Complete three (and only three) of the following four problems, showing all your work. For each problem, state the system of units in which you are working.

1. An infinite cylindrical wire of radius a carries a total current I in the $+\hat{z}$ direction. The current density, given in cylindrical coordinates (s, ϕ, z), is $\vec{J}(\vec{r}) = k\sqrt{s/a}\ \hat{z}$ for $s \leq a$ and $\vec{J}(\vec{r}) = 0$ for $s > a$, where k is a constant. Solve for k and find the magnetic field at *all points* in space.

2. (a) An insulating sphere of radius a has a uniform volume charge density ρ except for a spherical cavity of radius $b(b < a)$ whose center is located at position vector $\vec{c}$, where $(0 \leq |\vec{c}| \leq a - b)$, with respect to the center of the larger sphere. What is the electrostatic field everywhere inside the cavity?

   (b) What are the defining properties of the Dirichlet Green function?

   (c) What are the defining properties of the Neumann Green function?

   (d) What is the Dirichlet Green function for the interior of a hemispherical cavity of radius $r = a$ and $z > 0$ completely surrounded by a grounded conductor?

   (e) Use the Dirichlet Green function to find the electrostatic potential inside an empty hemisphere in which the curved surface is at potential zero while the plane surface is held at potential V_o.

3. Two infinite wires of linear charge densities $\pm\lambda$ lie parallel to the z-axis along the lines $x = \pm a, y = 0$.

   (a) What is the volume charge density $\rho(\vec{r})$ in cylindrical polar coordinates?

   (b) What is the electrostatic potential everywhere, with the choice $\Phi(\vec{r}) = 0$ on the yz-plane?

   (c) The equipotential surfaces are circular cylinders. What are the radius and coordinates of the center of the circle at potential $V_o > 0$?

   (d) If an infinite wire of linear charge density λ lies parallel to and a distance b from the axis of an infinite conducting cylinder of radius $R(R < b)$ held at potential V_o, what are the magnitude and location of the image volume charge density for the exterior problem?

   (e) Is the wire attracted to or repelled from the conducting cylinder? What is the force per unit length?

4. Consider two tiny metal spheres separated by a distance d and connected by a thin wire so that a current may flow from one sphere to the other. The upper sphere has a time-dependent charge $+q(t)$ and the lower sphere has a time-dependent charge $-q(t)$, where $q(t) = q_0 \cos(\omega t)$. This system is an *oscillating electric dipole,* $\vec{p}(t) = p_0 \cos(\omega t)\hat{z}$, where $p_0 = q_0 d$, the origin of coordinates is between the spheres, and $\hat{z}$ points from the lower sphere to the upper sphere.

   At field point $\vec{r}$ and frequency of oscillation satisfying the conditions $d << \frac{c}{\omega} << |\vec{r}|$ (called the "radiation zone"), the scalar and vector potentials are approximately

$$V(r, \theta, t) = -\frac{p_0 \omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega\left(t - \frac{r}{c}\right)\right]$$

   and

$$\vec{A}(r, \theta, t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin\left[\omega\left(t - \frac{r}{c}\right)\right]\hat{z}.$$

   In the radiation zone,

   (a) Calculate the time-dependent electric and magnetic fields at point $\vec{r}$.

   (b) Next calculate the Poynting vector at point $\vec{r}$.

   (c) Finally, average the Poynting vector over one complete cycle.