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B) Magnetic Fields in Ponderable Media

Macroscopic matter is composed of molecules. These microscopic entities often possess magnetic dipole moments, due primarily to electrons. To see how this comes about, consider an atom made up of electrons orbiting the nucleus. The magnetic dipole moment is:

$$\vec{m} = \frac{1}{2c}\int dV\ \vec{r}\times\vec{J}(\vec{r})$$

with $\vec{J}(\vec{r}) = \sum_{i=1}^{N} q_i \vec{v}_i\, \delta^3(\vec{r}-\vec{r}_i) = -e\sum_{i=1}^{N} \vec{v}_i\, \delta^3(\vec{r}-\vec{r}_i)$

so that

$$\vec{m} = -\frac{e}{2c}\sum_{i=1}^{N} \vec{r}_i\times\vec{v}_i = -\frac{e}{2m_ec}\sum_{i=1}^{N} m_e\vec{r}_i\times\vec{v}_i$$

(m_e: electron mass)

$$= -\frac{e}{2mc}\vec{L}$$

where $\vec{L}$ is the total orbital angular momentum of the electrons in the atom. Electrons also possess an intrinsic angular momentum called spin (poorly viewed as the angular momentum of electronic balls of charge rotating about an axis through the center of mass — but with a mass density different from the charge density).

Experiment and Quantum Mechanics (Dirac Equation) give us

$$\vec{m}_{spin} = -g \frac{e}{2mc} \vec{S} \qquad \text{and } g = 2$$

is the gyromagnetic ratio

[Actually, when one takes into account the coupling of the electron to the electromagnetic field (quantum electrodynamics – QED) one finds that $g \neq 2$ exactly, but the difference $g-2$ is $\sim 10^{-3}$ and is readily measurable experimentally; the extra bit is called the electron's <u>anomalous</u> magnetic moment.]

$$\left.\frac{g-2}{2}\right|_{experiment} = (11\ 659.22 \pm 0.09) \times 10^{-7}$$

$$\left.\frac{g-2}{2}\right|_{theory} = (11\ 659.19 \pm 0.10) \times 10^{-7}$$

At any rate, the presence of angular momentum generally implies the existence of a magnetic dipole moment.