

27. (a) Expand $\cos^4(\theta)$ in Legendre polynomials, $P_\ell(\cos \theta)$
(b) Expand $\cos(4\theta)$ in Legendre polynomials, $P_\ell(\cos \theta)$
28. In lecture, we introduced the orthonormal functions for a full cylinder of radius a and length L .

$$\psi_{nms}(\rho, \phi, z) = \sqrt{\frac{2}{\pi L}} \frac{1}{a J_{n+1}(u_{ns})} e^{in\phi} J_n\left(u_{ns} \frac{\rho}{a}\right) \sin\left(\frac{m\pi z}{L}\right)$$

Then we claimed that the Dirichlet Green function was

$$G_D(\vec{r}, \vec{r}') = \sum_{nms} G_{nms} \psi_{nms}^*(\rho', \phi', z') \psi_{nms}(\rho, \phi, z)$$

Show that $G_{nms} = \frac{4\pi}{\left(\frac{u_{ns}}{a}\right)^2 + \left(\frac{m\pi}{L}\right)^2}$

29. This is a practice problem for the final exam. Solve it for homework. A point charge q is placed on the positive z axis a distance d from the xy plane. The potential in this plane is specified to be

$$\Phi(x, y, 0) = V_0 e^{-(x^2+y^2)/R^2}$$

- (a) What is the charge density in the upper half space?
(b) What is the Dirichlet Green function for the upper half space?
(c) Find the potential in the upper half space. Set up the problem completely. Carry out any differentiation, but do **NOT** evaluate any integrals.