An application of nonperturbative Pauli – Villars regularization to QED

Sofia Chabysheva

Light Cone 2007 Relativistic Hadronic and Nuclear Physics
May 14-18 2007, Columbus
Electron’s magnetic moment

\[ \frac{g e \hbar}{4mc} \]

\( g \) — gyromagnetic ratio

Theoretical prediction, computed perturbatively up to order \( \alpha^4 \), is

\[ \frac{g - 2}{2} = \frac{\alpha}{2\pi} - (0.328 \ 478 \ 965 \ldots) \times \left( \frac{\alpha}{\pi} \right)^2 + (1.176 \ 11 \ldots) \times \left( \frac{\alpha}{\pi} \right)^3 - (1.434 \ldots) \times \left( \frac{\alpha}{\pi} \right)^4 = 0.001 \ 159 \ 652 \ 140 \ \ldots \]
Light-cone quantization regulated with $\mathcal{PV}$ fields in $3+1$ dimensions

- Light-cone gauge $A^+ = 0$, 3 $\mathcal{PV}$ electrons
- Feynman gauge, 1 $\mathcal{PV}$ electron $+$ 1 $\mathcal{PV}$ photon
- Again light-cone gauge, 3 $\mathcal{PV}$ electrons $+$ 1 $\mathcal{PV}$ photon, higher-order derivatives
Three problems must be solved to produce useful calculation:

✠ Problem of maintaining gauge invariance (exact solution exists and has all symmetries and a close approximation can safely break symmetries)

✠ New singularities (when the bare mass is less than the physical mass, as is the case in QED, then can be zero in energy denominator) — principal value prescription

✠ Uncancelled divergencies (missing of corrections due to truncation of Fock state) — keep at least on PV mass finite
\[ \sum_{i=0}^{1} \left( -\frac{1}{4} (-1)^i F^\mu_\nu F_{i,\mu\nu} + (-1)^i \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i) \psi_i + B_i \partial_\mu A^\mu_i + \frac{1}{2} B_i B_i \right) - e \bar{\psi} \gamma^\mu \psi A_\mu, \]

where \[ A^\mu = \sum_{i=0}^{1} A^\mu_i, \quad \psi = \sum_{i=0}^{1} \psi_i, \quad F^\mu_\nu = \partial^\mu A^\nu_i - \partial^\nu A^\mu_i \]

\[ P^- = \sum_{i,s} \int dp \frac{m_i^2 + p_i^2}{p^+} (-1)^i b^\dagger_i,s(p) b_{i,s}(p) + \sum_{i,\mu} \int dk \frac{m_i^2 + k_\mu^2}{k^+} (-1)^l \epsilon^\mu a^\dagger_i(l) a_i^\mu(l) + \]

\[ + \sum_{i,j,l,s,\mu} \int dp dq \left\{ b^\dagger_i,s(p) \left[ b_{j,s}(q) Q^\mu_{ij,2s}(p,q) + b_{j,-s}(q) R^\mu_{ij,-2s}(p,q) \right] a^\dagger_{l\mu}(q-p) + h.c. \right\} \]

\[ \Phi_+(P) = \sum_i z_i b^\dagger_{i+}(P)|0\rangle + \sum_{ijs} \int dq f_{ijs}(q) b^\dagger_{is}(P-q) a^\dagger_j(q)|0\rangle + \]

\[ + \sum_{ijkl} \int dq_1 dq_2 f_{ijkl}(q_1, q_2) \frac{1}{\sqrt{1 + \delta_{jk}}} b^\dagger_{is}(P-q_1-q_2) a^\dagger_j(q_1) a^\dagger_k(q_2)|0\rangle + \ldots \]
The anomalous moment of the electron in units of the Schwinger term \( \left( \frac{\alpha}{2\pi} \right) \) plotted versus the PV photon mass, \( \mu_1 \).
\[ P^+ P^- |\Phi_+\rangle = M^2 |\Phi_+\rangle \]
The equation for two-particle amplitudes only:

\[
\left[ M^2 - \frac{m_i^2 + q_{\bot}^2}{1 - y} - \frac{\mu_j^2 + q_{\bot}^2}{y} \right] G_{ijs}^{\lambda}(y, q_{\bot}) = \\
\frac{e^2}{8\pi^2} \sum_{l} I_{ijl}(y, q_{\bot}) G_{ijs}^{\lambda}(y, q_{\bot}) + \\
+ \frac{e^2}{8\pi^2} \sum_{n,k,s,\lambda'} \int_{0}^{1} dy' \int_{0}^{+\infty} q_{\bot}' dq_{\bot}' J_{ijs,nks'}^{(0)\lambda\lambda'}(y, q_{\bot}; y', q_{\bot}') G_{nks'}^{\lambda'}(y', q_{\bot}') + \\
+ \frac{e^2}{8\pi^2} \sum_{n,k,s,\lambda'} \int_{0}^{1-y} dy' \int_{0}^{+\infty} q_{\bot}' dq_{\bot}' J_{ijs,nks'}^{(2)\lambda\lambda'}(y, q_{\bot}; y', q_{\bot}') G_{nks'}^{\lambda'}(y', q_{\bot}')
\]
\[ \sim \frac{e^2}{8\pi^2} \sum_{n,k} \int_0^{1-y} \int_0^\infty d\gamma' \int_0^{q'_\perp} dq'_\perp \]
\[-M^2 y (1 - y) + m^2 y + \mu^2 (1 - y) + q^2\]
Logarithmic singularity of longitudinal momentum
Changing of variables
One-photon truncated wave function was obtained analytically and anomalous electron’s magnetic moment is within 14% accuracy of Schwinger term.

For two-photon truncated state anomalous magnetic moment is expected to get close to Sommerfield-Peterman term, but this case demands huge numerical calculation. Currently computer code is being checked for consistency with analytical solution derived from one-photon truncated state.