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Extended Hamiltonian Formalism of the Pure Space-Like Axial Gauge Schwinger Model II

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Abstract

Canonical methods are not sufficient to properly quantize space-like axial gauges. In this paper, we obtain guiding principles which allow the construction of an extended Hamiltonian formalism for pure space-like axial gauge fields. To do so, we clarify the general role residual gauge fields play in the space-like axial gauge Schwinger model. In all the calculations we fix the gauge using a rule, $n \cdot A = 0$, where n is a space-like constant vector and we refer to its direction as x_-. Then, to begin with, we construct a formulation in which the quantization surface is space-like but not parallel to the direction of n. The quantization surface has a parameter which allows us to rotate it, but when we do so we keep the direction of the gauge field fixed. In that formulation we can use canonical methods. We bosonize the model to simplify the investigation. We find that the antiderivative, $(\partial_-)^{-1}$, is ill-defined whatever quantization coordinates we use as long as the direction of n is space-like. We find that the physical part of the dipole ghost field includes infrared divergences. However, we also find that if we introduce residual gauge fields in such a way that the dipole ghost field satisfies the canonical commutation relations, then the residual gauge fields are determined so as to regularize the infrared divergences contained in the physical part. The propagators then take the form prescribed by Mandelstam and Leibbrandt. We make use of these properties to develop guiding principles which allow us to construct consistent operator solutions in the pure space-like case where the quantization surface is parallel to the direction of n and canonical methods do not suffice.

§1. Introduction

In a previous paper, which is hereafter referred to as I,[1] we constructed an extended Hamiltonian formalism with which we found a family of solutions to the Schwinger model. The solutions were of the axial or temporal gauge type. To consider the problem generally, we specified the gauge fixing direction by the constant vector $n^\mu = (n^0, n^3) = (\cos\theta, \sin\theta)$. At the same time we introduced $+-$-coordinates $x^\mu = (x^+, x^-)$, where

$$x^+ = x^0 \sin\theta + x^3 \cos\theta, \quad x^- = x^0 \cos\theta - x^3 \sin\theta \tag{1·1}$$

With those definitions, the gauge fixing condition

$$A_- = n{\cdot}A = A_0 \cos\theta - A_3 \sin\theta = 0 \tag{1·2}$$

is that of an axial or temporal gauge. In our formulation, the temporal and axial gauges in ordinary coordinates correspond, respectively, to $\theta = 0$ and $\theta = \frac{\pi}{2}$, while the light-front formulation corresponds to $\theta = \frac{\pi}{4}$. We found that in the region $0 \leqq \theta < \frac{\pi}{4}$, x^- should be taken as the evolution parameter and we constructed the canonical temporal gauge solutions. In that case, we found that there exist residual gauge fields which depend only on x^+. These residual gauge fields are therefore static canonical variables. By continuation, we obtained an operator solution in the axial region, $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, where x^+ should be taken as the evolution parameter. In that case, we find that there are infrared divergences associated with the physical degrees of freedom. These infrared divergences are regularized by the residual gauge fields. Among other results, we found that the Hamiltonian for the residual gauge fields must be calculated by integrating the divergence equation of the energy-momentum tensor over a suitable closed surface. Because the residual gauge fields do not depend on the initial value surface, x^-, the ($x^- \to \pm\infty$) contributions from these fields have to be kept.[2] In that way, we obtained the Hamiltonian, which includes a part from integrating a density involving the residual gauge fields over $x^- =$ constant.

In I, we found the solutions in the axial gauge region only by continuation from the temporal gauge region. In this paper we consider the problem of finding the axial gauge solutions directly; by quantizing on the surface $x^+ = 0$. This axial gauge formulation involves constrained fields and traditionally these constrained fields are eliminated in terms of physical degrees of freedom. That elimination requires that we introduce antiderivatives which can introduce infrared divergences.[3] In spite of extensive studies,[4] overcoming the infrared difficulties has remained as an open issue. In the present work we find that the residual gauge fields are essential to controlling the infrared divergences. These fields may be viewed as integration constants associated with solving the constraint equations and they are necessary to give the correct prescription for the required antiderivatives. Quantizing the residual gauge fields is itself an interesting subject. This is because they depend on the evolution parameter in such a way that they cannot be canonical variables. A first step in this direction was made by McCartor and Robertson[5)] in the light-front formulation of free abelian gauge fields.

To begin with, we consider a generalization of the models considered in I. We shall keep the constant vector at the fixed space-like direction and take the quantization surface to be space like, but will not have the constant vector lie parallel to the quantization surface. In that framework we can implement the canonical procedure. We then use the operator solutions found in such cases to clarify the dependence of the operator solutions on the quantization coordinates. We find that there are the residual gauge fields allowed by the fixed gauge choice and we can also use the operator solutions to clarify the general roles the residual gauge fields play in these axial gauge solutions. To implement these ideas we introduce another set of coordinates, $x^\mu = (x^\tau, x^\sigma)$, defined by

$$x^\tau = x^0 \sin\phi + x^3 \cos\phi, \quad x^\sigma = x^0 \cos\phi - x^3 \sin\phi. \tag{1·3}$$

In these coordinates the gauge fixing condition and the constant vector are expressed, respectively, as

$$A_- = \sin(\phi - \theta) A_\tau + \cos(\phi - \theta) A_\sigma = 0, \tag{1·4}$$

$$n^\mu = (n^\tau, n^\sigma) = (\sin(\phi - \theta), \cos(\phi - \theta)). \tag{1·5}$$

To simplify our investigation, we bosonize the Schwinger model and avoid quantizing the coupled system of fermi fields and gauge fields. The solutions contain a dipole ghost field, X, which contains both physical fields and residual gauge fields. In the space-like formulations where the constant vector is not parallel to the quantization surface we can employ A_σ and the dipole ghost field X as canonical variables and construct a canonical formulation without encountering any of the difficulties inherent in the pure space-like (PSL) axial gauge formulations where the constant vector is proportional to the quantization surface. We show that the physical part of X is uniquely determined by the gauge choice, while the residual gauge part, which reveals manifest quantization coordinate dependence, is determined by requiring that X satisfy the canonical commutation conditions. It turns out that $(n{\cdot}\partial)^{-1} = (\partial_-)^{-1}$ is ill-defined irrespective of the quantization coordinates as long as the gauge fixing direction is space-like ($n^2 < 0$). It follow from this that the physical part of X gives rise to infrared divergences irrespective of the quantization coordinates as long as the gauge fixing direction is space-like. However, if we introduce the residual gauge fields in such a way that X

satisfies the canonical commutation conditions, then the residual gauge part automatically regularizes the infrared divergences resulting from the physical part. As a consequence, the x^τ-time ordered propagator for X takes the form prescribed by Mandelstam[6)] and by Leibbrandt[7)] (ML prescription). In this way we see that the residual gauge degrees of freedom are indispensable to formulate the space-like axial gauge Schwinger model in a way that is free from infrared divergences.

We remark here that canonical formulations in ordinary coordinates were constructed for the case $n^2 = 0$ by Bassetto et al[8)] and for the case $n^0 \neq 0,\ n^2 < 0$ by Lazzizzera.[9)] These authors showed that to implement the ML prescription and to regularize the infrared divergences, residual gauge fields are indispensable.

Having found the solutions to the axial gauge formulations in the cases where the constant vector is not parallel to the the quantization surface, we turn to the the pure space-like case. The PSL case cannot be reached by taking the limit $\phi \to \theta$. This reflects the fact that we cannot construct the canonical formulation in the PSL case because residual gauge fields cannot be canonical fields; only X and its conjugate remain as unconstrained canonical fields. We circumvent this difficulty by using the properties of the dipole ghost fields found in § 2 as guiding principles. We show that operator solutions can be constructed by following these guiding principles. When these operator solutions are constructed they agree with ones given in I.

The paper is organized as follows: In § 2, we bosonize the space-like axial gauge Schwinger model and construct the canonical formulation in $\tau\sigma$-coordinates. In § 3 we show that our canonical formulation is free from infrared difficulties. In § 4, we carry out the quantization of the PSL case and construct the solution. Section 5 is devoted to concluding remarks.

In this paper we keep ϕ in the axial region $\frac{\pi}{4} < \phi \leqq \frac{\pi}{2}$ and use the following conventions in $\tau\sigma$-coordinates:

$$g_{\tau\tau} = -\cos 2\phi, \quad g_{\sigma\tau} = g_{\tau\sigma} = \sin 2\phi, \quad g_{\sigma\sigma} = \cos 2\phi,$$

$$g^{\tau\tau} = -\cos 2\phi, \quad g^{\sigma\tau} = g^{\tau\sigma} = \sin 2\phi, \quad g^{\sigma\sigma} = \cos 2\phi,$$

$$\gamma^0 = \sigma_1,\ \gamma^3 = i\sigma_2,\ \gamma^5 = -\sigma_3,$$

$$\gamma^\tau = \gamma^0 \sin\phi + \gamma^3 \cos\phi, \quad \gamma^\sigma = \gamma^0 \cos\phi - \gamma^3 \sin\phi.$$

§2. Equivalent bosonization of space-like axial gauge Schwinger model

2.1. *Field equation of the dipole ghost field*

The space-like axial gauge Schwinger model is defined by the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - B(n{\cdot}A) + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi \tag{2·1}$$

where B is the Nakanishi-Lautrup field in noncovariant formulations.[10] From the Lagrangian we derive the field equations

$$\partial_{\mu}F^{\mu\nu} = n^{\nu}B + J^{\nu}, \quad J^{\nu} = e\bar{\psi}\gamma^{\nu}\psi \tag{2·2}$$

$$i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi = 0, \tag{2·3}$$

and the gauge fixing condition (1·4). The field equation of B,

$$(n{\cdot}\partial)B = \partial_{-}B = (\sin(\phi-\theta)\partial_{\tau} + \cos(\phi-\theta)\partial_{\sigma})B = 0, \tag{2·4}$$

follows from operating on (2·2) with ∂_{ν}.

Let's first obtain the field equation of the dipole ghost field X. We now know that consistent operator solutions of Schwinger model[11] can be constructed by regularizing the vector current by means of the gauge invariant point-splitting procedure.[12] We will therefore regularize J^{μ} in the same manner in the present paper. With that regularization, the vector current is given by

$$J^{\mu} = j^{\mu} - m^2A^{\mu} \tag{2·5}$$

where $m^2 = \frac{e^2}{\pi}$ and j^{μ} is the part given as the bilinear product of the ψ. We now observe that Eq.(2·3) is massless; that is, j^{μ} satisfies $\varepsilon^{\mu\nu}\partial_{\mu}j_{\nu} = 0$. Therefore we can define j^{μ} as the gradient of the dipole ghost field X:

$$j_{\mu} = m\partial_{\mu}X. \tag{2·6}$$

Substituting (2·6) into (2·5) and then using current conservation, $\partial_{\mu}J^{\mu} = 0$, we obtain

$$m\Box X = m^2\partial^{\mu}A_{\mu}. \tag{2·7}$$

Substituting (2·5), (2·6) and (2·7) into (2·2) we get

$$(\Box + m^2)(A^{\nu} - \frac{1}{m}\partial^{\nu}X) = n^{\nu}B. \tag{2·8}$$

Finally, operating with n_{ν} on (2·8) and using $n{\cdot}A = 0$ we derive the field equation of the dipole ghost field X

$$(\Box + m^2)(\partial_{-}X) = -mn^2B. \tag{2·9}$$

2.2. *Bosonization of the generalized axial gauge Schwinger model*

Now we can employ Eqs.(2·7) and (2·8) as a guiding principles to obtain the Lagrangian for the equivalent bosonized model. These equations are derived from

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - B(n{\cdot}A) + \frac{1}{2}\partial_\mu X\partial^\mu X - m\partial_\mu XA^\mu + \frac{m^2}{2}A_\mu A^\mu, \tag{2·10}$$

which justifies the use of (2·10) as the Lagrangian in the present variables. From this Lagrangian, we see that in the axial region, $\pi/4 < \phi \leqq \pi/2$, where x^τ is chosen as the evolution parameter, the fundamental fields are A_σ and X. A_τ is a dependent field as long as $\phi \neq \theta$.

Canonical conjugate momenta are found from the Lagrangian to be

$$\pi^\tau = 0,\ \ \pi^\sigma = F_{\tau\sigma},\ \ \pi_B = 0,\ \ \pi_X = \partial^\tau X - mA^\tau. \tag{2·11}$$

Therefore we can choose A_σ, X, π^σ and π_X as independent canonical variables and express the dependent degrees of freedom as

$$A_\tau = \cot(\theta - \phi)A_\sigma,\ B = (\partial_\sigma\pi^\sigma - m\pi_X)/n^\tau. \tag{2·12}$$

Consequently, equal x^τ-time canonical quantization conditions can be imposed on the independent canonical variables; the nonvanishing commutators are

$$[A_\sigma(x), \pi^\sigma(y)] = i\delta(x^\sigma - y^\sigma),\ \ [X(x), \pi_X(y)] = i\delta(x^\sigma - y^\sigma). \tag{2·13}$$

For later convenience we give here the equal x^τ-time commutation relations of B:

$$\begin{gathered}[\pi^\sigma(x), B(y)] = [\pi_X(x), B(y)] = [B(x), B(y)] = 0,\\ [X(x), B(y)] = -i\frac{m}{n^\tau}\delta(x^\sigma - y^\sigma),\ \ [A_\sigma(x), B(y)] = -\frac{i}{n^\tau}\partial_\sigma\delta(x^\sigma - y^\sigma).\end{gathered} \tag{2·14}$$

2.3. *Expression of the dipole ghost field*

Now that the canonical formulation is given, we proceed to solving Eq.(2·8). To obtain a particular solution, we make use of the fact that, due to (2·4), B satisfies

$$(\Box + m^2)B = \left(m^2 - \frac{n^2\partial_\sigma^{\,2}}{\sin^2(\phi - \theta)}\right)B = (m^2 - n^2\partial_+^2)B. \tag{2·15}$$

Here and in what follows, we denotes, for brevity, $-\frac{\partial_\sigma}{\sin(\phi-\theta)}$ as ∂_+ when it is applied to operators dependent on only x^+. It follows immediately that a particular solution to equation (2·8), for the quantity $A^\nu - \frac{1}{m}\partial^\nu X$, is $\frac{n^\mu}{m^2-n^2\partial_+^2}B$.

To specify the remaining homogeneous part, which satisfies the free D'Alembert's equation of mass m, we take account of the fact that $F_{\tau\sigma}$ is gauge invariant and satisfies $F_{\tau\sigma} = F_{+-}$. We see from this that $F_{\tau\sigma}$ is independent of the quantization coordinates and therefore agrees with one given by the solution in I

$$F_{\tau\sigma} = m\tilde{\Sigma} + \frac{n^2}{m^2 - n^2\partial_+^2}\partial_+ B \tag{2·16}$$

where $\tilde{\Sigma}$ is the Schwinger field of mass m. We can easily see that (2·16) can be derived from the following expression for $A^\nu - \frac{1}{m}\partial^\nu X$:

$$A^\mu - \frac{1}{m}\partial^\mu X = \frac{n^\mu}{m^2 - n^2\partial_+^2}B - \varepsilon^{\mu\nu}\frac{\partial_\nu\tilde{\Sigma}}{m}. \tag{2·17}$$

where $\varepsilon^{\tau\sigma} = -\varepsilon^{\sigma\tau} = 1, \varepsilon^{\tau\tau} = \varepsilon^{\sigma\sigma} = 0$.

It is useful to point out here that the right hand side of (2·17) can be written in the following, divergence free, form

$$\frac{n^\mu}{m^2 - n^2\partial_+^2}B - \varepsilon^{\mu\nu}\frac{\partial_\nu\tilde{\Sigma}}{m} = -\frac{1}{m}\varepsilon^{\mu\nu}\partial_\nu\lambda \tag{2·18}$$

where

$$\lambda = \tilde{\Sigma} - \frac{mn^\tau}{m^2 - n^2\partial_+^2}\partial_\sigma^{-1}B \tag{2·19}$$

and the operator ∂_σ^{-1} is defined by

$$(\partial_\sigma)^{-1}f(x) = \frac{1}{2}\int_{-\infty}^{\infty} dy^\sigma \varepsilon(x^\sigma - y^\sigma) f(x^\tau, y^\sigma) \tag{2·20}$$

which imposes, in effect, the principal value regularization. It follows from (2·5), (2·6), (2·17) and (2·18) that A^μ and J^μ can be written as

$$A^\mu = \frac{1}{m}(\partial^\mu X - \varepsilon^{\mu\nu}\partial_\nu\lambda), \quad J^\mu = m\varepsilon^{\mu\nu}\partial_\nu\lambda. \tag{2·21}$$

We can now verify that $\tilde{\Sigma}$ and $\partial^\tau\tilde{\Sigma}$ satisfy canonical equal x^τ-time commutation relations

$$[\tilde{\Sigma}(x), \tilde{\Sigma}(y)] = [\partial^\tau\tilde{\Sigma}(x), \partial^\tau\tilde{\Sigma}(y)] = 0, \quad [\tilde{\Sigma}(x), \partial^\tau\tilde{\Sigma}(y)] = i\delta(x^\sigma - y^\sigma), \tag{2·22}$$

$$[B(x), \tilde{\Sigma}(y)] = [B(x), \partial^\tau\tilde{\Sigma}(y)] = 0 \tag{2·23}$$

by using their expressions in terms of the canonical variables:

$$\tilde{\Sigma} = \frac{1}{m}(\pi^\sigma - \frac{n^2}{m^2 - n^2\partial_+^2}\partial_+ B), \quad \partial^\tau\tilde{\Sigma} = \partial_\sigma X - mA_\sigma + \frac{mn_\sigma}{m^2 - n^2\partial_+^2}B. \tag{2·24}$$

Let's next obtain an expression for X. To this aim we multiply (2·17) by n_μ and use $n{\cdot}A = A_- = 0$ and $n{\cdot}\partial = \partial_-$. We then get

$$\partial_- X = -\frac{mn^2}{m^2 - n^2\partial_+^2}B + \varepsilon^{\mu\nu} n_\mu \partial_\nu \tilde{\Sigma} = -\frac{mn^2}{m^2 - n^2\partial_+^2}B + \partial^+ \tilde{\Sigma} \qquad (2{\cdot}25)$$

and see that X is obtained by integrating (2·25) with respect to x^-. The first term has to be carefully integrated. At first sight it seems that a linear function of x^- is included because the first term depends on only x^+. However it turns out that if X has such term, then the equal x^τ-time canonical commutation relations of X are not satisfied. We use the possibility of adding arbitrary functions of x^+ to write the integral of the first term as $-\frac{x^\tau}{n^\tau}\frac{mn^2}{m^2-n^2\partial_+^2}B$. To integrate the second term, we make use of the antiderivative $(\partial_-)^{-1}$ defined by

$$\frac{1}{\partial_-}\tilde{\Sigma} = -\frac{n^\tau \partial^\tau + n_\sigma \partial_\sigma}{m^2 \sin^2(\phi - \theta) - n^2 \partial_\sigma^2}\tilde{\Sigma}. \qquad (2{\cdot}26)$$

We can show that (2·26) is correct by operating on both sides with $\partial_- = n^\tau \partial_\tau + n^\sigma \partial_\sigma = \frac{n^\tau \partial^\tau - n_\sigma \partial_\sigma}{-\cos 2\phi}$ and using mass shell condition $\{(\partial^\tau)^2 - \partial_\sigma^2 - \cos 2\phi m^2\}\tilde{\Sigma} = 0$. We thus obtain the general solution which we write in the form

$$X = -\frac{x^\tau}{n^\tau}\frac{mn^2}{m^2 - n^2\partial_+^2}B + \frac{\partial^+}{\partial_-}\tilde{\Sigma} + \text{integration constant}.$$

The integration constant is determined in the following way: To obtain the first commutation relation in the second line of (2·14), we need an operator which does not commute with B; that is because B commutes with both $\tilde{\Sigma}$ and $\partial^\tau \tilde{\Sigma}$ as seen in (2·23) and so with $\frac{\partial^+}{\partial_-}\tilde{\Sigma}$, which is described as

$$\frac{\partial^+}{\partial_-}\tilde{\Sigma} = -\frac{m^2 n^\tau n_\sigma + n^2 \partial^\tau \partial_\sigma}{m^2 \sin^2(\phi - \theta) - n^2 \partial_\sigma^2}\tilde{\Sigma}. \qquad (2{\cdot}27)$$

Therefore we must introduce another field, C, which depends on only x^+. To obtain the relation $[X(x), X(y)] = 0$ when $x^\tau = y^\tau$, we need an extra term. That is because it is natural to assume that C commutes with $\tilde{\Sigma}$ and $\partial^\tau \tilde{\Sigma}$, and because the commutator $[\frac{\partial^+}{\partial_-}\tilde{\Sigma}(x), \frac{\partial^+}{\partial_-}\tilde{\Sigma}(y)]$ does not vanish when $x^\tau = y^\tau$. We find that if we parameterize the integration constant in the form

$$X = \frac{\partial^+}{\partial_-}\tilde{\Sigma} + \frac{m}{m^2 - n^2\partial_+^2}\left(C - \frac{n^2 x^\tau}{n^\tau}B + \frac{n^2 n_\sigma}{m^2 \sin^2(\phi - \theta) - n^2\partial_\sigma^2}\partial_\sigma B,\right) \qquad (2{\cdot}28)$$

then the canonical commutation conditions yield the following equal x^τ-time commutation relations for C:

$$[C(x), C(y)] = 0, \;\; [B(x), \frac{1}{m^2 - n^2\partial_+^2}C(y)] = i\frac{1}{n^\tau}\delta(x^\sigma - y^\sigma),$$
$$[C(x), \tilde{\Sigma}(y)] = [C(x), \partial^\tau \tilde{\Sigma}(y)] = 0. \qquad (2{\cdot}29)$$

Substituting (2·28) into (2·17) then yields an explicit expression for A_μ:

$$A_\mu = \varepsilon_{\mu\nu} n^\nu \frac{m}{\partial_-} \tilde{\Sigma} + \frac{n_\mu}{m^2 - n^2 \partial_+^2} B$$
$$+ \frac{\partial_\mu}{m^2 - n^2\partial_+^2} \left(C - \frac{n^2 x^\tau}{n^\tau} B + \frac{n^2 n_\sigma}{m^2 \sin^2(\phi - \theta) - n^2 \partial_\sigma^2} \partial_\sigma B \right). \tag{2·30}$$

In this way we see that the residual gauge fields are indispensable to preserve the canonical commutation relations and that the residual gauge part of X must include an explicit dependence on the quantization coordinates. We close this subsection by pointing out how infrared divergences appear in our formulation. As is seen from (2·30), the inverse of the operator $m^2 \sin^2(\phi - \theta) - n^2 \partial_\sigma^2$ is applied to both $\tilde{\Sigma}$ and to the residual gauge fields. This inverse operator gives rise to infrared divergences because $n^2 = \cos 2\theta < 0$ in the range $\frac{\pi}{4} < \theta < \frac{\pi}{2}$. So that operator becomes singular in our range. We show in next section that the infrared divergences resulting from the physical field are cancelled by infrared divergences from the residual gauge part.

2.4. *Fermion field operator*

Now that we have the explicit expression for A_μ, we can construct the fermion field operators in the same way as in I. From the expression for A^μ in (2·21), we see that the fermion operators are formally given by

$$\psi_\alpha(x) = \frac{Z_\alpha}{\sqrt{(\gamma^0\gamma^\tau)_{\alpha\alpha}}} \exp[-i\sqrt{\pi}\Lambda_\alpha(x)], \quad (\alpha = 1, 2)$$

where Z_α is normalization constant and

$$\Lambda_\alpha(x) = X(x) + (-1)^\alpha \lambda(x). \tag{2·31}$$

We need to rewrite the formal solution into a normal ordered product. However, if we simply normal order the exponential and then calculate the canonical anticommutation relations, we find another infrared divergence inherent in two-dimensional massless scalar fields. In our formulation it results from the singular operator, $\partial_\sigma^{-1} B$, in λ in (2·19). We overcome this difficulty by not rewriting the infrared parts of the singular operator and its conjugate operator into normal ordered form.[13] In what follows, we keep $\phi > \theta$ and, to incorporate the ML prescription, we employ the following representations of B and C:

$$B(x) = \frac{m}{n^\tau \sqrt{2\pi}} \int_{-\infty}^{\infty} dk_\sigma \theta(-k_\sigma) \sqrt{|k_\sigma|} \{ B(k_\sigma) e^{-ik\cdot x} + B^*(k_\sigma) e^{ik\cdot x} \},$$
$$\frac{m}{m^2 - n^2 \partial_+^2} C(x) = \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{dk_\sigma}{\sqrt{|k_\sigma|}} \theta(-k_\sigma) \{ C(k_\sigma) e^{-ik\cdot x} - C^*(k_\sigma) e^{ik\cdot x} \}, \tag{2·32}$$

where $k_\tau = \cot(\theta - \phi)k_\sigma$, creation and annihilation operators satisfy

$$[B(k_\sigma), C^*(q_\sigma)] = [C(k_\sigma), B^*(q_\sigma)] = -\delta(k_\sigma - q_\sigma), \tag{2·33}$$

and all other commutators are zero. These relations allow us to define the physical subspace, V, by

$$V = \{\ |\mathrm{phys}\rangle\ \mid\ B(k_\sigma)|\mathrm{phys}\rangle = 0\ \}. \tag{2·34}$$

and to define the infrared part, $\varLambda_\alpha^{(0)}$, of $\varLambda_\alpha$ by

$$\varLambda_\alpha^{(0)} = \frac{i}{\sqrt{2\pi}} \int_{-\kappa}^{0} \frac{dk_\sigma}{\sqrt{|k_\sigma|}} \{C(k_\sigma) - C^*(k_\sigma) + (-1)^\alpha (B(k_\sigma) - B^*(k_\sigma))\} \tag{2·35}$$

where κ is a small positive constant.

Now we can define the fermion field operators to be

$$\psi_\alpha(x) = \frac{Z_\alpha}{\sqrt{(\gamma^0\gamma^\tau)_{\alpha\alpha}}} \exp[-i\sqrt{\pi}\varLambda_{\alpha r}^{(-)}(x)]\sigma_\alpha \exp[-i\sqrt{\pi}\varLambda_{\alpha r}^{(+)}(x)] \tag{2·36}$$

where $\varLambda_{\alpha r}^{(-)}$ and $\varLambda_{\alpha r}^{(+)}$ are creation and annihilation operator parts of $\varLambda_{\alpha r} \equiv \varLambda_\alpha - \varLambda_\alpha^{(0)}$ and

$$\sigma_\alpha = \exp\left[-i\sqrt{\pi}\left(\varLambda_\alpha^{(0)} - (-1)^\alpha \frac{Q}{2m}\right)\right]. \tag{2·37}$$

Here, $Q = -n^\tau \int_{-\infty}^{\infty} dx^\sigma B(x)$; note that Q in σ_α constitutes a Klein transformation. We refer to σ_α as the spurion operator.[13)]

We enumerate the properties of the ψ_α which show that the bosonized model is actually equivalent to the original model defined by the Lagrangian (2·1). We note that the symmetric energy-momentum tensor (2·41)∼(2·43) given below follows directly from the Lagrangian (2·10).

(1) The Dirac equation is satisfied:

$$i\gamma^\mu(\partial_\mu + ieA_\mu)\psi = 0 \tag{2·38}$$

(2) The canonical commutation relations with A_σ and π^σ and anticommutation relations are satisfied.

(3) By applying the gauge invariant point-splitting procedure to $e\bar{\psi}\gamma^\mu\psi$, we obtain the vector current $J^\mu = m\partial^\mu X - m^2 A^\mu = m\varepsilon^{\mu\nu}\partial_\nu\lambda$. This result verifies that j^μ is given by $j^\mu = m\partial^\mu X$ so that it satisfies $\varepsilon^{\mu\nu}\partial_\mu j_\nu = 0$. The charge operator, Q, is given by

$$Q = \int_{-\infty}^{\infty} dx^\sigma J^\tau(x) = -n^\tau \int_{-\infty}^{\infty} dx^\sigma B(x), \tag{2·39}$$

where the derivative terms integrate to zero.

(4) Applying the gauge invariant point-splitting procedure to the fermi products in the symmetric energy-momentum tensor and subtracting a divergent c-number (we denote this procedure[14)] by R), we get

$$\Theta_\tau{}^\sigma = iR(\bar\psi\gamma^\sigma\partial_\tau\psi) - A_\tau J^\sigma - n^\sigma A_\tau B = \partial_\tau\lambda\partial^\sigma\lambda - n^\sigma A_\tau B, \tag{2·40}$$

$$\begin{aligned}\Theta_\tau{}^\tau &= -iR(\bar\psi\gamma^\sigma\partial_\sigma\psi) + A_\sigma J^\sigma + \frac{1}{2}(F_{\tau\sigma})^2 - n^\tau BA_\tau \\ &= -\frac{\cos 2\phi}{2}\Big\{(\partial_\tau\lambda)^2 + (\partial_\sigma\lambda)^2\Big\} + \frac{1}{2}(F_{\tau\sigma})^2 - n^\tau BA_\tau,\end{aligned} \tag{2·41}$$

$$\begin{aligned}\Theta_\sigma{}^\sigma &= iR(\bar\psi\gamma^\sigma\partial_\sigma\psi) - A_\sigma J^\sigma + \frac{1}{2}(F_{\tau\sigma})^2 - n^\sigma BA_\sigma \\ &= \frac{\cos 2\phi}{2}\{(\partial_\tau\lambda)^2 + (\partial_\sigma\lambda)^2\} + \frac{1}{2}(F_{\tau\sigma})^2 - n^\sigma BA_\sigma,\end{aligned} \tag{2·42}$$

$$\Theta_\sigma{}^\tau = iR(\bar\psi\gamma^\tau\partial_\sigma\psi) - A_\sigma J^\tau - n^\tau BA_\sigma = \partial_\sigma\lambda\partial^\tau\lambda - n^\tau BA_\sigma. \tag{2·43}$$

(5) Translational generators consist of those of the constituent fields:

$$\begin{aligned}P_\tau &= \int_{-\infty}^{\infty} dx^\sigma : \Theta_\tau{}^\tau := \int_{-\infty}^{\infty} dx^\sigma : \Big[-\frac{\cos 2\phi}{2}\Big\{(\partial_\tau\tilde\Sigma)^2 + (\partial_\sigma\tilde\Sigma)^2\Big\} + \frac{m^2}{2}(\tilde\Sigma)^2 \\ &\qquad + \frac{1}{2}B\frac{n^2}{m^2 - n^2\partial_+^2}B + B\frac{n^\sigma}{m^2 - n^2\partial_+^2}\partial_\sigma C\Big] :, \\ P_\sigma &= \int_{-\infty}^{\infty} dx^\sigma : \Theta_\sigma{}^\tau := \int_{-\infty}^{\infty} dx^\sigma : \Big\{\partial_\sigma\tilde\Sigma\partial^\tau\tilde\Sigma - B\frac{n^\tau}{m^2 - n^2\partial_+^2}\partial_\sigma C\Big\} : .\end{aligned} \tag{2·44}$$

§3. Cancellation of Infrared divergences resulting from ∂_-^{-1}

We begin this section by pointing out that X incorporates Higgs phenomena and that the possible infrared singularities in A_μ are in X. Therefore we confine ourselves to showing that X is free from infrared divergences. More precisely, we show that commutator function and the propagator for X are free from infrared divergences. To this aim we represent $\tilde\Sigma$ as

$$\tilde\Sigma(x) = \frac{1}{\sqrt{4\pi}}\int_{-\infty}^{\infty}\frac{dp_\sigma}{\sqrt{p^\tau}}\{a(p_\sigma)e^{-ip\cdot x} + a^*(p_\sigma)e^{ip\cdot x}\}. \tag{3·1}$$

Here, $p^\tau = \sqrt{p_\sigma^2 + m_0^2}$ with $m_0^2 = -\cos 2\phi m^2$ and

$$[a(p_\sigma), a(q_\sigma)] = 0, \quad [a(p_\sigma), a^*(q_\sigma)] = \delta(p_\sigma - q_\sigma). \tag{3·2}$$

We first show that the commutator function of X includes the commutator function, $E(x)$, characteristic of a dipole ghost field. From (2·28), (2·32) and (3·1) we obtain

$$[X(x), X(y)] = i\{\Delta(x-y; m^2) + n^2m^2E(x-y)\} \tag{3·3}$$

where $\Delta(x;m^2)$ is the commutator function of the free boson field of mass m and

$$E(x) = \frac{1}{\partial_-^2}\Delta(x;m^2) - \frac{x^\tau}{m^2(n^\tau)^2 - n^2\partial_\sigma^2}\delta(x^\sigma - \cot(\theta-\phi)x^\tau) + \frac{2n^\tau n_\sigma}{(m^2(n^\tau)^2 - n^2\partial_\sigma^2)^2}\partial_\sigma\delta(x^\sigma - \cot(\theta-\phi)x^\tau). \quad (3{\cdot}4)$$

When $x^\tau = y^\tau$, the commutator $[X(x), X(y)]$ vanishes. We see that as follows: The first term of (3·3) vanishes trivially. The second term of $E(x-y)$, which is proportional to $x^\tau - y^\tau$, also vanishes trivially. If we evaluate the first term of (3·4) using ∂_-^{-1} as defined in (2·26), then we get a nonvanishing term; however, that term is cancelled by the third term of (3·4).

The following are properties of $E(x)$:

$$(\Box + m^2)E(x) = -\frac{x^\tau}{(n^\tau)^2}\delta(x^\sigma - \cot(\theta-\phi)x^\tau), \quad \partial_-^2 E(x) = \Delta(x;m^2), \quad (3{\cdot}5)$$

$$E(x)|_{x^\tau=0} = 0, \quad \partial_- E(x)|_{x^\tau=0} = 0, \quad \partial_-^2 E(x)|_{x^\tau=0} = 0, \quad (3{\cdot}6)$$

$$(\Box + m^2)E(x)|_{x^\tau=0} = 0, \quad (\Box + m^2)\partial_- E(x)|_{x^\tau=0} = -\frac{1}{n^\tau}\delta(x^\sigma). \quad (3{\cdot}7)$$

Next we show that the vacuum expectation value, $\langle 0|X(x)X(y)|0\rangle$, does not diverge when $x^\tau = y^\tau$. We will need to use $\langle 0|X(x)X(y)|0\rangle$ evaluated at $x^\tau = y^\tau$ to calculate the equal x^τ-time anticommutatuion relations of the fermion field operators. It is straightforward to obtain

$$\langle 0|X(x)X(y)|0\rangle = \Delta^{(+)}(x-y;m^2) + m^2 n^2 E^{(+)}(x-y) \quad (3{\cdot}8)$$

where $\Delta^{(+)}(x;m^2)$ is the positive frequency part of $i\Delta(x;m^2)$ and

$$E^{(+)}(x) = \frac{1}{\partial_-^2}\Delta^{(+)}(x;m^2) - \frac{ix^\tau}{2\pi}\int_{-\infty}^0 dk_\sigma \frac{1}{m^2(n^\tau)^2 + n^2 k_\sigma^2}\mathrm{e}^{-ik\cdot x} + \frac{1}{2\pi}\int_{-\infty}^0 dk_\sigma \frac{2n^\tau n_\sigma k_\sigma}{(m^2(n^\tau)^2 + n^2k_\sigma^2)^2}\mathrm{e}^{-ik\cdot x}. \quad (3{\cdot}9)$$

A logarithmic divergence appears in the second term but we regularize it with the principal value prescription. In addition, linear divergences appear in the first and third terms when $x^\tau = 0$. We set $x^\tau = 0$ and divide the integration region of the first term into a region where the integration variable is positive and a region where the integration variable is negative. We then combine the integration in the negative region with third term and obtain

$$\frac{-1}{4\pi}\int_{-\infty}^0 \frac{dp_\sigma}{p^\tau}\frac{1}{p_-^2}\mathrm{e}^{-ip_\sigma x^\sigma} + \frac{1}{2\pi}\int_{-\infty}^0 dk_\sigma \frac{2n^\tau n_\sigma k_\sigma}{(m^2(n^\tau)^2 + n^2k_\sigma^2)^2}\mathrm{e}^{-ik_\sigma x^\sigma}$$
$$= \frac{-1}{4\pi}\int_{-\infty}^0 \frac{dp_\sigma}{p^\tau}\frac{(n^\tau p^\tau - n_\sigma p_\sigma)^2}{(m^2(n^\tau)^2 + n^2p_\sigma^2)^2}\mathrm{e}^{-ip_\sigma x^\sigma} = \frac{-1}{4\pi}\int_{-\infty}^0 \frac{dp_\sigma}{p^\tau}\frac{(-\cos 2\phi)^2}{(n^\tau p^\tau + n_\sigma p_\sigma)^2}\mathrm{e}^{-ip_\sigma x^\sigma}. \quad (3{\cdot}10)$$

It is useful to recall here that ϕ and θ lie in the regions $(\frac{\pi}{4} < \theta < \phi \leqq \frac{\pi}{2})$ so that $n^\tau = \sin(\phi - \theta) > 0$ and $n_\sigma = \cos(\phi + \theta) < 0$. As a result, no infrared divergences appear from (3·10). Furthermore, changing the integration variable from p_σ to $-p_\sigma$ verifies that (3·10) is equal to the positive integration part of the first term of $E^{(+)}(x)$. It follows that $E^{(+)}(x-y)$ is well defined at $x^\tau = y^\tau$, which implies that we can incorporate the equal x^τ-time anticommutation relations of the fermion field operators in the same way as § 3 in I.

Finally, we show that the factors $(m^2(n^\tau)^2 + n^2 p_\sigma^2)^{-1}$ relevant to the infrared divergences drop out completely from the propagator for X, which is given by

$$\langle 0|T(X(x)X(y))|0\rangle = \varDelta_F(x-y;m^2) + m^2 n^2 E_F(x-y) \tag{3·11}$$

where $\varDelta_F(x-y;m^2)$ is the propagator for the free boson field of mass m and $E_F(x-y)$ is defined by

$$\begin{aligned} E_F(x-y) &= \theta(x^\tau - y^\tau)E^{(+)}(x-y) + \theta(y^\tau - x^\tau)E^{(+)}(y-x) \\ &= \frac{1}{(2\pi)^2}\int d^2q E_F(q) \mathrm{e}^{-iq\cdot(x-y)}. \end{aligned} \tag{3·12}$$

Substituting the expression in (3·9) into (3·12) and then Fourier transforming it provides us with

$$\begin{aligned} E_F(q) = &-\frac{i}{q^2 - m^2 + i\varepsilon}\frac{(n^\tau)^2(q_\sigma^2 - \cos 2\phi m^2) + n_\sigma^2 q_\sigma^2 + 2n^\tau n_\sigma q^\tau q_\sigma}{(m^2(n^\tau)^2 + n^2 q_\sigma^2)^2} \\ &+ \frac{i}{(q_- + i\varepsilon\mathrm{sgn}(q_+))^2}\frac{(n^\tau)^2}{m^2(n^\tau)^2 + n^2 q_\sigma^2} + \frac{i}{q_- + i\varepsilon\mathrm{sgn}(q_+)}\frac{2(n^\tau)^2 n_\sigma q_\sigma}{(m^2(n^\tau)^2 + n^2 q_\sigma^2)^2} \end{aligned} \tag{3·13}$$

where $q_+ \equiv -\frac{q_\sigma}{n^\tau}$. The term on the first line comes from the physical degrees of freedom whereas the terms on the second line come from the residual gauge degrees of freedom. It is remarkable that if we combine them into one term, then we obtain $E_F(q) = -\frac{i}{q^2 - m^2 + i\epsilon} \times \frac{1}{(q_- + i\varepsilon\mathrm{sgn}(q_+))^2}$ and hence

$$\langle 0|T(X(x)X(y))|0\rangle = \frac{1}{(2\pi)^2}\int d^2q \frac{i}{q^2 - m^2 + i\epsilon}\left(1 - \frac{n^2 m^2}{(q_- + i\varepsilon\mathrm{sgn}(q_+))^2}\right)\mathrm{e}^{-iq\cdot(x-y)}. \tag{3·14}$$

In this way, the infrared divergences are eliminated and the singularity associated with the gauge fixing is prescribed in such a way that causality is preserved in complex q_τ coordinates. It can be shown that the same is true of the propagator for A_μ and we get

$$\int d^2x \langle 0|T(A_\mu(x)A_\nu(0))|0\rangle \mathrm{e}^{iq\cdot x} = \frac{iP_{\mu\nu}}{q^2 - m^2 + i\epsilon} \tag{3·15}$$

where

$$P_{\mu\nu} = -g_{\mu\nu} + \frac{n_\mu q_\nu + n_\nu q_\mu}{q_- + i\varepsilon\mathrm{sgn}(q_+)} - n^2\frac{q_\mu q_\nu}{(q_- + i\varepsilon\mathrm{sgn}(q_+))^2}. \tag{3·16}$$

§4. Pure space-like case

We begin by noting that the limit $\phi \to \theta$ of the residual gauge part of the operator solution given in § 2, which has factors dependent on quantization coordinates, is not well-defined. We see from this that an operator solution in the (PSL) case, $\phi = \theta$, is not constructed in the same manner as that given in § 2.

The Lagrangian and the equations of motion for A_μ and X in the PSL case are given respectively by transforming (2·1),(2·7) and (2·8) into those in $+-$-coordinates. We obtain two new constraints

$$\pi^- + \partial_- A_+ = 0, \quad \partial_- \pi^- - m\pi_X = 0 \tag{4·1}$$

in addition to the gauge fixing condition $A_- = 0$. As a result only X and π_X are left as independent canonical variables. This reflects the fact that the residual gauge fields depend on only x^+ so they cannot be canonical variables. Therefore we cannot obtain their quantization conditions from the Dirac procedure.[15] Instead we employ the following items as guiding principles to introduce them in the PSL case:

(1) X and π_X satisfy the canonical commutations conditions.

(2) The residual gauge fields commute with the massive field.

(3) B satisfies $[B(x), X(y)] = im\delta(x^+ - y^+)$ and so, generates c-number residual gauge transformations.

(4) The infrared divergences which come from the physical part of X are regularized by infrared divergences from the residual gauge fields.

We start constructing an operator solution by solving Eq.(2.8) and obtain an expression similar to (2.17). The massive field obtained will be identified below as $\tilde{\Sigma}$. So, since $A_- = 0$, we can write

$$\partial_- X = \partial^+ \tilde{\Sigma} - \frac{mn^2}{m^2 - n^2\partial_+^2} B. \tag{4·2}$$

We see from this that X is given by

$$X = \frac{\partial^+}{\partial_-}\tilde{\Sigma} - \frac{mn^2 x^-}{m^2 - n^2\partial_+^2} B + \text{integration constant.}$$

The massive, physical part of X is now known and π_X can be written as

$$\pi_X = \partial^+ X - mA^+ = \partial_- \lambda = \partial_- \tilde{\Sigma}, \tag{4·3}$$

¿From this we see that if we impose the canonical commutation conditions on X and π_X, that will imply the following equal x^+ -time commutation relations

$$[\tilde{\Sigma}(x), \tilde{\Sigma}(y)] = 0, \;\; [\tilde{\Sigma}(x), \partial^+\tilde{\Sigma}(y)] = i\delta(x^- - y^-), \;\; [\partial^+\tilde{\Sigma}(x), \partial^+\tilde{\Sigma}(y)] = 0. \tag{4·4}$$

The integration constant is determined in the following way: To implement the residual gauge transformation, we add $\frac{m}{m^2-n^2\partial_+^2}C$ to X, where C is the conjugate to B and satisfies the following commutation relations

$$[C(x),C(y)]=0,\ \ [B(x),\frac{1}{m^2-n^2\partial_+^2}C(y)]=i\delta(x^+-y^+). \tag{4·5}$$

Furthermore we require that the infrared divergence resulting from $\frac{\partial^+}{\partial_-}\tilde{\varSigma}$ be cancelled through the mechanism worked out in § 2. If we consider a surface of constant x^-, then we can write

$$\frac{\partial^+}{\partial_-}\tilde{\varSigma}=\frac{n_+m^2+n_-\partial_+\partial^-}{m^2-n^2\partial_+^2}\tilde{\varSigma} \tag{4·6}$$

and see that the equal x^--time commutator, $[\frac{\partial^+}{\partial_-}\tilde{\varSigma}(x),\frac{\partial^+}{\partial_-}\tilde{\varSigma}(y)]$, does not vanish. To correct for this we must add $\frac{mn_+n_-}{(m^2-n^2\partial_+^2)^2}\partial_+B$ to X. Summing all the terms, we obtain

$$X=\frac{\partial^+}{\partial_-}\tilde{\varSigma}+\frac{m}{m^2-n^2\partial_+^2}\left(C-n^2x^-B+\frac{mn_+n_-}{m^2-n^2\partial_+^2}\partial_+B\right), \tag{4·7}$$

This is exactly the solution given in I. Therefore, we need not repeat the construction of the fermion field operators or the description of their properties.

It remains to be shown that infrared divergences do not appear when we evaluate $\langle 0|X(x)X(y)|0\rangle$ at $x^+=y^+$ or when we calculate the x^+-time ordered propagator for X. By using the representations for the constituent operators given in I, we obtain the following vacuum expectation value

$$\langle 0|X(x)X(y)|0\rangle=\varDelta^{(+)}(x-y;m^2)+n^2m^2E^{(+)}_{PSL}(x-y) \tag{4·8}$$

where

$$\begin{aligned}E^{(+)}_{PSL}(x)=&-\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{dp_-}{p^+}\frac{\mathrm{e}^{-ip\cdot x}}{p_-^2}\\&-\frac{ix^-}{2\pi}\int_0^{\infty}dk_+\frac{\mathrm{e}^{-ik_+x^+}}{m^2+n^2k_+^2}+\frac{1}{2\pi}\int_0^{\infty}dk_+\frac{2n_+k_+\mathrm{e}^{-ik_+x^+}}{(m^2+n^2k_+^2)^2}.\end{aligned} \tag{4·9}$$

Here, p^+ and p_+ are defined, respectively, as $p^+=\sqrt{p_-^2+m_0^2}$, $(m_0^2=-n^2m^2)$, $p_+=\frac{p^+-n_+p_-}{-n_-}$. The integral on the first line comes from the $\tilde{\varSigma}$, while the integrals on the second line come from the residual gauge fields. The value of $E^{(+)}_{PSL}(x)$ at $x^+=0$ is formally given by

$$\begin{aligned}E^{(+)}_{PSL}(x)|_{x^+=0}=&-\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{dp_-}{p^+}\frac{\mathrm{e}^{-ip_-x^-}-1}{p_-^2}\\-\frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{dp_-}{p^+}\frac{1}{p_-^2}&-\frac{ix^-}{2\pi}\int_0^{\infty}dk_+\frac{1}{m^2+n^2k_+^2}+\frac{1}{2\pi}\int_0^{\infty}dk_+\frac{2n_+k_+}{(m^2+n^2k_+^2)^2}\end{aligned} \tag{4·10}$$

where we have divided the first term into a finite term and a diverging term. It should be noted here that p_- is conjugate to the spatial variable x^-, while k_+ is conjugate to the temporal variable x^+. To make the infrared divergence cancellation mechanism work as in § 3, both integration variables have to be spatial or temporal. Therefore, we change the integration variable from the spatial p_- to the temporal $p_+ = \frac{\sqrt{p_-{}^2+m_0^2}-n_+p_-}{-n_-}$. At the same time we denote k_+ as p_+. If we take account of the fact that p_+ is two-valued function of p_-, then we can rewrite the diverging integral into the following form

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{dp_-}{p^+}\frac{1}{p_-^2} &= \int_{m_0}^{\infty}\frac{dp_+}{\sqrt{p_+^2-m_0^2}}\left(\frac{-n_-}{n_+p_+-\sqrt{p_+^2-m_0^2}}\right)^2 \\ &\quad + \int_{m_0}^{\infty}\frac{dp_+}{\sqrt{p_+^2-m_0^2}}\left(\frac{-n_-}{n_+p_++\sqrt{p_+^2-m_0^2}}\right)^2 \end{aligned} \tag{4·11}$$

where the first term diverges, but second term is finite. Now we see that if we combine the first integral in (4·11) with the third one on the second line of (4·10), we obtain the following finite integrals:

$$\begin{aligned} &-\int_{m_0}^{\infty}\frac{dp_+}{\sqrt{p_+^2-m_0^2}}\left(\frac{-n_-}{n_+p_+-\sqrt{p_+^2-m_0^2}}\right)^2 + \int_0^{\infty}dp_+\frac{4n_+p_+}{(m^2+n^2p_+^2)^2} \\ &= -\int_{m_0}^{\infty}\frac{dp_+}{\sqrt{p_+^2-m_0^2}}\left(\frac{n_+p_++\sqrt{p_+^2-m_0^2}}{m^2+n^2p_+^2}\right)^2 + \int_0^{\infty}dp_+\frac{4n_+p_+}{(m^2+n^2p_+^2)^2} \\ &= -\int_{m_0}^{\infty}\frac{dp_+}{\sqrt{p_+^2-m_0^2}}\left(\frac{n_+p_+-\sqrt{p_+^2-m_0^2}}{m^2+n^2p_+^2}\right)^2 + \int_0^{m_0}dp_+\frac{4n_+p_+}{(m^2+n^2p_+^2)^2} \\ &= -\int_{m_0}^{\infty}\frac{dp_+}{\sqrt{p_+^2-m_0^2}}\left(\frac{-n_-}{n_+p_++\sqrt{p_+^2-m_0^2}}\right)^2 + \int_0^{m_0}dp_+\frac{4n_+p_+}{(m^2+n^2p_+^2)^2}. \end{aligned} \tag{4·12}$$

After tedious but straightforward calculations, we finally obtain

$$E_{PSL}^{(+)}(x)|_{x^+=0} = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{dp_-}{p^+}\frac{1-\cos(p_-x^-)}{p_-^2} + \frac{1}{2\pi m^2}\frac{-n_-}{1+n_+}. \tag{4·13}$$

Finally, without demonstration, we give the x^+-time ordered propagator for X. The necessary demonstration can be carried out in parallel with that given in Appendix A in I. It turns out that

$$\langle 0|T(X(x)X(y))|0\rangle = \frac{1}{(2\pi)^2}\int d^2q\frac{i}{q^2-m^2+i\varepsilon}\left(1-\frac{n^2m^2}{(q_-+i\varepsilon\mathrm{sgn}(q_+))^2}\right)\mathrm{e}^{-iq\cdot(x-y)}. \tag{4·14}$$

It is remarkable to see that in spite of the fact that all factors depending on the quantization coordinates drop out, we have the same propagator that we have obtained in (3·14).

§5. Concluding remarks

In this paper the framework used in I has been generalized by introducing the $\tau\sigma$-coordinates and at the same time simplified by bosonizing the model. The new framework has allowed us to investigate the way in which operator solutions develop a dependence on the quantization coordinates. In the new framework we can take the dipole ghost field, X, and the component of the gauge field, A_σ, as canonical variables. We have given special attention to the determination of X, because we know that it cannot be a manifest Lorentz scalar since it develops an explicit dependence on the quantization coordinates. We have found that the physical part of X is determined uniquely by the gauge choice, while the residual gauge part, which contains the manifest dependence on the quantization coordinates, is determined by requiring that X satisfy the canonical commutation conditions. The main findings of this paper are:

(1)the residual gauge fields are indispensable ingredients of the space-like axial gauge Schwinger model.

(2) $(n{\cdot}\partial)^{-1} = (\partial_-)^{-1}$ is ill-defined irrespective of the quantization coordinates, as long as the gauge fixing direction is space-like ($n^2 < 0$).

(3) As a consequence, the physical part of X includes infrared divergences irrespective of the quantization coordinates, as long as the gauge fixing direction is space-like.

(4) If we introduce the residual gauge fields in such a way that X satisfies the canonical commutation relations, then the residual gauge part is determined so as to regularize the infrared divergences resulting from the physical part.

In the PSL case the residual gauge fields cannot be canonical variables due to the fact that they depend on the evolution parameter x^+. So the operator solution for this case cannot be constructed purely by canonical methods. We have overcome this difficulty by employing the items described in § 4 as guiding principles supplement canonical methods. The operator solution we obtain by the extended methodology is satisfactory in every aspect. In particular, all ill-defined factors drop out from the x^+-time ordered propagators for X and A^μ so that we have the same ML form for the propagators irrespective of the quantization coordinates.

The light-cone gauge, $\theta = \frac{\pi}{4}$, is exceptional. In that case, n^2 is zero, so the manifest dependence on the quantization coordinates disappears whatever coordinates we may have. Therefore, the light-cone axial gauge formulation is obtained by simply setting $\theta = \phi = \frac{\pi}{4}$ and we obtain

$$X = \tilde{\Sigma} + m^{-1}C, \quad A_+ = 2\frac{\partial_+}{m}\tilde{\Sigma} + \frac{\partial_+}{m^2}(C + \partial_+^{-1}B). \tag{5·1}$$

On comparing these operators with the corresponding operators we gave in a previous paper,[16] we find that B and C are related to η and ϕ by $C = m(\eta + \phi)$, $B = m\partial_+(\eta - \phi)$.

We end this paper by pointing out that in axial gauge quantizations in 4-dimensions, the same infrared divergence cancellation mechanism works. We already have some work in this direction.[17] We hope to report more in subsequent studies.

References

1) Y. Nakawaki and G. McCartor, Prog. Theor. Phys. **106** (2001), 167.
2) This procedure was first devised in G. McCartor, Z. Phys. **C41** (1988), 271.
3) N. Nakanishi, Phys. Lett. **131B** (1983), 381. N. Nakanishi, *Quantum Electrodynamics*, ed.T. Kinoshita (World Scientific, Singapore, 1990), p. 36.
4) N. Nakanishi, Prog. Theor. Phys. **67** (1982), 965.
   K. Haller, Phys. Lett. **251B** (1990), 575.
   P. V. Landshoff and P. van Nieuwenhuizen, Phys. Rev. **D50** (1994), 4157.
   M. Morara and R. Soldati, Phys. Rev. **D58** (1998), 105011.
5) G. McCartor and D.G. Robertson, Z. Phys. **C62** (1994), 349; **C68** (1995), 345.
6) S. Mandelstam, Nucl. Phys. **B213** (1983), 149.
7) G. Leibbrandt, Phys. Rev. **D29** (1984), 1699.
8) A. Bassetto, M. Dalbosco, I. Lazziera and R. Soldati, Phys. Rev. **D31** (1985), 2012.
   A. Bassetto, G. Nardelli and R. Soldati, *Yang-Mills Theories in Algebraic Non-Covariant Gauges* (World Scientific, Singapore, 1991).
9) I. Lazzizzera, Phys. Lett. **210B** (1988), 188; Nuovo Cim. **102A** (1989), 1385.
10) B. Lautrup, Mat. Fys. Medd. Dan. Vid. Selsk. **35**(1967), No.11.
    N. Nakanishi, Prog. Theor. Phys. Suppl. No.51 (1972),1.
11) J. H. Lowenstein and J. A. Swieca, Ann. of Phys. **68** (1971), 172.
    N. Nakanishi, Prog. Theor. Phys. **57** (1977), 580.
12) J. Schwinger, Phys. Rev. Lett. **3** (1959), 296.
13) Y. Nakawaki, Prog. Theor. Phys. **72** (1984), 134.
14) Y. Nakawaki, Prog. Theor. Phys. **64** (1980), 1828.
15) P. A. M. Dirac, *Lectures on Quantum Mechanics*, Belfer Graduate School of Science-Yeshiva University (Academic Press, New York, 1964).
    A. Hanson, T. Regge and C. Teitelboim, *Constrained Hamiltonian Systems* (Accad. Naz. deiLincei, Rome, 1976).
16) Y. Nakawaki and G. McCartor, Prog. Theor. Phys. **103** (2000), 161.
17) Y. Nakawaki and G. McCartor, Prog. Theor. Phys. **103** (1999), 149; Nucl. Phys. **B108(proc.Suppl.)** (2002), 201.