DØ - note 5171-Conf

Measurement of $m_{\rm top}$ in $e\mu$ Events with Neutrino Weighting in Run II at DØ

The DØ Collaboration
URL http://www-d0.fnal.gov

We present a measurement of the mass of the top quark, $m_{\rm top}$, in $e\mu$ final states from top pair production. We utilize expected neutrino rapidity distributions to solve an otherwise underconstrained kinematic fit. We use three different choices of parameters extracted from these weight distributions to obtain a top mass measurement in data: 1. the first two moments of the weight distribution. 2. the weight distribution rebinned in 5 bins, and 3. the mass with the maximum weight, A data sample of approximately 835 pb^{-1} of DØ Run-II data in the $e\mu$ channel is used for this analysis. The top quark mass extracted from 28 $e\mu$ candidate events is measured to be

$$m_{\rm top}^{\rm mom} = 171.6 \pm 7.9 \text{ (stat.)}^{+5.1}_{-4.0} \text{ (syst.) GeV}$$

using the main Moments Method. The alternative Binned Template Method gives 173.6 ± 6.7 (stat.)$^{+5.1}_{-4.0}$ (syst.) GeV. A further cross-check using the Maximum Method gives 165.7 ± 9.7 (stat.)$^{+4.4}_{-4.7}$ (syst.) GeV.

Preliminary Results for Summer 2006 Conferences

Contents

I. INTRODUCTION

In the Standard Model, mass is generated from the spontaneous breaking of electroweak symmetry via the Higgs mechanism. A precision measurement of the mass of the top quark provides information about the Higgs boson mass via corrections to the W boson mass. In this analysis, the top quark mass is measured in $e\mu$ decays of $t\bar{t}$ events; that is, events in which each top quark decays to a b quark and a W boson, and in which one W boson decays into an electron and a neutrino and one W boson decays into a muon and a neutrino.

The top quark mass is measured for events collected at DØ since August 2002 of Run II at the Tevatron, comprising approximately 835 pb^{-1} of data. The DØ detector is described in detail in [1].

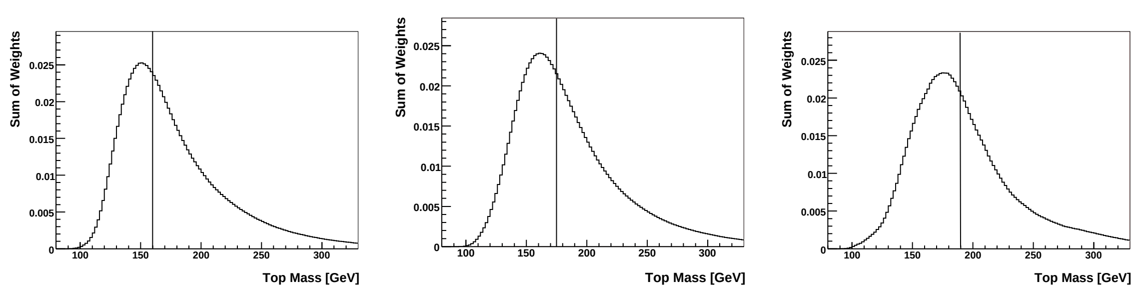
II. METHOD

In dilepton decays, the final state consists of six particles: two charged leptons, two jets from b quarks, and two neutrinos. The mass of each final particle is known *a priori*, so this results in 18 independent kinematic quantities in the final state. Fourteen of these parameters – the momenta of the charged leptons and jets and the transverse energy components of the neutrino pair – are measured directly in the detector. Three additional constraints are added by requiring that the invariant mass of each lepton and neutrino pair equal the W mass and that the mass of the top and anti-top quarks be equal. This leads to a total of seventeen constraints, which is one constraint short of allowing a solution for the system.

A solution is found by ignoring the observed missing transverse energy ($\not\!\!E_T$) from the neutrinos and instead assuming a top quark mass and a pseudorapidity for each neutrino. From this information, the four-momentum of each neutrino may be determined. The measured missing energy is then used to assign a weight to the solution, based on the agreement of the calculated transverse momentum of the neutrinos with the observed missing transverse energy ($\not\!\!E_T$):

$$\omega = \frac{1}{N_{\rm iter}} \sum_{i=1}^{N_{\rm iter}} \exp\left(\frac{-(\not\!\!E_{x,i}^{\rm calc} - \not\!\!E_x^{\rm obs})^2}{2\sigma^2_{\not E_x}} \right) \exp\left(\frac{-(\not\!\!E_{y,i}^{\rm calc} - \not\!\!E_y^{\rm obs})^2}{2\sigma^2_{\not E_y}} \right), \tag{1}$$

where the sum over $N_{\rm iter}$ indicates the sum over all solutions for all neutrino η assumptions and all permutations. This procedure is repeated for 10 pseudorapidity choices of each neutrino at the assumed top quark mass. The bins of pseudorapidities were chosen in a way that they are filled with equal statistics. A weight is formed by summing over the weights for each neutrino choice. At each mass, the jets and lepton momenta are smeared 150 times within detector resolutions, and the solution algorithm is repeated for each smearing. The weights for all smearings are summed together to form a total event weight at the assumed top quark mass. Weights are calculated in 2 GeV increments for top quark masses between 80 and 330 GeV. When these weights are summed over a large number of Monte Carlo events, the distributions produce a peak near the input top quark mass, as shown in Fig. 1.



(a) (b) (c)

FIG. 1: Sum of event weights for a few thousand $t\bar{t}$ Monte Carlo events with top quark mass of (a) 160, (b) 175, and (c) 190 GeV. The vertical lines indicate the mass of the Monte Carlo top quarks.

TABLE I: Expected and observed yields for signal and background, after all selection cuts are applied.

$t\bar{t} \rightarrow e\mu$	WW	$Z \rightarrow \tau\tau$	fake e	background sum	total	observed
20.2 ± 2.7	$1.24^{+2.2}_{-0.5}$	$2.7^{+1.5}_{-1.3}$	0.4 ± 0.2	$4.4^{+2.6}_{-1.4}$	$24.6^{+3.8}_{-3.0}$	28

III. EVENT SELECTION

Dilepton events from the 835 pb^{-1} data were analyzed. All events are required to pass the selection criteria of the dilepton cross section analyses described in [2]. Each event is required to be triggered at least by one of six different combined electron muon triggers. These analyses require one identified isolated electron with pseudorapidity $|\eta| < 1.1$ or $1.5 < |\eta| < 2.5$ and $p_T > 15$ GeV, and one identified isolated muon with pseudorapidity $|\eta| < 2.0$ and $p_T > 15$ GeV. Selected muons need to be matched to a central track and to pass timing cuts against cosmics. Two jets with $p_T > 20$ GeV, as well as significant total energy from the jets and leading lepton (H_T^{ℓ}) in the event are required. The cut on the combined H_T^{ℓ} from the jets and leading lepton is chosen to be 120 GeV.
A cut on the 'electron likelihood' is used in place of the electron likelihood fit described in [2]. The electron likelihood for a candidate electron is formed from a combination of:

- f_{em}, the fraction of the candidate's energy deposited in the electromagnetic calorimeter
- E_T/p_T, the ratio of tranverse energy in the calorimeter cluster to the tranverse momentum of the matched central track
- $Prob(\chi^2_{\text{spatial}})$ of the match between the calorimeter cluster and a central track
- Distance of Closest Approach (DCA), which measures the shortest distance of the selected track to the line parallel to the $z-$axis which passes through the primary vertex position.
- Number of tracks in a $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} = 0.05$ cone, around and including the candidate track
- Total track p_T in a $\Delta R = 0.4$ cone around, but excluding the candidate track
- The χ^2 of a covariance matrix built up with a set of variables describing the shower shapes.

This likelihood variable distinguishes between electrons from physics process (which have a likelihood value near 1), and "fake" electron signals from pions (which have a value near 0).
Electrons with electron likelihood values less than 0.85 are rejected for the mass analysis. This dramatically reduces the background from fake electrons, while only minimally reducing the selection efficiency on real electrons.

Events which pass all selection cuts are then required to have at least one solution in the event-weighting scheme detailed in Section I. In that scheme, each neutrino momentum is obtained from the solution to a quadratic equation. Thus, it is possible that the solved momentum for one or both neutrinos is imaginary. In some events, there is no choice of top mass or neutrino rapidity that yields a real solution for both neutrinos. The requirement of at least one real event weight solution acts as a *de facto* additional cut on the selected events, further reducing the expected event yield. All 28 events that pass the kinematic selection cuts produce real solutions. The expected number of signal and background events are shown in Table I.

IV. TEMPLATES

For each event, a weight distribution as function of the top quark mass hypothesis is generated from the weighted neutrino solutions (Neutrino Weighting Method). An "event weight vector" filled with parameters characterizing the weight distribution is produced. We use three different choices of variables to form this vector:

1. **Moments method (main):** a 2-dimensional event weight vector consists of the first two moments (mean and root-mean-square (RMS)) of the weight distribution.

2. **Binned Template method (alternative):** a 5-dimensional event weight vector is produced by rebinning the event weight distribution coarsely into 5 bins.

3. **Maximum method (cross check):** a 1-dimensional event weight vector consists of the maximum of the weight distribution.

Thus, every event is characterised by a d-dimensional vector $\vec{w}_{ev}$ of weight distributions, with $d = 2$ for the **Moments Method**, $d = 5$ for the **Binned Template Method** and $d = 1$ for the **Maximum Method**.

Weight vector sets for the three above methods are generated for Monte Carlo samples of $t\bar{t}$ to $e\mu$ decays for various top quark masses. We call them templates. Each Monte Carlo sample contains events that were generated with PYTHIA [5]. Fragmentation and hadronization and decays of short-lived particles were modeled with PYTHIA, as well. A full detector simulation was done with GEANT [6]. Ten independent Monte Carlo samples were used, for top quark masses ranging from 155 GeV to 200 GeV in steps of 5 GeV. Template sets are also generated for physics background events. All events are required to pass the selection criteria described in Section II.

V. MAXIMUM LIKELIHOOD

Each individual weight vector $\vec{w}_{ev}$ from data is compared to weight vectors $\vec{w}_i$ from the MC templates. For the **Moments Method** and **Binned Template Method** an estimate of the signal probability density f_s is made by placing a Gaussian kernel of width h at the event weight of each MC event, so that:

$$f_s(\vec{w}_{ev} \mid m_{\rm top}^{\rm MC}) = \frac{1}{(h\sqrt{2\pi})^d N^{\rm MC}(m_{\rm top}^{\rm MC})} \sum_{i=1}^{N^{\rm MC}(m_{\rm top}^{\rm MC})} \prod_{j=1}^{d} e^{\left[\frac{-(w_{ev,j}-w_{i,j})^2}{2h^2}\right]} \tag{2}$$

A similar estimate is made of the background probability density. For multiple background sources, each source is scaled by a relative weight b_k, which is defined by the expected number of background events $\bar{n}_k$ for each background k and the number of Monte Carlo weight vectors for each background:

$$\frac{b_k N_k^{\rm MC}}{\sum_{k=1}^{N_{\rm source}} b_k N_k^{\rm MC}} = \frac{\bar{n}_{b,k}}{\sum_{k=1}^{N_{\rm source}} \bar{n}_{b,k}}. \tag{3}$$

The probability density estimate (PDE) for the background may thus be written as:

$$f_b(\vec{w}_{ev}) = \frac{1}{(h\sqrt{2\pi})^d} \frac{1}{\sum_{k=1}^{N_{\rm source}} b_k N_k^{\rm MC}} \sum_{k=1}^{N^{\rm source}} b_k \sum_{i=1}^{N_k^{\rm MC}} \prod_{j=1}^{d} e^{\left[\frac{-(w_{ev,j}-w_{ik,j})^2}{2h^2}\right]}. \tag{4}$$

For the **Maximum Method** the signal probability density distribution f_s is determined by fitting an analytic function to the 2-dimensional distribution of maxima of the weight distributions $m_{\rm top}^{\rm hypothesis}$ and the generated top quark mass $m_{\rm top}^{\rm generated} \equiv m_{\rm top}^{\rm MC}$. This procedure accounts for correlations between Monte Carlo samples for different generated top quark masses and minimizes fit errors when performing fits to the likelihood. The background probability function f_b depends only $m_{\rm top}^{\rm hypothesis}$ and is fitted by an analytic function as well.

A likelihood function is formed at each top quark mass point $m_{\rm top}^{\rm MC}$ by combining the signal and background probability density functions f_s and f_b with the number of signal (n_s) and background (n_b) events. A gaussian constraint $\mathcal{L}_{\rm gaus}(n_b, \bar{n}_b, \sigma_b)$ forces consistency between the observed and expected number of background events, and a poisson constraint $\mathcal{L}_{\rm poisson}(n_s + n_b, N)$ ensures agreement between the observed number of events N and the sum of n_s and n_b. The combined likelihood function is:

$$\mathcal{L}(\vec{w_{ev}}, \bar{n_b}, N \mid m_{\rm top}^{\rm MC}, n_s, n_b) = \mathcal{L}_{\rm gaus}(n_b, \bar{n}_b, \sigma_b)\mathcal{L}_{\rm poisson}(n_s + n_b, N) \prod_{i=1}^{N} \frac{n_s f_s(\vec{w_i} \mid m_{\rm top}^{\rm MC}) + n_b f_b(\vec{w_i})}{n_s + n_b}. \tag{5}$$

The values of n_s and n_b at each Monte Carlo top mass are not fixed, and the likelihood function is minimized with respect to these variables at each generated top quark mass. A polynomial fit is performed on the minimized points, and an overall minimum is determined from the fit, which is our estimate for the top quark mass $m_{\rm top}$. The statistical error $\sigma_{m_{\rm top}}$ is estimated by calculating the masses at which the $-\ln\mathcal{L}$ value is half a unit above its minimum.

The performance of the 3 methods is evaluated using ensemble testing techniques: the top quark mass $m_{\rm top}$ is extracted in 500 pseudo-experiments performed on ensembles of 28 events each. The events are chosen randomly from the signal and background Monte Carlo samples, so that the average number of background events per source matches the expected yield.

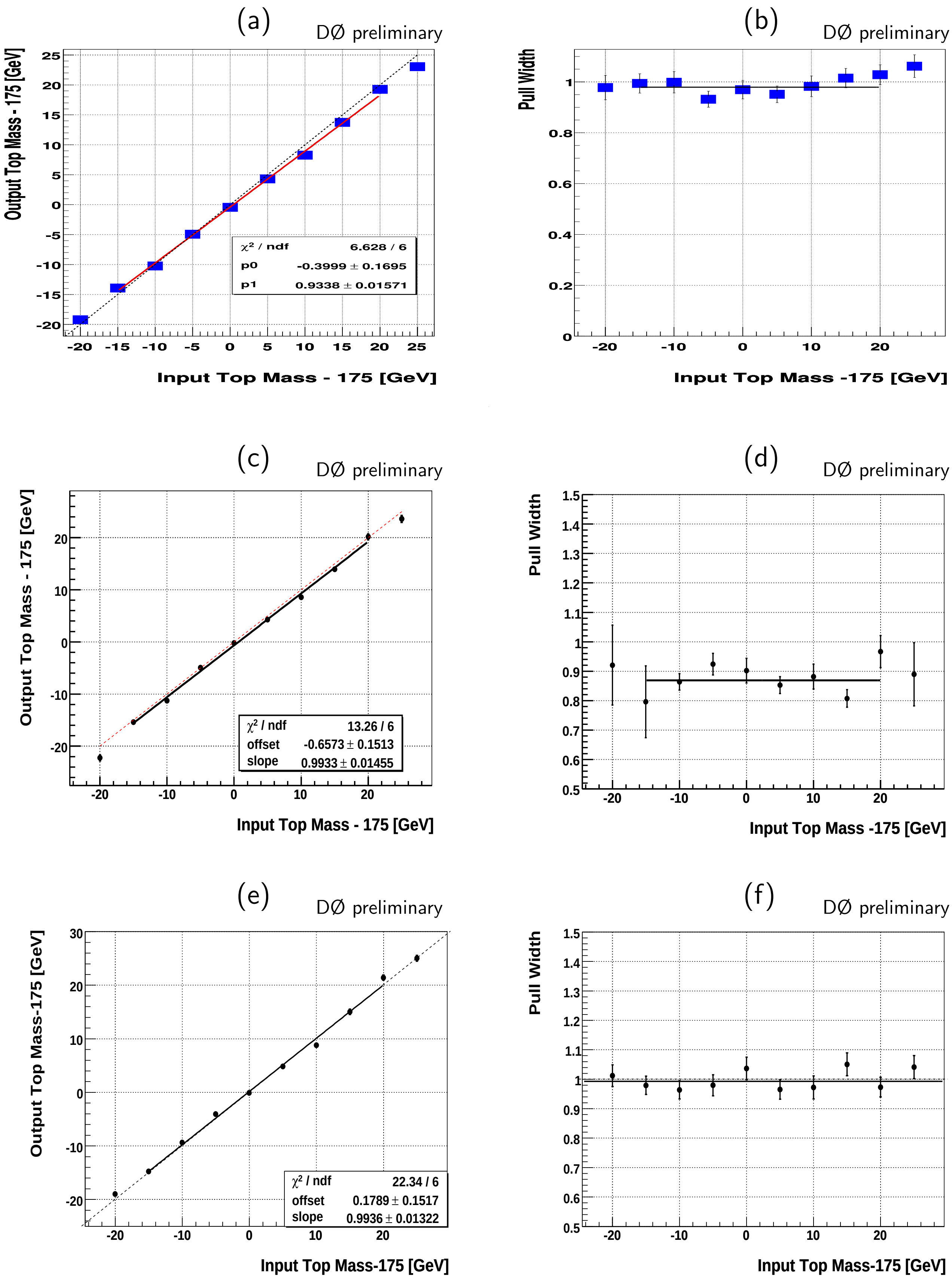


FIG. 2: Calibration Curves: top quark mass estimate as function of generated MC input top quark mass for the **Moments Method** (a), **Binned Template Method** (c), and **Maximum Method** (e). Pull width distributions for the **Moments Method** (b), **Binned Template Method** (d), and **Maximum Method** (f).

Method	slope	offset [GeV]	$\langle$pull width$\rangle$	$\langle \sigma_{m_{\text{top}}} \rangle$ [GeV]
Moments Method	0.93 ± 0.02	-0.30 ± 0.18	0.97 ± 0.01	9.2
Binned Template Method	0.99 ± 0.01	-0.66 ± 0.15	0.86 ± 0.01	8.2
Maximum Method	0.99 ± 0.01	0.18 ± 0.15	0.99 ± 0.01	8.8

TABLE II: Slope and offset of the calibration curves in Figure 2, the pull width after calibration, and the mean value of the statistical uncertainty after calibration and correction for the pull width for the three methods.

Figure 2 shows a good agreement between the output and input top quark mass and demonstrates the sanity of the statistical error estimation with widths of pull distributions near their expected value of 1.0. The linear fits in Figure 2 are used as a calibration tool on the results from data, mapping the output minimum to an input top quark mass.

The results of the fits are summarized in Tab. II.

VI. RESULTS

The results of the likelihood fits are shown in Figure 3. At each test top quark mass point, data event weight distributions are compared to Monte Carlo background event templates and the Monte Carlo signal template for that particular mass. The top quark mass estimates and uncertainties are corrected to account for the calibration from ensemble tests and the non-unit pull widths.

The measured top mass for the three listed methods after calibration yields: 171.6±7.9 GeV for the **main Moments Method**, 173.6±6.7 GeV for the **alternative Binned Template Method** and 165.7±9.7 GeV for the **cross-check Maximum Method**. It has been checked that the differences in the output top mass for identical ensembles between the **Binned Template Method** and the **Maximum Method** are significantly smaller than the estimated statistical uncertainties. Furthermore, it has been shown that a difference as in the final result is within the 2σ expectation.

A. Statistical Uncertainties

The statistical uncertainties quoted above are calculated from the quadratic fit to the likelihood minima. The distribution of expected statistical errors for the three Methods are shown in Fig 4.

B. Systematic Uncertaintiess

A summary of all systematics is given below.

- Jet energy scale. The main systematic uncertainty is expected to arise from the uncertainty in the jet energy scale. To evaluate its impact on our final result, the following procedure is used. First, the signal and background probability density distributions are generated with regular, unshifted Monte Carlo events. In the second step, pseudo-experiments with their jet energy scale shifted by 1σ using the original probability density functions are analyzed. This is done for all generated Monte Carlo top quark masses and a new calibration curves are produced. To estimate the error due to unprecise knowledge of the jet energy scale, the difference between the calibration curve for events with jet energy scale shifted up by $+1\sigma$ and down by -1σ is taken and divided by two.

- *Jet resolution.* Jets from a Monte Carlo sample with m_{top}=175 GeV are smeared by $\pm 1\sigma$, where σ is the width of the measured jet resolution distribution. Event selection is performed on the smeared events, and event weights are generated for the selected events. These weights are compared to the original Monte Carlo templates.

- *Background yields.* The yields for background has some uncertainty and there is a systematic error due to it. Also, control samples with low jet multiplicities have more events than expected. To account for this we make ensemble tests assuming nominal background yields and then double the dominant background yield ($Z \to \tau\tau$). The difference in expectations was taken as systematic error due to uncertainty on background yield. It is equal to 1.0 GeV.

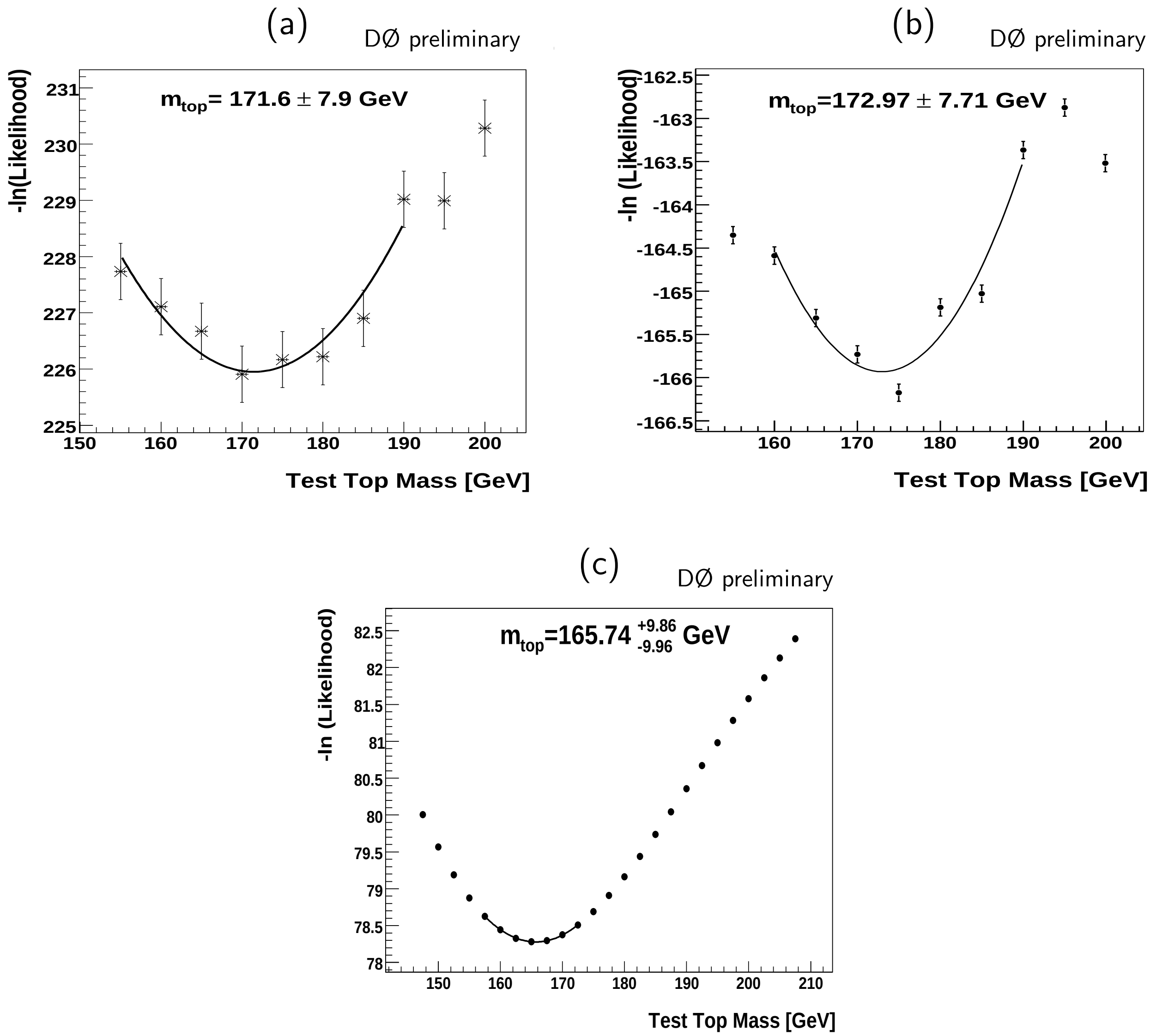


FIG. 3: The negative logarithmic likelihood for the 28 data events in the $e\mu$ channel for the 835 pb^{-1} dataset collected in part of Run IIa at the DØ detector for the **Moments Method** (a), **Binned Template Method** (b) and **Maximum Method** (c).

- *Background Template Shape Uncertainty.* To estimate the systematic error of mass measurement due to varying background shape, a very conservative approach was used. We replaced the templates for the dominant $Z \to \tau\tau$ background with those generated for the WW background when making ensembles. The difference in fitted top mass for the generated of 175 GeV was taken as a systematic error.

A list of all evaluated systematic uncertainties appears in Table III. The systematic uncertainties are dominated by jet energy scale uncertainties. The combined systematic uncertainty is $^{+5.1}_{-4.0}$ GeV for the **Moments Method** and **Binned Template Method** and $^{+4.4}_{-4.7}$ GeV for the **Maximum Method**.

VII. CONCLUSION

An analysis of 835 pb^{-1} of proton-antiproton collision data at a center-of-mass energy of $\sqrt{s} = 1.96$ TeV recorded by the DØ experiment at the Tevatron is presented. Using the neutrino weighting method a top mass estimate is

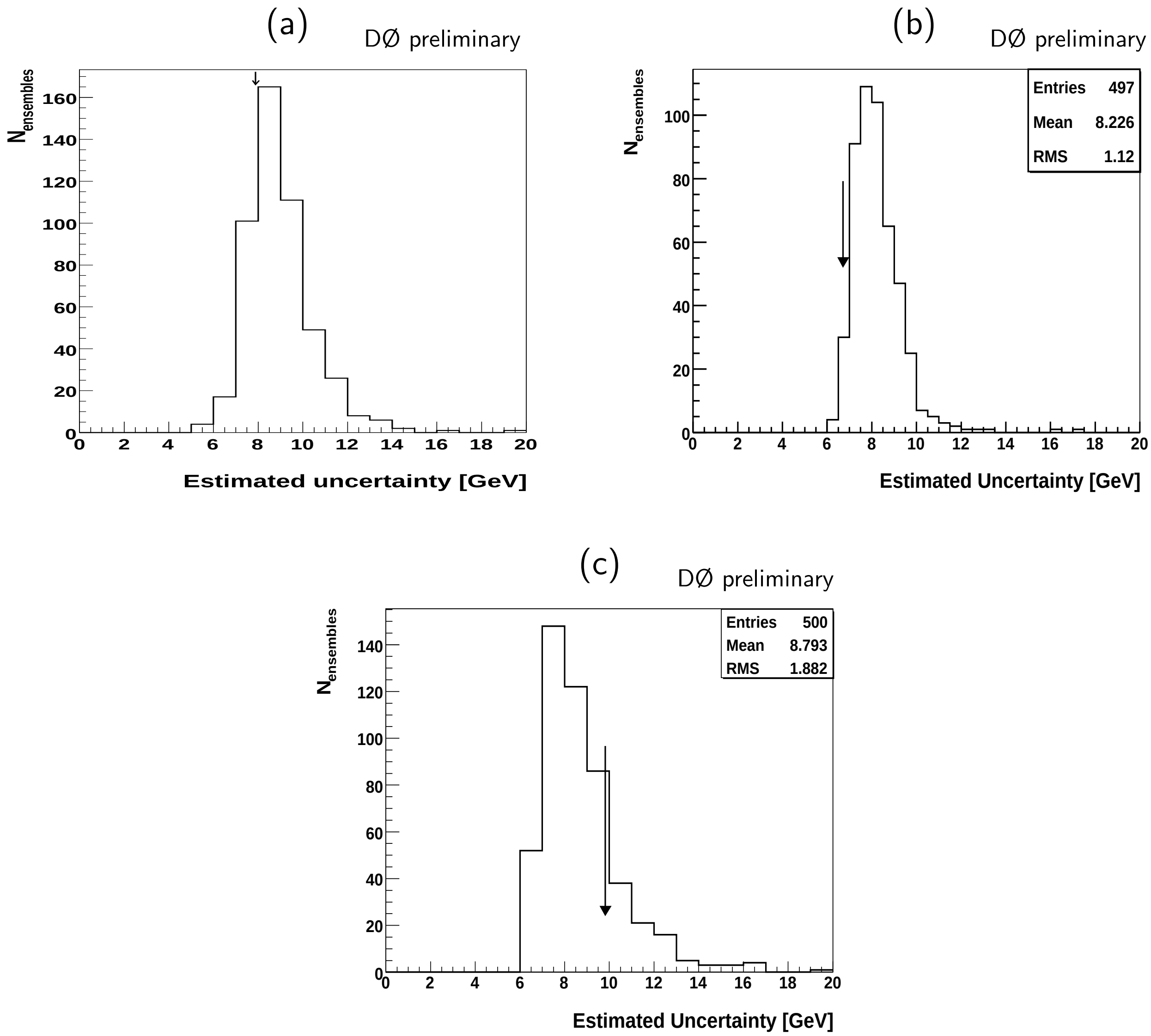


FIG. 4: Distribution of statistical uncertainties after correcting for the pull width for $m_{\rm top}^{\rm MC}$=175 GeV. Results are shown for the **Moments Method** (a), the **Binned Template Method** (b) and the **Maximum Method** (c).

extracted from $e\mu$ final states in top pair production. Three different treatments of the underlying weight distributions are utilized for the kinematic fits. These methods are found to provide similar expected sensitivity to the actual top mass. The results of these fits are the following:

$$\begin{aligned}
\textbf{Moments Method}: m_{\rm top} &= 171.6 \pm 7.9\ ({\rm stat.})\ ^{+5.1}_{-4.0}\ ({\rm syst.})\,{\rm GeV} \\
\textbf{Binned Template Method}: m_{\rm top} &= 173.6 \pm 6.7\ ({\rm stat.})\ ^{+5.1}_{-4.0}\ ({\rm syst.})\,{\rm GeV} \\
\textbf{Maximum Method}: m_{\rm top} &= 165.7\ \ ^{+9.7}_{-9.8}\ ({\rm stat.})\ ^{+4.4}_{-4.7}\ ({\rm syst.})\,{\rm GeV}
\end{aligned}$$

Due to earlier studies which produced an optimization to the Moments method, that method is quoted as the result. The alternative Binned Template method confirms this result. The Maximum method provides a valuable additional cross-check.

TABLE III: Summary of systematic uncertainties for a combined measurement.

Source	Error (GeV)
Jet Energy Scale (**Moments** and **Binned Template Method**)	$^{+4.4}_{-3.0}$
Jet Energy Scale (**Maximum Method**)	$^{+3.6}_{-3.9}$
Jet Resolution	± 0.4
Muon Resolution	± 0.4
$t\bar{t}$ + jets	± 2.0
PDF variation	± 0.7
Background Template Shape	± 0.3
Background Yields	± 1.0
Template fit statistics	± 0.9
Total Systematic Error (**Moments** and **Binned Template Method**)	$^{+5.1}_{-4.0}$
Total Systematic Error (**Maximum Method**)	$^{+4.4}_{-4.7}$

[1] V.M Abazov, et al., "The Upgraded DØ Detector," Fermilab-Pub-05/341-E (2005).
[2] M. Besancon, F. Deliot, and V. Shary, "Measurement of the $t\bar{t}$ Production Cross Section at $\sqrt{s} = 1.96$ TeV in Electron Muon Final States, v1.5," DØ Preliminary Note 4877 (February 2005)
[3] O. Brandt et. al. "Measurement of m_{top} in $e\mu$ Events with Neutrino Weighting" D0 Note 5162
[4] V. Barger, J. Ohnemus, and R.J.N. Phillips, Phys. Rev. D **48**, 3953 (1993).
[5] T. Sjöstrand, L. Lönnblad, and Stephen Mrenna, "PYTHIA 6.2 Physics and Manual," arXiv:hep-ph/0108264.
[6] R. Brun *et al.*, "GEANT - detector description and simulation tool," CERN Program Library Vers. 3.21, W5013 (1993).
[7] A.G Frodesen, O. Skjeggstad, and H. Tøofhte, *Probability and Statistics in Particle Physics*, Universitetsforlaget (1979).
[8] The DØ Collaboration, "Top Quark Mass Measurement with the Matrix Element Method in the Lepton+Jets Final State at DØ Run II," DØ Note 4874-CONF.