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Light-Cone Quantization of Gauge Theories

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In this paper I shall discuss the problem of specifying the boundary conditions that must be used in the solution of light-cone constraint equations. The problem may be expressed as the specification of integration constant fields to be added to the commonly used solutions to these constraint equations. These integration constant fields are unphysical and nondynamical; but they affect the dynamics of physical fields and change the results of the calculations of measurable quantities. I shall illustrate these remarks with examples from QED and QCD.

1. INTRODUCTION

In this paper I shall report research that I have done with Yuji Nakawaki [1] and with Simon Dalley [2]. The work attempts to specify the correct boundary conditions that should be used in the solution of light-cone constraint equations. We still do not have a complete specification of these boundary conditions for all cases of interest, but we do have some results for QED and also some for QCD.

Light cone quantization usually involves the solution of differential constraint equations. If the theory includes fermi fields we have the constraint equation for ψ_-:

$$\mathrm{i}\partial_- \psi_- = \mathrm{i}\gamma_\perp \cdot D_\perp \gamma^0 \psi_+. \tag{1}$$

That equation is of first order so we have one integration constant, which, below, is referred to as $\psi_-^0(x^+, x^\perp)$. If the theory is a gauge theory and both light-cone quantiation and light-cone gauge are used, we have the constraint equation for the field A^-:

$$\partial_-^2 A^- + \partial_- \partial_i A^i = -e\Psi_+^\dagger \Psi_+. \tag{2}$$

That equation is of second order and we have two integration constants, which, below, are referred to as $B(x^+, x^\perp)$ and $C(x^+, x^\perp)$

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In section 2 I shall consider the constraint equation for A^-. I shall provide a specification for B and C and shall include them in a calculation of the free gauge propagator . I shall then perform a calculation of the one-loop electron self energy in QED. It is the first successful calculation of that process in a formulation that uses light-cone quantization and light-cone gauge, where the constraint equation (2) must be solved. In section 3 I shall consider the case of QCD and shall specify the integration constant, ψ_-^0, that is associated with the constraint equation for the quark field. We shall see that ψ_-^0 provides states that can, and do, mix with the light-cone vacuum; that mixing induces new operators into the dynamics that break chiral symmetry. In general, the integration constants are unphysical, nondynamical fields; but they affect the dynamics of physical fields and change the results of calculations of physical quantities.

2. QED

The fact that light-cone gauge requires the use of unphysical fields that are functions only of x^+ and $x^\perp$ has been know for some time. Bassetto [3] and his coworkers quantized QCD using equal-time quantization but light-cone gauge. They found that to get perturbative agreement with covariant gauges two auxiliary functions were needed. Since they did not use light-cone quantization they did not have a constraint equation to solve but found the two auxiliary functions as a part of normal (careful) canonical quantization. The two auxiliary functions modified the time ordered gauge field propagator and the modified propagator did give agreement with covariant guage calculations for the many loop processes that they calculated. Their auxiliary functions are the same as the B and C that we shall find below as integration constants for the light-cone constraint equation when we use light-cone quantization.

Some time later, Dave Robertson and I [4] quantized QED on the light-cone and found B and C as integration constants for equation (2). We then showed that the final result for free QED was the same operator solution found by Bassetto et. al. and therefore it had the same time-ordered propagator. We did not calculate the x^+-ordered propagator nor did we calculate any loop processes.

Later, Morara and Soldati [5] quantized QED in light-cone gauge on the surface $x^- = 0$ (that is equivalent to quantizing on $x^+ = 0$ using the gauge $A^- = 0$). They did not have a constraint equation to solve but they found B and C as a part of the application of the Dirac procedure. They calculated the x^--ordered propagator and also the one loop electron self energy. The electron self energy calculation agreed with the usual Feynman answer. Morara and Soldati believed that the operator solution given in [4] did not have a well defined x^+-ordered propagator.

Recently, Yuji Nakawaki and I [1] have quantized QED in a family of gauges and on a family of surfaces. Each case is and axial gauge of either the spatial or temporal type. Limiting cases give the case of light-cone gauge quantized on $x^- = 0$ (Morara and Soldati's case) or $x^+ = 0$ (the case I studied with Dave Robertson). We have shown that the propagator ordered in the direction away from the quantization surface is well defined in all cases if the constants B and C are properly included. We also performed a calculation of the electron's self energy in the case of light-cone gauge on the surface $x^+ = 0$ where the constraint equation (2) had to be solved. I am now going to discuss those calculations.

The operator solution to the free theory is easily found to be

$$A_\mu = a_\mu - \frac{\partial_\mu}{\partial_-} a_- - \frac{\delta_{\mu +}}{\partial_\perp^2} B - \frac{\partial_\mu}{\partial_\perp^2} C. \tag{3}$$

Here, B and C are the integration constants and the a's, B and C are given by

$$a_+(x) = \frac{-1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \frac{k_+}{k_\perp} \left\{ a_1(k_+) e^{-ik\cdot x} + a_1^\dagger(k_+) e^{ik\cdot x} \right\}, \tag{4}$$

$$a_-(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \frac{k_-}{k_\perp} \left\{ a_1(k_+) e^{-ik\cdot x} + a_1^\dagger(k_+) e^{ik\cdot x} \right\}, \tag{5}$$

$$a_i(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \epsilon_i^{(2)}(k) \left\{ a_2(k_+) e^{-ik\cdot x} + a_2^\dagger(k_+) e^{ik\cdot x} \right\}, \tag{6}$$

$$B(x) = \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} k_\perp^2 \left\{ B(k_+) e^{-ik\cdot x} + B^\dagger(k_+) e^{ik\cdot x} \right\} |_{x^-=0}, \tag{7}$$

$$C(x) = \frac{i}{\sqrt{(2\pi)^3}} \int d^3k_+ \sqrt{k_+} \left\{ C(k_+) e^{-ik\cdot x} - C^\dagger(k_+) e^{ik\cdot x} \right\} |_{x^-=0}, \tag{8}$$

where the ϵ's are standard polarization tensors:

$$\epsilon_\mu^{(2)}(k) = (0,\ 0,\ -\frac{k_2}{k_\perp},\ \frac{k_1}{k_\perp}), \tag{9}$$

and d^3k_+ is shorthand for $dk_+ d^2k_\perp$. The algebra of the a operators can be determined by standard canonical methods and is found to be

$$[a_\lambda(k_+),\ a_{\lambda'}(q_+)] = 0, \tag{10}$$

$$[a_\lambda(k_+),\ a_{\lambda'}^\dagger(q_+)] = \delta_{\lambda\lambda'} \delta^{(3)}(k_+ - q_+). \tag{11}$$

The algebra of the integration constant fields cannot be determined by standard canonical methods. We have determined that algebra by two methods: in ref. [4] it was determined by forcing agreement between the Heisenberg equations and the equations of motion; in [1] it was determined by analytically continuing from quantization on $x^- = 0$, where canonical methods can be used, to the quantization surface $x^+ = 0$. In either case, the algebra turns out to be

$$\begin{aligned}[B(k_+),\ C^\dagger(q_+)] &= [C(k_+),\ B^\dagger(q_+)] \\ &= -\delta^{(3)}(k_+ - q_+). \end{aligned} \tag{12}$$

Without the integration constants the x^+-ordered propagator is not a well defined object. The presence of B and C softens the singularities that arise from the part of the propagator derived using only the a fields. The worst singularity occurs in the $++$ component of the propagator. I now want to consider that component of the propagator and show how the singularity in the terms from the physical fields is partly cancelled by a singularity in the terms from B and C. To do that we need to do a change of variables in the above integrals by defining

$$k_+ \equiv \frac{k_\perp^2}{2k_-} \ ; \ d^3k_- \equiv dk_- d^2k_\perp. \tag{13}$$

We then find that

$$a_+(x) - \frac{\partial_+}{\partial_-} a_- = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_-}{\sqrt{k_-}} \frac{k_\perp}{k_-} \left\{ a_1(k_-) e^{-i(k_- x^- + k_\perp x^\perp + \frac{k_\perp^2}{2k_-} x^+} + CC \right\}. \tag{14}$$

With this result we can easily calculate that the contribution to the $++$ component of the propagator from the physical fields is given by

$$\langle 0|TA_+(x)A_+(y)|0\rangle|_a = \frac{1}{16\pi^3} \int \frac{d^3k_- k_\perp^2}{k_-^3} \left\{ e^{-i(k_-(x^- - y^-) + k_\perp(x^\perp - y^\perp) + \frac{k_\perp^2}{2k_-}(x^+ - y^+)} + CC \right\}. \tag{15}$$

Here, the T symbol indicates ordering with respect to the x^+ direction. We notice the very bad singularity at $k_- = 0$. In a similar way we find that the contribution from B and C is given by

$$\langle 0|TA_+(x)A_+(y)|0\rangle|_{BC} = \frac{-1}{16\pi^3} \int \frac{d^3k_- k_\perp^2}{k_-^3} \left\{ e^{-i(k_\perp(x^\perp - y^\perp) + \frac{k_\perp^2}{2k_-}(x^+ - y^+)} + CC \right\}. \tag{16}$$

Again we have a bad singularity at $k_- = 0$. Putting these two results together we find that the full $++$ component of the propagator is given by

$$D_{++}(x,y) = \langle 0|TA_+(x)A_+(y)|0\rangle = \frac{1}{16\pi^3} \int \frac{d^3k_- k_\perp^2}{k_-^3} \left\{ e^{-i(k_\perp(x^\perp - y^\perp) + \frac{k_\perp^2}{2k_-}(x^+ - y^+)} \left[e^{-i(k_-(x^- - y^-)} - 1 \right] + CC \right\}. \tag{17}$$

As $k_- \to 0$ the quantity in brackets goes to zero, which softens the singularity and the propagator becomes well defined. The full calculation of the propagator gives the following results:

$$D_{ij}(q) = \frac{i\delta_{ij}}{q^2 + i\epsilon}, \tag{18}$$

$$D_{+i}(q) = D_{i+}(q) = \frac{i}{q^2 + i\epsilon} \cdot \frac{q_i}{q_- + i\epsilon \mathrm{sgn}(q_+)}, \tag{19}$$

$$D_{++}(q) = \frac{i}{q^2 + i\epsilon} \cdot \frac{2q_+}{q_- + i\epsilon\varepsilon(q_+)} - \frac{i}{2} \left\{ \frac{1}{(q_- + i\epsilon)^2} + \frac{1}{(q_- - i\epsilon)^2} \right\}. \tag{20}$$

The auxiliary fields B and C also make contributions to the dynamical operator P^-. For the free part of the operator we find

$$P_0^- = \int d^3x^- \left\{ \frac{1}{2}(f_{-+})^2 + \frac{1}{2}(f_{12})^2 + i\bar{\psi}\gamma^-\partial_-\psi \right\} + \int d^3x^+ B \frac{\partial_+}{\nabla_\perp^2} C, \quad (21)$$

where d^3x^- stands for $dx^- d^2x^\perp$. The interacting part of the operator is given by:

$$P_I^- = \int d^3x^- \left\{ J^\mu A_\mu + e^2 \bar{\psi}\gamma^\mu A_\mu \frac{\gamma^+}{2i\partial_-}\gamma^\nu A_\nu \psi - \frac{1}{2} J^+ \frac{1}{\partial_-^2} J^+ \right\}. \quad (22)$$

While B and C do not appear explicitly in this expression, they are present since they are a part of A_μ. In the calculation we are about to do they will affect the outcome in that they change the propagator, as we have seen.

2.1. THE ELECTRON'S SELF ENERGY

With all the work that has been done on the light-cone quantization of gauge theories it comes as a surprise to many people that the one-loop electron self energy had never, prior to ref. [1], been successfully calculated in the formulation using light-cone quantization and light-cone gauge such that we have to solve the constraint equation (2). That is the most common formulation discussed in the literature. The successful calculations of the one-loop electron self energy that are based on light-cone quantization and that I know of are as follows: Morara and Soldati [5] did a calculation using the anti-light-cone gauge ($A^- = 0$). Brodsky, Franke, Hiller, McCartor, Paston and Prokhvatilov [6] did two calculations: one in Feynman gauge; the other in light-cone gauge, but with a higher derivative included as a regulator so that A^- was a degree of freedom and equation (2) did not have to be solved. Langnau and Burkardt [7]also did a calculation in Feynman gauge; they did not start from the light-cone but took the amplitude over from equal-time quantization and did the p^+ integral to get a light-cone integral. All of these calculations have in common that A^- is a degree of freedom and equation (2) was not solved.

All of these calculations required regulation. Morara and Soldati used two Pauli-Villars photons. Brodsky, Franke, Hiller, McCartor, Paston and Prokhvatilov used one Pauli-Villars photon and one Pauli-Villars electron for the Feynman gauge calculation, but for the light-cone gauge calculation had to use three Pauli-Villars electrons three auxiliary photons and two momentum cutoffs.

If the standard light-cone gauge formulation, where the constraint equation (2) is solved to write A^- in terms of ψ and $A^\perp$, is used, and if the integration constants B and$C$ are set to zero, one obtains the following expression for the amplitude [6]

$$\frac{\alpha}{4\pi} \int dx dz \frac{1}{(1-x)} \frac{(\frac{2}{x^2} - \frac{2}{x} + 1)z + m^2x^2}{m^2x(1-x) - m^2x - \mu^2(1-x) - z}. \quad (23)$$

The very strong singularity at $x = 0$ renders this expression meaningless. Whatever regulators are used to control the divergence, the answer has a very strong dependence on the regulators and the expression (23) has no known use. A very useful point of comparison can be found in the paper by Brodsky, Roskies and Suaya [8]. They wrote down the amplitude in equal-time quantization then boosted the resulting expression to the infinite momentum frame. They obtained

$$\frac{\alpha}{4\pi} \int dx dz \frac{1}{(1-x)} \frac{z + m^2(-2 + 2x + x^2)}{m^2x(1-x) - m^2x - \mu^2(1-x) - z}. \quad (24)$$

If this expression is regulated with one Pauli-Villars photon one obtains the usual log dependence on the Pauli-Villars photon mass and the answer agrees with the usual Feynman answer. There is no doubt that (24) is the expression one should obtain in a correct light-cone procedure. In ref. [1] Nakawaki and I performed the calculation in light-cone gauge but included B and C in our calculation. I now want to discuss that calculation. For the lagrangian we take

$$\sum_{i=0}^{1}(-1)^{i+1}\frac{1}{4}F^{i\mu\nu}F^{i}{}_{\mu\nu} - NA_{-} + \sum_{i}\frac{1}{\nu_i}\bar{\psi}_i(i\gamma^\mu\partial_\mu - m_i)\psi_i - e\bar{\psi}\gamma^\mu\psi A_\mu, \quad (25)$$

where we have included two Pauli-Villars electrons and one Pauli-Villars photon and defined

$$\psi = \sum_{i=0}^{2}\psi_i \quad ; \quad A_\mu = \sum_{i=0}^{1}A^i_\mu, \quad (26)$$

so that the interaction contains flavor changing currents; the flavor changing currents are necessary in the present formulation. The flavor changing currents break gauge invariance but that breaking will be removed when we take the masses of the Pauli-Villars electrons to infinity. In the Lagrangian, N is the Nakanishi-Lautrup field, which is usually referred to as B, but which I have here labelled N to avoid confusion with the integration constant B. For conditions on the Pauli-Villars fields we take

$$\sum_{i=0}^{2}\nu_i = 0, \quad \delta m|_{m_0=0} = 0. \quad (27)$$

The first of these conditions is standard; the meaning of the second one is that we shall use the flexibility of having the second Pauli-Villars electron to set the shift in the electron's mass to zero if the bare mass of the electron is zero. That is, we shall use the second Pauli-Villars electron to impose the requirement of chiral symmetry. It is common experience that chiral symmetry, which is a dynamical symmetry in light-cone quantization, must be imposed by hand. These regulators are not quite enough: we must still regulate the spurious gauge singularity. In QCD it is crucial to define that singularity by the Mandelstam-Leibbrandt prescription. Here, the singularity is essentially cancelled and many ways of regulating it will work; we shall use the simple replacement

$$\frac{1}{x} \to \frac{1}{x+\epsilon}. \quad (28)$$

It is straightforward to write down the amplitude as

$$\Sigma(p) = \frac{ie^2}{(2\pi)^4}\int d^4q \sum_{i,j}\nu_i(-1)^j \frac{\gamma^\mu\{\gamma\cdot(p-q)+m_i\}\gamma^\nu}{(p-q)^2 - m_i^2 + i\varepsilon}\cdot\frac{-g_{\mu\nu} + \frac{n_\mu q_\nu + n_\nu q_\mu}{q_- + i\varepsilon \mathrm{sgn}(q_+)}}{q^2 - \mu_j^2 + i\varepsilon}. \quad (29)$$

If we use the gauge propagator derived above and the electron propagator that was derived long ago:

$$S_F = \frac{i}{(2\pi)^4}\int d^4p \;\; e^{-ip\cdot x}\left[\frac{(\not{p}+m)}{p^2 - m^2 + i\varepsilon} - \frac{1}{2}\frac{\gamma^+}{p^+}\right], \quad (30)$$

and after the integration is done we take the (finite) limit of the Pauli-Villars electron masses going to infinity so that the final answer is regulated by only one Pauli-Villars photon, a long calculation gives

$$\delta m = \frac{e^2}{(2\pi)^3 m_0}\int d^2q_\perp \int_0^1 dx \sum_j (-1)^j \frac{m_0^2(1+x)}{m_0^2x^2 + \mu_j^2(1-x) + q_\perp^2}. \quad (31)$$

While this integral does not have the same integrand as (24), an identity discovered by John Hiller [9] can be used to show that the integrals are the same. I want to mention a few noteworthy features of the calculation: the contribution from the second line of (20) (these come from the first term in (22)) cancel the contribution from the last term in (22). The contributions from the second term in (22) are independent of masses and sum to zero in the sum over Pauli-Villars fields. Similarly, the contributions from the second term in (30) are independent of masses and cancel in the sum over Pauli-Villars fields.

The important point is that the inclusion of the auxiliary fields, B and C, transforms the useless expression (23) into an expression that, using standard regulation techniques, gives the standard answer. It is worthwhile to note that the method of regulation used in the calculation presented here is more complicated than that needed in Feynman gauge, but considerably simpler than that needed in the higher-derivative-regulated light-cone gauge calculation. Since no one has yet managed to formulate the Light-cone quantization of QCD in Feynman gauge, if the methods presented here for QED can be extended to QCD, that might result in the simplification of practical calculations in QCD.

3. QCD

I now want to turn to QCD and turn from the constraint equation for the gauge field to the constraint equation for ψ_-. That constraint equation is

$$\mathrm{i}\partial_-\psi_- = \mathrm{i}\gamma_\perp \cdot D_\perp\gamma^0\psi_+. \tag{32}$$

The general solution to this equation can be written as

$$\psi_- = \psi_-^0(x^+, x^\perp) + \int dx^-\gamma_\perp \cdot D_\perp\gamma^0\psi_+, \tag{33}$$

where the antiderivative, $\int$, is to be taken to mean the result of the replacement

$$\mathrm{e}^{\mathrm{i}kx} \longrightarrow \frac{1}{\mathrm{i}k}\mathrm{e}^{\mathrm{i}kx} \tag{34}$$

in the Fourier transform of the integrand. In other words, we have written the general solution as the solution most commonly used in the literature plus the integration constant $\psi_-^0(x^+, x^\perp)$. In this section I want to discuss the fact that $\psi_-^0(x^+, x^\perp)$ provides states that can mix with the vacuum and that mixing will induce new operators into the dynamics. Those new operators will break chiral symmetry [2].

To see how this works, we need to rewrite the standard expansions of the fields in Fourier modes. We begin with the standard expansion for ψ_+ except that we do the fourier expansion only in x^-

$$\psi_{+,s}^{(a)}(0, x^-, x_\perp) = \frac{1}{\sqrt{\Omega_q}}\int_0^\infty dk^+ \left(b_s^{(a)}(k^+, x_\perp)\mathrm{e}^{-ik^+x^-} + d_{-s}^{(a)*}(k^+, x_\perp,)\mathrm{e}^{ik^+x^-}\right). \tag{35}$$

Here, a is a color index, s is a spin index and the modes satisfy the standard anticommutation relations

$$\{b_{s_1}^{(a)}(k^+, x_\perp), b_{s_2}^{(b)*}(p^+, y_\perp)\} = \delta(k^+ - p^+)\delta(x_\perp - y_\perp)\delta_{s_1s_2}\delta_{ab}\ , \tag{36}$$

with similar relations for anti-fermions d. The independent fermion ψ_-^0 is independent of x^- and may be expanded as

$$\psi_{-,s}^{(0)(a)}(x^+, x_\perp) = \frac{1}{\sqrt{\Omega_q}}\int_0^\infty dk^- \left(\beta_s^{(a)}(k^-, x_\perp)\mathrm{e}^{-ik^-x^+} + \delta_{-s}^{(a)*}(k^-, x_\perp,)\mathrm{e}^{ik^-x^+}\right). \tag{37}$$

For gluons we write

$$A_s^{(c)}(0, x^-, x_\perp) = \frac{1}{\sqrt{\Omega_g}}\int_0^\infty dk^+ \frac{1}{\sqrt{2k^+}} \left(a_s^{(c)}(k^+, x_\perp)\,\mathrm{e}^{-ik^+x^-} + a_{-s}^{(c)^*}(k^+, x_\perp,)\,\mathrm{e}^{ik^+x^-}\right), \tag{38}$$

where

$$[a_{s_1}^{(b)}(k^+, x_\perp), a_{s_2}^{(c)*}(p^+, y_\perp)] = \delta(k^+ - p^+)\delta(x_\perp - y_\perp)\delta_{s_1s_2}\delta_{ab}\ . \tag{39}$$

In the discrete case the above integrals over $k^\pm$ become discrete sums for integers n of $\pi n/L$ (bosons) or $\pi(2n-1)/2L$ (fermions).

At fixed $x^\perp$ each of the fermi fields expanded above is just a one dimensional fermi field and can be bosonized in the standard way. For the ψ_+ field we write

$$\psi_{+,s}^{(a)}(0, x^-, x^\perp) = Z_+\mathrm{e}^{-\lambda_s^{(a)(-)}(x^-,x_\perp)}\sigma_{+,s}^{(a)}(x_\perp)\mathrm{e}^{-\lambda_s^{(a)(+)}(x^-,x_\perp)}, \tag{40}$$

where Z_+ is a renormalization constant, $\sigma_{+,s}^{(a)}(x_\perp)$ is a spurion and

$$\lambda_s^{(a)(+)}(x^-, x_\perp) = -\int_0^\infty dk^+ \frac{1}{k^+}C_{+,s}^{(a)}(k^+, x_\perp)(\mathrm{e}^{-ik^+x^-} - \theta(k - k^+)), \tag{41}$$

with

$$\lambda_s^{(a)(-)}(x^-, x_\perp) = -\lambda_s^{(a)(+)^*}, \tag{42}$$

and

$$C_{+,s}^{(a)}(k^+, x_\perp) = \int_0^{k^+} dq^+ d_{-s}^{(a)}\left(q^+, x_\perp\right) b_s^{(a)}\left(k^+ - q^+, x_\perp\right) + \int_0^{\infty} dq^+ b_s^{(a)*}\left(q^+, x_\perp\right) b_s^{(a)}\left(k^+ + q^+, x_\perp\right) - \int_0^{\infty} dq^+ d_{-s}^{(a)*}\left(q^+, k_\perp\right) d_{-s}^{(a)}\left(k^+ + q^+, x_\perp\right). \tag{43}$$

In the discrete case the $\theta(k - k^+)$ term is missing. We make a similar expansion for ψ_- but in that case the operators corresponding to the $C_{+,s}^{(a)}$ are all unphysical and we only need the spurion, $\sigma_{-,s}^{(a)}(x_\perp)$.

The only operators allowed to dress the light-cone bare vacuum are the spurions: the $C_{+,s}^{(a)}$ all carry nonzero + momentum and cannot dress the vacuum for the usual reasons. The corresponding operators from the ψ_- field are unphysical and cannot appear in physical states. If chiral symmetry is to be spontaneously broken we must dress the vacuum. We shall now assume that there is a component of the vacuum that contains only fermions, no gluons. There will be sectors of the vacuum which do contain glue so we are discussing only part of the vacuum state. For the component that contains only fermions, color symmetry and Lorentz invariance determine that the state must be of the form

$$|\Omega_f\rangle = F[\sigma_{-,s}^{(a)*}(x_\perp)\sigma_{+,-s}^{(a)}(x_\perp), \sigma_{+,-s}^{(a)*}(x_\perp)\sigma_{-,s}^{(a)}(x_\perp)]|0\rangle. \tag{44}$$

Notice that a state of this form will give a contribution to the chiral condensate $\langle\bar{\psi}\psi(x_\perp)\rangle$ of the form

$$\langle\bar{\psi}\psi(x_\perp)\rangle \supset \langle\Omega_f|\sum_{s,a}\sigma_{-,s}^{(a)*}(x_\perp)\sigma_{+,-s}^{(a)}(x_\perp) + c.c.|\Omega_f\rangle. \tag{45}$$

That expression may not be equal to the chiral condensate because sectors of the vacuum that contain glue may also contribute the the chiral condensate.

The fact that ψ_-^0 is not zero induces new operators into P^-. Basically every term in P^- that contains ψ_- will obtain one of these induced operators. If the quark bare mass is zero the only term that contains ψ_- is $-\bar{\psi}\gamma_\perp iD_\perp\psi$. Here, I shall discuss the operators that come from that term. There is also an induced operator associated with the $\bar{\psi}\psi$ term; some discussion of that term can be found in my talk at the Amsterdam meeting in 2004 [10]. With the quark bare mass taken to be zero there are two induced operators. A straightforward calculation gives them as

$$I_1 = \int dx^-\ d^2x^\perp \sum_a (i\partial_\uparrow\psi_{+,\downarrow}^{(a)*})\psi_{-,\downarrow}^{0(a)} + c.c. - [\downarrow\leftrightarrow\uparrow], \tag{46}$$

and

$$I_2 = g\int dx^-\ d^2x^\perp \sum_{abc} \lambda_{ab}^c\psi_{+,\downarrow}^{(a)*}\psi_{-,\downarrow}^{0(b)}A_\uparrow^{(c)} + c.c. - [\downarrow\leftrightarrow\uparrow], \tag{47}$$

where we have defined

$$\partial_\uparrow = (\partial_1 - i\partial_2)/\sqrt{2}\ , \qquad \partial_\downarrow = \partial_\uparrow^*. \tag{48}$$

In terms of the spurions these are

$$I_1 = Z_-\int dx^-\ d^2x^\perp \sum_a \Big\{(i\partial_\uparrow\psi_{+,\downarrow}^{(a)*})\sigma_{+,\uparrow}^{(a)}\left[\sigma_{+,\uparrow}^{(a)*}\sigma_{-,\downarrow}^{(a)}\right] + \sigma_{+,\downarrow}^{(a)*}(i\partial_\uparrow\psi_{+,\uparrow}^{(a)})\left[\sigma_{-,\uparrow}^{(a)*}\sigma_{+,\downarrow}^{(a)}\right] - \sigma_{+,\uparrow}^{(a)*}(i\partial_\downarrow\psi_{+,\downarrow}^{(a)})\left[\sigma_{-,\downarrow}^{(a)*}\sigma_{+,\uparrow}^{(a)}\right] - (i\partial_\downarrow\psi_{+,\uparrow}^{(a)*})\sigma_{+,\downarrow}^{(a)}\left[\sigma_{+,\downarrow}^{(a)*}\sigma_{-,\uparrow}^{(a)}\right]\Big\}; \tag{49}$$

$$I_2 = gZ_- \int dx^- \, d^2x^\perp \sum_{abc}$$
$$\lambda^c_{ab} \Big\{ A^{(c)}_\uparrow \psi^{(a)*}_{+,\downarrow} \sigma^{(b)}_{+,\uparrow} \left[\sigma^{(b)*}_{+,\uparrow} \sigma^{(b)}_{-,\downarrow}\right]$$
$$+ A^{(c)}_\uparrow \sigma^{(a)*}_{+,\downarrow} \psi^{(b)}_{+,\uparrow} \left[\sigma^{(a)*}_{-,\uparrow} \sigma^{(a)}_{+,\downarrow}\right]$$
$$- A^{(c)}_\downarrow \sigma^{(a)*}_{+,\uparrow} \psi^{(b)}_{+,\downarrow} \left[\sigma^{(a)*}_{-,\downarrow} \sigma^{(a)}_{+,\uparrow}\right]$$
$$- A^{(c)}_\downarrow \psi^{(a)*}_{+,\uparrow} \sigma^{(b)}_{+,\downarrow} \left[\sigma^{(b)*}_{+,\downarrow} \sigma^{(b)}_{-,\uparrow}\right] \Big\}. \quad (50)$$

I now want to try to explain why, in spite of the presence of ψ^0_- the eigenvalue equation for bound states can be solved using only the usual light-cone subspace. I shall denote any of the usual light-cone operators — the ones that do not involve ψ^0_- — with a subscript P and call them physical operators. These operators are functionals of the independent physical fields $\psi_=$ and $A_\perp$: $\mathcal{O}_P \equiv F(\psi_+, A_\perp)$. I define a subspace S_0 to consist of all states that result from the application of a physical operator to the bare light-cone vacuum: $S_0 \equiv \mathcal{O}_P|0\rangle$. I define a projection operator $\mathcal{P}$ to project onto S_0. The auxiliary operators are unphysical and the eigenvectors will be formed by physical operators on the physical vacuum:

$$(P^+P^- - P_\perp^2)|\Psi\rangle = M^2|\Psi\rangle, \quad (51)$$

where

$$|\Psi\rangle = \mathcal{O}_P|\Omega\rangle. \quad (52)$$

The full P^- will consist of physical operators, the induced operators we have found above and possibly other induced operators associated with the sectors of the vacuum that contain glue. I shall not discuss this last possibility in the present paper. The induced operators we have found in this paper have the form $I = I_P[\sigma^*_\pm \sigma_\mp]$ where I_P is a physical operator. We now need two results that depend on the specific form of our vacuum. The first is that, since the quanta from ψ_- appear in the vacuum pared with quanta from ψ_+, the only component of the vacuum in S_0 is the bare light-cone vacuum and we have that $\mathcal{P}|\Omega\rangle = c|0\rangle$. We can choose c to be zero. Also, if an operator of the form $[\sigma^*_\pm \sigma_\mp]$ is such a way as to remove all of the ψ_- quanta, it must also remove all of the ψ_+ quanta; so we have that $\mathcal{P}\sigma^{(a)*}_{-,s}(x_\perp)\sigma^{(a)}_{+,-s}(x_\perp)|\Omega\rangle = \kappa|0\rangle$, where κ is a real constant independent of s, a, and $x_\perp$. Therefore, if we write the eigenvector equation in the form

$$(P^-_P + I)\mathcal{O}_P|\Omega\rangle = M^2\mathcal{O}_P|\Omega\rangle, \quad (53)$$

then act with $\mathcal{P}$, we get that

$$\left(P^-_P + \kappa\, I_P\right)\mathcal{O}_P|0\rangle = M^2\mathcal{O}_P|0\rangle. \quad (54)$$

This is an equation formulated entirely in the usual light-cone subspace, S_0. It has the same eigenvalues as the full equation (51) and the eigenvectors of (54) are the projections of the eigenvectors of (54) onto S_0.

The induced operator, I will break quark helicity symmetry and split the masses of the pion and the rho. It is a soft breaking of the symmetry in the sense that if the vacuum is symmetric then κ is zero and there is no induced operator. In the light-cone representation, the dynamic effects of soft symmetry breaking occur as operators in the dynamics whose coefficients are condensates. I conclude this section with Fig. 1 showing the action of the induced operator, I as vertices; to do this it is convenient to define an effective coupling h as $h = gZ_-\kappa$ where Z_- is the wave function renormalization constant for the field ψ^0_- and will depend on the regulators used to define the theory.

4. FURTHER STUDIES

There is still much to be done in the area of finding and using the integration constants associated with light-cone constraint equations. Here I shall give only a few examples, on which I am working or hope to work:

The calculation of the one-loop electron self energy given above was done in the interaction representation. It is not straightforward to extend the calculation to nonperturbative calculations. The issue is the proper treatment of the term coupling the auxiliary fields to the physical fields. That term appears to be very singular, and while we know how to treat it in the perturbative calculation presented here, the proper treatment in a

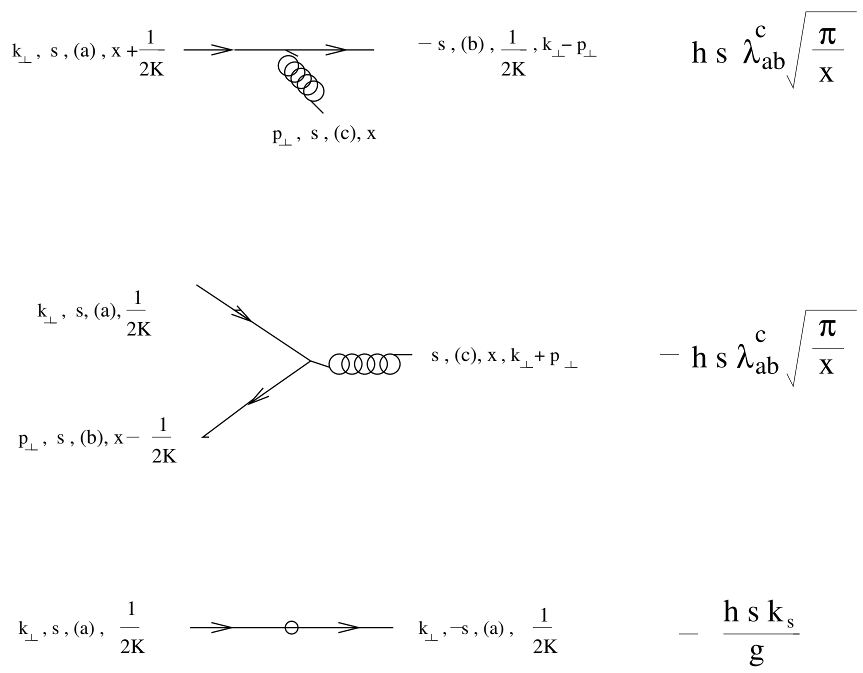

Figure 1. The new QCD vertices created by the induced operators I_1 and I_2.

nonperturbative calculation has not been worked out. That should be done. In a similar vein, the specification of the fields corresponding to B and C in the nonabelian case needs to be worked out.

In this paper I discussed one of the induced operators in QCD. There are no doubt more. In particular, there must be one associated with the glue in the vacuum that gives rise to the gluon condensate $\langle G^{\mu\nu\,(a)} G^{(a)}_{\mu\nu} \rangle$. That operator needs to be found. I believe that it is possible to derive the form of the vacuum functional, including the gluon contribution, from considerations of residual gauge transformations; but I do not now know how to do it. It would be a very big step forward to find such a derivation. It is also important to include the induced operators in calculation of the QCD spectrum in order to study their effects. Such calculations are just now being planned.

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