_2) \qquad \text{where}$$

$$\vec{B}_1(\vec{r}_2) = q_1 \frac{\vec{v}_1}{c} \times \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} \longleftarrow \text{source point}$$

field point

RESERVE

$\vec{B}_1(\vec{r}_2)$ is the magnetic field __at__ $\vec{r}_2$ due to the moving charge q_1 __at__ $\vec{r}_1$.

The vector magnetic force obeys the superposition principle:

$\vec{r}$ = field point
$\vec{r}_i$ = source points

$$\vec{B}(\vec{r}) = \sum_{i=1}^{N} q_i \frac{\vec{v}_i}{c} \times \frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3}$$

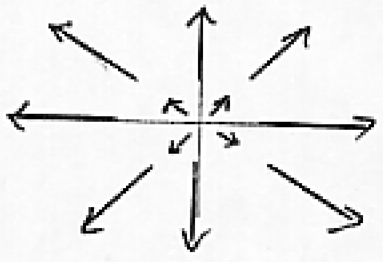
$$= -\sum_{i=1}^{N} q_i \frac{\vec{v}_i}{c} \times \vec{\nabla} \frac{1}{|\vec{r}-\vec{r}_i|}$$

We now use a vector calculus identity for a scalar field $\psi(\vec{r})$ and a vector field $\vec{w}(\vec{r})$;

$$\vec{\nabla} \times \left[\psi(\vec{r})\, \vec{w}(\vec{r})\right] = \psi(\vec{r})\, \vec{\nabla} \times \vec{w}(\vec{r}) - \vec{w}(\vec{r}) \times \vec{\nabla} \psi(\vec{r})$$

let $\psi = \frac{1}{|\vec{r}-\vec{r}_i|}$ and $\vec{w} = \vec{v}$

and notice that $\vec{\nabla} \times \vec{r} = 0$. You can prove this easily in Cartesian coordinates, or graph the vector function $\vec{r}$:



and __see__ that this field has __no curl__.

then since $\vec{v} = \frac{d\vec{r}}{dt}$, we have

$$\vec{\nabla} \times \vec{v} = 0$$

$$\vec{B}(\vec{r}) = \sum_{i=1}^{N} q_i \frac{1}{c} \vec{\nabla} \times \left(\frac{\vec{v}_i}{|\vec{r}-\vec{r}_i|} \right)$$

$$= \vec{\nabla} \times \vec{A}(\vec{r})$$

where

$$\vec{A}(\vec{r}) = \frac{1}{c} \sum_{i=1}^{N} q_i \frac{\vec{v}_i}{|\vec{r}-\vec{r}_i|}$$ is the <u>vector potential</u>

End lecture #18