Zero momentum modes in discrete light-cone quantization

Sofia S. Chabysheva

Department of Physics
Southern Methodist University
Dallas, TX 75275
and
Department of Physics
University of Minnesota-Duluth
Duluth, Minnesota 55812

John R. Hiller

Department of Physics
University of Minnesota-Duluth
Duluth, Minnesota 55812

(Dated: March 6, 2009)

Abstract

We consider the constrained zero modes found in the application of discrete light-cone quantization (DLCQ) to the nonperturbative solution of quantum field theories. These modes are usually neglected for simplicity, but we show that their inclusion can be relatively straightforward, and, what is more, that they are useful for nonperturbative calculations of field-theoretic spectra. In particular, inclusion of zero modes improves the convergence of the numerical calculation and makes possible the direct calculation of vacuum expectation values, even when the zero modes are determined dynamically.

PACS numbers: 11.10.Ef,11.30.Qc

I. INTRODUCTION

The technique of discrete light-cone quantization (DLCQ) [1, 2, 3, 4] has been employed many times for the nonperturbative solution of various field theories, particularly in two dimensions. This includes recent calculations of eigenstates in supersymmetric Yang–Mills theories [5] and ϕ^4 theory [6, 7], as well as the early applications to Yukawa theory [1], ϕ^3 and ϕ^4 theories [8, 9], QED [10], and QCD [11]. The method is based on light-front quantization [12] and discretization of the single-particle momentum modes. Those modes with zero momentum, zero modes, are not dynamical but are instead constrained by the spatial average of the Euler–Lagrange equation for the field [13, 14]. As such, the zero modes are usually neglected, because the constraint equation is considered too difficult to solve.

The neglect of zero modes can have various consequences, ranging from the benign, such as slowed convergence of a numerical calculation, to the serious, an absence of understanding of vacuum effects, particularly symmetry breaking [13, 14, 15, 16, 17]. Can they instead be included in some straightforward way? We argue here that they can. Although the constraint equation can be a nonlinear operator equation, the DLCQ approximation requires only a finite expansion of the solution in inverse powers of the resolution K, where discrete longitudinal momentum fractions are measured in multiples of $1/K$. The solution to the constraint equation can then be generated analytically order by order. For simplicity, we formulate the discussion in two dimensions, but the approach can be immediately extended to modes with zero longitudinal momentum in any number of dimensions.

The DLCQ approximation is equivalent to numerical quadrature for the field-theoretic mass eigenvalue problem, where the eigenstate is expanded in Fock states with momentum wave functions as coefficients. The eigenvalue problem can be reduced to coupled integral equations for these wave functions. The quadrature points, in terms of momentum fractions, are equally spaced by $1/K$. The zero-mode contributions are contributions from the numerical approximation where a momentum is zero but the integrand is nonzero, even though the wave function itself may be zero. To neglect such contributions is an error of order $1/K$, which delays convergence as $K \to \infty$, relative to the nominal trapezoidal quadrature error of $1/K^2$. Within the DLCQ approximation, such contributions appear as zero-mode contributions in the Hamiltonian. These typically take the form of effective interactions that include zero-mode exchange between dynamical modes [18, 19]. Higher-order quadrature schemes can also generate effective interactions, which can be derived from the numerical approximation to the coupled integral equations for the wave functions

In the remainder of the paper, we develop these ideas more fully. A general discussion is given in Sec. II, followed by three specific applications in Sec. III. The approach is summarized in the final Sec. IV.

II. ZERO MODES IN DLCQ

We consider various two-dimensional theories and the contributions to their light-front Hamiltonians from modes of zero momentum. Much of the notation is taken from earlier work on zero modes [16]. As light-front coordinates [12], we use $x^\pm = (x^0 \pm x^1)/\sqrt{2}$.

Each Lagrangian is of the general form

$$\mathcal{L} = \partial_+\tilde{\phi}\partial_-\tilde{\phi} \mp \frac{\mu^2}{2}\tilde{\phi}^2 - V(\tilde{\phi}) + \mathcal{L}_{\text{free}}^{\text{other}}, \tag{2.1}$$

with $\tilde{\phi}$ the scalar field of interest, V a generic interaction term which may include other fields, and $\mathcal{L}_{\rm free}^{\rm other}$ the free Lagrangian for any other fields. The mass term includes the possibility of a plus sign, in order to consider a ϕ^4 theory with tree-level symmetry breaking. For this Lagrangian, the Hamiltonian density is

$$\mathcal{H} = \pm\frac{\mu^2}{2}\tilde{\phi}^2 + V(\tilde{\phi}) + \mathcal{H}_{\rm free}^{\rm other}. \tag{2.2}$$

We apply DLCQ [1, 4] by imposing periodic boundary conditions on $\tilde{\phi}$ in the box $-L/2 < x^- < L/2$. A constant, zero-momentum mode ϕ_0 is separated from the other modes, so that we have $\tilde{\phi} = \phi + \phi_0$ and

$$\phi = \sum_{n>0} \frac{1}{\sqrt{4\pi n}} \left[e^{ik_n^+ x^-} a_n + e^{-ik_n^+ x^-} a_n^\dagger \right], \tag{2.3}$$

with $k_n^+ = 2\pi n/L$ and $[a_n, a_n^\dagger] = 1$. The zero mode, written $\phi_0 = \frac{1}{\sqrt{4\pi}} a_0$, is constrained [13, 14] by the Euler-Lagrange equation

$$(2\partial_+\partial_- \pm \mu^2)\tilde{\phi} = -V'(\tilde{\phi}), \tag{2.4}$$

which, after integration over the length of the box, yields

$$\mp \mu^2\phi_0 = \frac{1}{L}\int_{-L/2}^{L/2} V'(\phi+\phi_0)dx^-. \tag{2.5}$$

This constraint is to be solved, to determine ϕ_0 and, therefore, a_0 in terms of the dynamical modes.

The DLCQ Hamiltonian operator for evolution in light-front time x^+ is

$$\mathcal{P}^- = \int_{-L/2}^{L/2} dx^- \mathcal{H} = \pm\mu^2 \frac{L}{4\pi}\left[\Sigma_2 + \frac{1}{2}a_0^2\right] + \int_{-L/2}^{L/2} dx^- \left[V(\phi+\phi_0) + \mathcal{H}_{\rm free}^{\rm other}\right]. \tag{2.6}$$

As in [16], we define

$$\Sigma_n = \frac{1}{n!}\sum_{i_1\cdots i_n \neq 0} \frac{\delta_{i_1+\cdots+i_n,0}}{\sqrt{|i_1\cdots i_n|}} : a_{i_1}\cdots a_{i_n} : . \tag{2.7}$$

The form of Σ_n is made compact by using a negative index to indicate a creation operator; when $i<0$, we have $a_i = a_{|i|}^\dagger$. The expression for $\mathcal{P}^-$ can be simplified once the full form is specified and the constraint equation is invoked. Following Rozowsky and Thorn [6], we work with a rescaled Hamiltonian

$$h = \frac{2\pi}{\mu^2 L}\mathcal{P}^- \tag{2.8}$$

The eigenstates of h are constructed as Fock-state expansions at fixed light-cone momentum $P^+ = 2\pi K/L$, where K is an integer that sets the resolution of the calculation [1]. The momenta $k_n^+ = 2\pi n/L$ of the particles in each Fock state must sum to P^+, and the indices n must then sum to K. The light-cone momenta, and the integer indices, must be positive; this limits the number of particles to a maximum of K.

The eigenvalue problem for h yields coupled equations for the wave functions of the Fock-state expansion. In this context, the zero-mode contributions come from integration end points where the momentum is zero. This can be tracked by starting from the continuum form and discretizing the integral equations for the wave functions. The discretization equivalent to the DLCQ approximation is a trapezoidal approximation to the integrals. In terms of the momentum fractions $x = k^+/P^+$, an integral from zero to one is replaced by a sum over discrete points in the integral, at $x_n = n/K = k_n^+/P^+$, multiplied by the interval size, $1/K$:

$$\int_0^1 dx f(x, 1-x) = \frac{1}{2K} f(0,1) + \frac{1}{K}\sum_{n=1}^{K-1} f(n/K, 1-n/K) + \frac{1}{2K} f(1,0) + \mathcal{O}(1/K^2). \quad (2.9)$$

The end-point corrections, which are the zero-mode contributions, are then of order $1/K$ higher than the bulk of the sum that approximates the integral. Thus, the zero-mode contributions are equivalent to the addition of effective interactions to the Hamiltonian, interactions that include explicit powers of $1/K$ [18], and have the effect, at a minimum, of improving numerical convergence by restoring the $1/K$ terms missed in ignoring the end-point corrections of integrals.

In DLCQ these terms are generated by the zero-mode part of the field as determined by the constraint equation. For consistency with the underlying trapezoidal approximation, these contributions should be kept to no higher in $1/K$ than the corrections expected. The order is measured relative to the non-zero-mode parts, which are typically proportional to $\Sigma_n = \bar{\Sigma}_n / K^{n/2}$, where

$$\bar{\Sigma}_n = \frac{1}{n!} \sum_{i_1 \cdots i_n \neq 0} \frac{\delta_{i_1+\cdots+i_n,0}}{\sqrt{|x_1 \cdots x_n|}} : a_{i_1} \cdots a_{i_n} : \quad (2.10)$$

is written explicitly in terms of momentum fractions x_i. A non-zero-mode contribution of Σ_n would then require zero-mode contributions of no more than order $1/K^{1+n/2}$. We illustrate this in the following section for various interaction models.

III. APPLICATIONS

A. Wick–Cutkosky Model

The simplest nontrivial case is an interaction $V = \lambda\tilde{\phi}|\chi|^2$ with a complex scalar field χ of mass m. The spectrum of this theory is unbounded from below [20]; this is obvious classically because the ϕ field can acquire a negative value and an arbitrarily strong χ field can then drive V to $-\infty$. An ordinary DLCQ calculation that excludes zero modes can detect this structure, but not without careful extrapolation [21]. Here we include zero modes, and a simple variational calculation is all that is needed to detect the unbounded behavior.

We use antiperiodic boundary conditions for the discretized χ field and thereby avoid its zero modes. This option is not available for the $\tilde{\phi}$ field, since it is coupled to the necessarily periodic square of the χ field. The mode expansion of the χ field is

$$\chi = \sum_{n>0} \frac{1}{\sqrt{4\pi n}} \left[c_n e^{ik_n^+ x^-} + d_n^\dagger e^{-ik_n^+ x^-} \right]. \quad (3.1)$$

The constraint equation (2.5) yields

$$a_0 = -2g\Sigma_2^\chi, \quad (3.2)$$

where $g = \lambda/\mu^2\sqrt{4\pi}$ and

$$\Sigma_2^\chi = \frac{1}{2}\sum_{n>0}\frac{1}{n}\left[c_n^\dagger c_n + d_n^\dagger d_n\right]. \tag{3.3}$$

The Hamiltonian reduces to

$$h = \frac{1}{2}\Sigma_2 + \frac{m^2}{\mu^2}\Sigma_2^\chi - g^2\left(\Sigma_2^\chi\right)^2 + \frac{g}{2}\sum_{klm\neq 0}\frac{\delta_{k+l+m,0}}{\sqrt{|klm|}}a_k\left[c_l c_m + d_l d_m\right], \tag{3.4}$$

where the sum in the last term includes both positive and negative indices. The $g^2\left(\Sigma_2^\chi\right)^2$ term has as its origin the zero-mode contributions ϕ_0^2 and $\phi_0|\chi|^2$ to the Hamiltonian density. The constraint equation specifies that ϕ_0 is proportional to the negative of the average of $|\chi|^2$, making a net negative-definite contribution to the energy density.

To isolate this negative contribution, consider the expectation value of h for the highest Fock state $\left(c_1^\dagger\right)^{K-l}\left(d_1^\dagger\right)^l|0\rangle$ of K χ-particles in any charge sector, each with the same momentum fraction $1/K$. The expectation value is

$$\langle h\rangle = \frac{m^2}{\mu^2}\frac{K}{2} - g^2\left(\frac{K}{2}\right)^2. \tag{3.5}$$

Since this tends to $-\infty$ as $K\to\infty$, the spectrum extends to $-\infty$ in the continuum limit.[1]

B. ϕ^3 Theory

In ϕ^3 theory there are, of course, no fields other than $\tilde{\phi}$, and the interaction is $V = \frac{\lambda}{3!}\tilde{\phi}^3$. The constraint equation (2.5) becomes

$$-a_0 = \frac{g}{2}a_0^2 + g\Sigma_2, \tag{3.6}$$

with $g = \lambda/\mu^2\sqrt{4\pi}$. On use of this constraint, the Hamiltonian can be written

$$h = \frac{1}{2}\Sigma_2 + \frac{g}{2}\Sigma_3 - \frac{1}{12}a_0^2 - \frac{g}{12}a_0^3. \tag{3.7}$$

The non-zero-mode piece of the Hamiltonian, the first two terms, can be written as $\frac{1}{2K}\bar{\Sigma}_2 + \frac{g}{2K^{3/2}}\bar{\Sigma}_3$, where the $\bar{\Sigma}_n$ are defined in Eq. (2.10). Thus the zero-mode contributions we seek are of order $1/K^{5/2}$ at most.

The constraint equation is solved to a consistent order by taking $a_0 = v_0 + v_1/K$ and finding v_0 and v_1. From the constraint equation we have

$$-v_0 = \frac{g}{2}v_0^2 \;\text{ and }\; -v_1 = v_0 v_1 + v_1 v_0 + g\bar{\Sigma}_2. \tag{3.8}$$

For v_0, the two possible solutions are $v_0 = 0$, the local minimum in the classical energy density, and $v_0 = -2/g$, the local maximum. Either solution is acceptable as a starting

[1] This fate is avoided in Yukawa theory, where χ is a fermi field, simply because the identical $x = 1/K$ states cannot be populated.

point, but the first leads to simpler expressions and is sufficient for our purposes. The second corresponds to a different choice for the perturbative vacuum, annihilated by the dynamical a_n, because it represents a different choice for $\phi = \tilde{\phi} - \phi_0$; however, this change in the perturbative vacuum is compensated by different terms in the Hamiltonian, with no net effect on the spectrum. With $v_0 = 0$, the solution for v_1 is then immediately $v_1 = -g\bar{\Sigma}_2$, and we find for the zero mode

$$a_0 = -\frac{g}{K}\bar{\Sigma}_2 + \mathcal{O}(\frac{1}{K^2}). \tag{3.9}$$

On substitution of this expansion for the zero mode, the Hamiltonian becomes

$$h = \frac{1}{2K}\bar{\Sigma}_2 + \frac{g}{2K^{3/2}}\bar{\Sigma}_3 - \frac{g^2}{12K^2}\bar{\Sigma}_2^2 \tag{3.10}$$

The expectation value of h for the state populated with K particles of momentum fraction $1/K$ is

$$\langle h \rangle = \frac{K}{2} - \frac{g^2}{12}K^2. \tag{3.11}$$

This tends to $-\infty$ in the $K \to \infty$ limit, and, as known for a cubic theory [20], the spectrum is unbounded from below.

C. ϕ^4 Theory

For ϕ^4 theory, with its interaction $V = \frac{\lambda}{4!}\tilde{\phi}^4$, the contributions of zero modes are more subtle than for the first two applications. Their inclusion will improve numerical convergence and may provide a means to understand vacuum structure and symmetry breaking. Previous calculations [6, 7] of the spectrum did not include zero modes, and previous studies of the constraint equation [16] attempted to solve it fully, rather than keeping a_0 only to an order consistent with the DLCQ Hamiltonian.

The constraint equation (2.5) in this case is

$$\mp a_0 = \frac{g}{3}a_0^3 + 2g\Sigma_3 + \frac{2g}{3}\left(a_0\Sigma_2 + \Sigma_2 a_0\right) + \frac{g}{3}\sum_{n\neq 0}\frac{1}{|n|}a_n a_0 a_{-n}, \tag{3.12}$$

where $g = \lambda/8\pi\mu^2$. The Hamiltonian is

$$\begin{aligned} h \;=\; & \pm\frac{1}{2K}\bar{\Sigma}_2 + \frac{g}{K^2}\bar{\Sigma}_4 - \frac{g}{24}a_0^4 + \frac{g}{24K^{3/2}}\sum_{klm\neq 0}\frac{\delta_{k+l+m,0}}{\sqrt{x_{|k|}x_{|l|}x_{|m|}}}(a_k a_l a_0 a_m + a_k a_0 a_l a_m) \\ & + \frac{g}{24K}\sum_{n\neq 0}\frac{1}{x_{|n|}}(a_n a_0^2 a_{-n} - a_0 a_n a_{-n} a_0). \end{aligned} \tag{3.13}$$

From the K dependence of the non-zero-mode pieces of the Hamiltonian, we see that we need zero-mode corrections to order $1/K^3$ and thus expand a_0 to order $1/K^{3/2}$.

To solve the constraint equation to this order, we write $a_0 = v_0 + v_1/K^{1/2} + v_2/K + v_3/K^{3/2}$ and solve for the v_i. Two of the three possible solutions are available only for the wrong-sign mass term. They have $v_0 = \pm\sqrt{3/g}$ and correspond to the local minima in the classical energy density. The third solution, $v_0 = 0$, yields the local maximum as well as simpler expressions; it is also the only solution in the case of the correct-sign mass term. For simplicity, we choose to work with this third solution, for which v_1 and v_2 are also zero and

$$a_0 = \mp \frac{2g}{K^{3/2}} \bar{\Sigma}_3 = \mp 2g\Sigma_3. \tag{3.14}$$

Here the upper (lower) sign corresponds to the upper (lower) sign in the mass term of the Hamiltonian.

In the Hamiltonian, this solution to the constraint equation, combined with the keeping of terms to consistent order, yields

$$h = \pm \frac{1}{2K} \bar{\Sigma}_2 + \frac{g}{K^2} \bar{\Sigma}_4 \mp \frac{g^2}{12K^3} \sum_{klm \neq 0} \frac{\delta_{k+l+m,0}}{\sqrt{x_{|k|} x_{|l|} x_{|m|}}} (a_k a_l \bar{\Sigma}_3 a_m + a_k \bar{\Sigma}_3 a_l a_m) + \mathcal{O}(1/K^4). \tag{3.15}$$

The first term is the (rescaled) mass term, and the second is the ordinary interaction term. The last term provides the zero-mode corrections to the mass eigenvalue problem at an order in $1/K$ that is consistent with the DLCQ approximation. The sign of the term is particularly significant. In the case of a positive mass term, this correction is negative, which will allow the spectrum to extend below zero and provide a nontrivial vacuum state and symmetry breaking at larger couplings. For the case of a negative mass term, where the symmetry breaking effect is built in, the zero-mode term is positive and can play a role in restoring the broken symmetry and the trivial vacuum at larger couplings.

In addition to the zero-mode corrections to the Hamiltonian, the zero mode also allows for a direct calculation of the DLCQ approximation for the vacuum expectation value. The eigenstates of h separate into sectors of odd and even particle number, and in the continuum limit the lowest state of each sector become degenerate. Let $|o\rangle$ and $|e\rangle$ be the odd and even ground states and form maximally mixed states $|\pm\rangle = (|o\rangle \pm |e\rangle)/\sqrt{2}$. The expectation value for the field is then

$$v_\pm = \frac{\langle \pm|\tilde{\phi}|\pm\rangle}{\langle \pm|\pm\rangle} \equiv \pm v. \tag{3.16}$$

Since $\tilde{\phi}$ changes particle number by one, only the cross terms contribute to the numerator, and since the states have the same momentum, only the zero mode of the field can contribute. Thus, we have

$$v = \frac{\langle e|\phi_0|o\rangle + \langle o|\phi_0|e\rangle}{\langle e|e\rangle + \langle o|o\rangle}. \tag{3.17}$$

On substitution of the solution (3.14) to the constraint equation, this becomes

$$v = \mp \frac{g}{\sqrt{\pi}} \frac{\langle e|\Sigma_3|o\rangle + \langle o|\Sigma_3|e\rangle}{\langle e|e\rangle + \langle o|o\rangle}. \tag{3.18}$$

So, one can calculate v if the mass eigenvalue problem is solved to find the lowest odd and even states, and one can study the continuum limit as the resolution $K \to \infty$.

IV. SUMMARY

Although zero modes are traditionally neglected in DLCQ calculations, they can actually be taken into account without much additional effort. Each mode satisfies a constraint equation that is the (light-front) spatial average of an Euler–Lagrange equation. The constraint connects the zero mode to the dynamical degrees of freedom and can be solved either explicitly or in terms of an expansion in inverse powers of the DLCQ resolution K. The latter expansion is truncated at the order consistent with the DLCQ approximation.

From the zero-mode solution, one can construct contributions to the Hamiltonian which will, at a minimum, repair the convergence of the DLCQ calculation. The DLCQ approximation is equivalent to a trapezoidal approximation to integral equations for Fock-state wave functions, with quadrature points spaced equally by $1/K$. The error for such an integral is of order $1/K^2$, but when the zero modes are neglected the error is $1/K$ and convergence as $K \to \infty$ is slowed. The zero-mode contributions to the Hamiltonian repair this and restore the $1/K^2$ behavior of the integration errors.

Inclusion of zero modes also makes possible the direct calculation of vacuum expectation values. Explicit symmetry breaking will yield a c-number contribution to the zero mode which trivially has a vacuum expectation value, but this is not the only way to have a nonzero value. The zero mode will, in general, have dynamical contributions. For these, the expectation value will be zero with respect to the perturbative vacuum. However, the spectrum of the theory can be such that the perturbative vacuum is not the eigenstate of lowest energy. Instead, some nontrivial eigenstate has a lower energy and, with respect to this state, the dynamical contributions to the zero mode can have a nonzero expectation value.

Acknowledgments

This work was supported in part by the Department of Energy through contract DE-FG02-98ER41087.

---

[1] H.-C. Pauli and S.J. Brodsky, Phys. Rev. D **32**, 1993 (1985); **32**, 2001 (1985).

[2] T. Maskawa and K. Yamawaki, Prog. Theor. Phys. **56**, 270 (1976).

[3] C.B. Thorn, Phys. Lett. B **70**, 85 (1977); Phys. Rev. D **17**, 1073 (1978); **59**, 270 (1999).

[4] For reviews, see M. Burkardt, Adv. Nucl. Phys. **23**, 1 (1996); S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky, Phys. Rep. **301**, 299 (1998).

[5] J. R. Hiller, S. Pinsky, Y. Proestos, N. Salwen, and U. Trittmann, Phys. Rev. D **76**, 045008 (2007).

[6] J.S. Rozowsky and C.B. Thorn, Phys. Rev. Lett. **85**, 1614 (2000).

[7] V. T. Kim, G. B. Pivovarov, and J. P. Vary, Phys. Rev. D **69**, 085008 (2004); D. Chakrabarti, A. Harindranath, L. Martinovic, G.B. Pivovarov, and J.P. Vary, Phys. Lett. B **617**, 92 (2005).

[8] A. Harindranath and J.P. Vary, Phys. Rev. D **37**, 1064 (1988).

[9] A. Harindranath and J.P. Vary, Phys. Rev. D **36**, 1141 (1987); **37**, 1076 (1988); 3010 (1988).

[10] T. Eller, H.-C. Pauli, and S.J. Brodsky, Phys. Rev. D **35**, 1493 (1987).

[11] M. Burkardt, Nucl. Phys. A **504**, 762 (1989); K. Hornbostel, S.J. Brodsky, and H.-C. Pauli, Phys. Rev. D **41**, 3814 (1990).

[12] P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[13] Th. Heinzl, St. Krusche, and E. Werner, Phys. Lett. B **272**, 54 (1991); —bf 275, 410 (1992); T. Heinzl, S. Krusche, S. Simburger, and E. Werner, Z. Phys. C **56**, 415 (1992).

[14] D.G. Robertson, Phys. Rev. D **47**, 2549 (1993).

[15] K. Hornbostel, Phys. Rev. D **45**, 3781 (1992).

[16] C.M. Bender, S.S. Pinsky, and B. van de Sande, Phys. Rev. D **48**, 816 (1993); S.S. Pinsky and B. van de Sande, Phys. Rev. D **49**, 2001 (1994); S.S. Pinsky, B. van de Sande, and J.R. Hiller, Phys. Rev. D **51**, 726 (1995).

[17] A. Borderies, P. Grangé, and E. Werner, Phys. Lett. B **319**, 490 (1993); **345**, 458 (1995); P. Grangé, P. Ullrich, and E. Werner, Phys. Rev. D **57**, 4981 (1998); S. Salmons, P. Grangé, and E. Werner, Phys. Rev. D **60**, 067701 (1999); S. Salmons, P. Grangé, and E. Werner, Phys. Rev. D **65**, 125014 (2002).

[18] J.J. Wivoda and J.R. Hiller, Phys. Rev. D **47**, 4647 (1993).

[19] M. Maeno, Phys. Lett. B **320**, 83 (1994).

[20] G. Baym, Phys. Rev. **117**, 886 (1960).

[21] J.B. Swenson and J.R. Hiller, Phys. Rev. D **48**, 1774 (1993).