Physics 7312 Professor Scalise
Homework Assignment #9

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21. Show that if A_ν is an arbitrary covariant 4-vector and B^μ is a contravariant 4-vector, and if
$$C^{\mu\nu} A_\nu = B^\mu$$
then $C^{\mu\nu}$ is a contravariant 4-tensor.

22. Simplify as much as possible:
$g^{\mu\nu}$ is the particle (or East Coast) metric tensor and $F^{\mu\nu}$ is the field strength tensor.
    (a) g^{11}
    (b) g_1^1
    (c) g_μ^μ
    (d) $\sum_{\mu=0}^{3} g^{\mu\mu}$
    (e) $g_{\mu\nu} F^{\mu\nu}$
    (f) $F^{\mu\nu} F_{\nu\alpha} F^{\alpha\beta} g_{\beta\mu}$
    (g) $F^{\mu\nu} F_{\nu\alpha} F^{\alpha\beta} F_{\beta\mu}$

23. (a) Maxwell's Equations can be written in covariant form as
$$\partial_\mu F^{\mu\nu} = \tfrac{4\pi}{c} J^\nu$$
$$\partial_\mu \mathcal{F}^{\mu\nu} = 0$$
where the dual field strength tensor is defined by
$$\mathcal{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$
Show explicitly how these equations relate to the more familiar 3-vector Maxwell Equations in vacuum with sources.

    (b) Show that $\partial_\mu \mathcal{F}^{\mu\nu} = 0$ is equivalent to $\partial^\alpha F^{\mu\nu} + \partial^\mu F^{\nu\alpha} + \partial^\nu F^{\alpha\mu} = 0$