Physics 7311 — Professor Scalise

Homework Assignment #10 — Due: 13 November 2002

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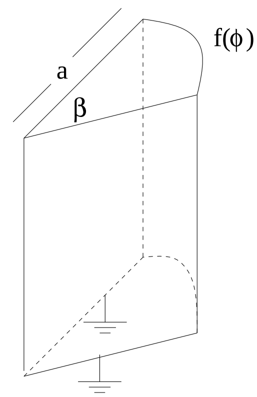
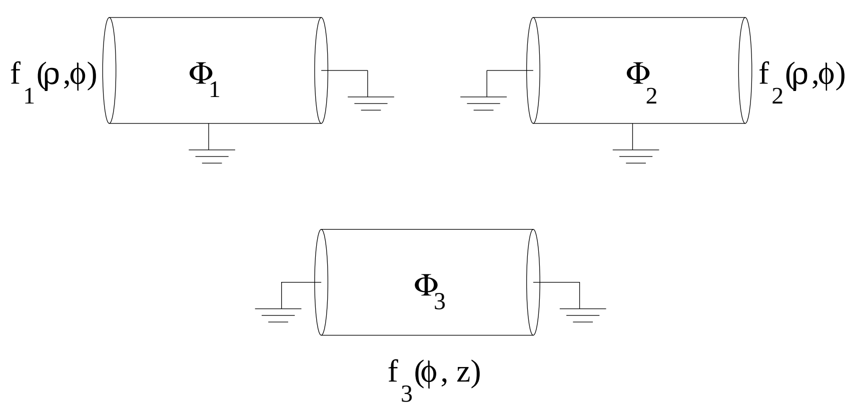
25. In lecture, we found that the potential inside an infinitely long sector of a circular cylinder of radius a and opening angle β with grounded planar faces, and Dirichlet boundary condition on the curved face $\Phi(a,\phi) = f(\phi)$ could be written as



$$\Phi(\rho,\phi) = \sum_{n=1}^{\infty} A_n \sin(\frac{n\pi\phi}{\beta})\rho^{n\pi/\beta} \quad .$$

Let $f(\phi) = V_o\frac{\phi}{\beta}$. Find the coefficients A_n by Fourier analysis of the curved face of the sector.

26. In lecture, we derived the partial solutions



$$\Phi_1(\rho,\phi,z) = \sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty} J_n\left(u_{ns}\frac{\rho}{a}\right)\left[A_{ns}\cos(n\phi) + B_{ns}\sin(n\phi)\right]\sinh\left(u_{ns}\frac{L-z}{a}\right)$$
$$\Phi_2(\rho,\phi,z) = \sum_{n=-\infty}^{\infty}\sum_{s=1}^{\infty} J_n\left(u_{ns}\frac{\rho}{a}\right)\left[A'_{ns}\cos(n\phi) + B'_{ns}\sin(n\phi)\right]\sinh\left(u_{ns}\frac{z}{a}\right)$$
$$\Phi_3(\rho,\phi,z) = \sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty} I_n\left(\frac{m\pi\rho}{L}\right)\left[A''_{nm}\cos(n\phi) + B''_{nm}\sin(n\phi)\right]\sin\left(\frac{m\pi z}{L}\right) \quad .$$

Find integral expressions for the expansion coefficients: A_{ns}, B_{ns}, A'_{ns}, B'_{ns}, A''_{nm}, and B''_{nm} in terms of the Dirichlet boundary conditions: $f_1(\rho,\phi)$, $f_2(\rho,\phi)$, and $f_3(\phi,z)$. Notice that the functions in the integrands of Φ_1 and Φ_2 are orthogonal for different n because of the orthogonality of $\sin(n\phi)$ and $\cos(n\phi)$, and orthogonal for same n and different s because of the orthogonality of the Bessel functions $J_n(u_{ns}\frac{\rho}{a})$. Therefore, the ϕ integrals must be performed first, then the ρ integrals.