Physics 7312 Professor Scalise
Homework Assignment #6

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15. (a) Show that the column vector $\vec{v}_{(1)}$ (defined in lecture) describes an ellipse with major axis along $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and minor axis along $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ with eccentricity e. Show that the column vector $\vec{v}_{(2)}$ describes an ellipse with major axis along $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and minor axis along $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ with eccentricity e.

    (b) Show that $\vec{v}^{\dagger}_{(i)} \cdot \vec{v}_{(j)} = \delta_{ij}$.

    (c) Show that $\mathbf{R}^{\dagger}\mathbf{R} = \mathbf{I}$ and hence that $\vec{u}^{\dagger}_{(i)} \cdot \vec{u}_{(j)} = \delta_{ij}$.

    (d) What geometric figure do the vectors $\vec{u}_{(i)}$ describe?

    (e) Write the density matrix ρ as a single matrix in terms of P, the polarization.

16. (a) Can the matrix $\begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix}$ be a density matrix?

    (b) Consider the following density matrices:

$$\rho_{\alpha} = \begin{pmatrix} +\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \end{pmatrix} \qquad \rho_{\beta} = \begin{pmatrix} +\frac{1}{4} & +\frac{i}{4} \\ -\frac{i}{4} & +\frac{3}{4} \end{pmatrix} .$$

    Find the eigenvalues, eigenvectors, polarizations, and the nature of the two independent polarization states for each of the density matrices. (You may find the previous problem useful for this.)