Parton distribution functions (PDFs) for Higgs boson studies

Pavel Nadolsky

with Fred Olness, Sean Doyle, Madeline Hamilton, Tim Hobbs, Bo-Ting Wang,

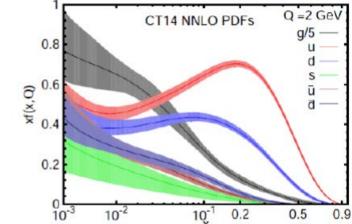
Keping Xie

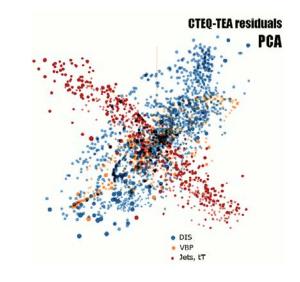
Southern Methodist University

and CTEQ-TEA (Tung et al.) working group

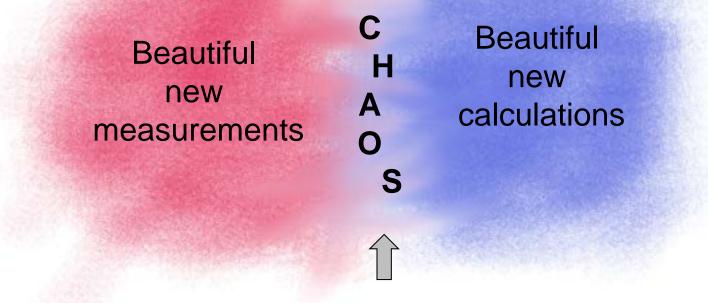
Kennesaw State U., Shanghai Jiao Tong U., Michigan State U., SMU, Xinjiang U.







Today's high-energy QCD is reminiscent of early 1990's



An interesting spot to be

QCD expectations for high-luminosity LHC

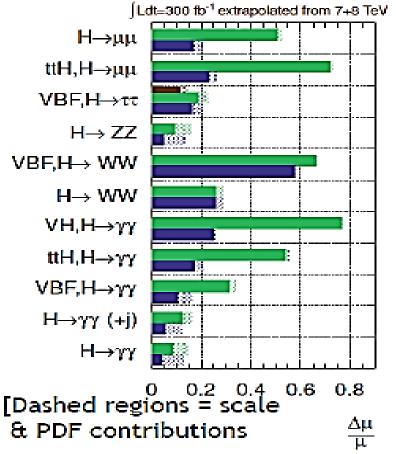
- New (N)NNLO calculations likely to be completed
- Measurements of Higgs cross sections/couplings become limited by PDFs in the HL-LHC era
- Searches for non-resonant production in TeV mass range will demand accurate predictions for sea PDFs at x > 0.1
- The target is to obtain PDFs that "achieve 1% accuracy for LHC predictions" within about a decade

Projected Experimental

<u>Uncertainties</u>

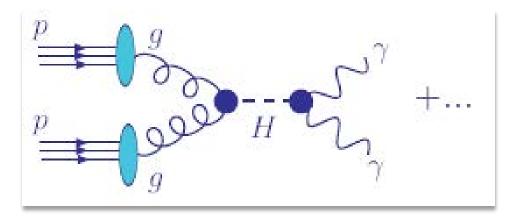
ATLAS Simulation





P. Newman, DIS'2016

Example: total cross section for gg \rightarrow Higgs \rightarrow \gamma\gamma

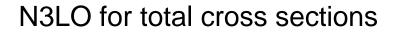


$$\sigma_{pp \to H \to \gamma\gamma X}(Q) = \sum_{a,b=g,q,\bar{q}} \int_0^1 d\xi_a \int_0^1 d\xi_b \hat{\sigma}_{ab \to H \to \gamma\gamma} \left(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, \frac{Q}{\mu_R}, \frac{Q}{\mu_F}; \alpha_s(\mu_R)\right) \times f_a(\xi_a, \mu_F) f_b(\xi_b, \mu_F) + O\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$

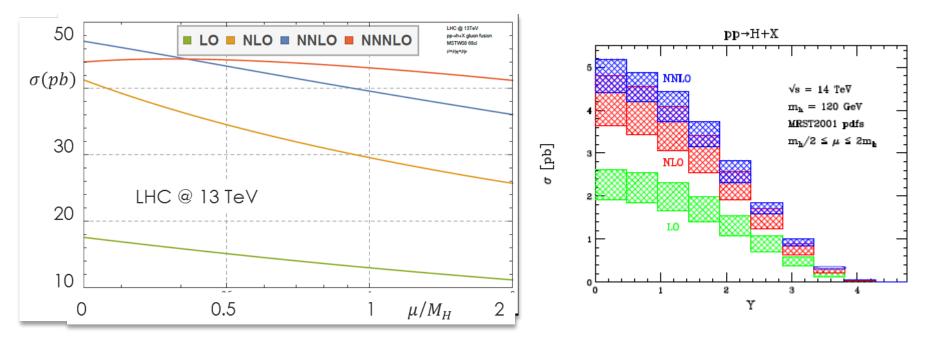
 $\hat{\sigma}_{ab \to H \to \gamma\gamma}$ is the perturbative cross section to produce a Higgs boson from partons a and b; $a, b = g, u, \bar{u}, d, \bar{d}, ...$

 $f_a(\xi,\mu)$ is a parton distribution function (PDF) associated with the probability for finding a parton *a* with the "+" momentum ξp^+ in a proton with the "+" momentum p^+ for $p^+ \to \infty$, at a factorization scale $\mu > 1$ GeV

Hard-scattering cross sections for $gg \rightarrow H \rightarrow \gamma \gamma$



NNLO for differential distributions

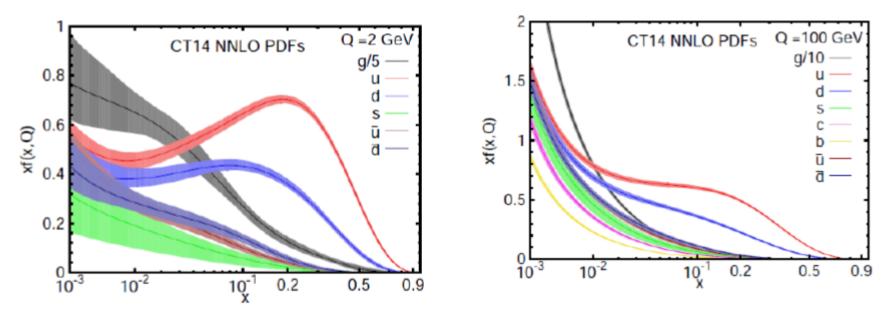


Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 1503.06056 N3LO corrections are of the order of +2.2%. The total scale variation at N3LO is 3% 2018-09-17

Anastasiou, Melnikov, Petriello, hep-ph/0409088, 0501130

Recent CT14 PDFs

(S. Dulat et al., arXiv:1506.07443)



Q= 2 GeV

Q= 100 GeV

Phenomenological parametrizations of PDFs are provided with estimated uncertainties of multiple origins (**uncertainties of measurement, theoretical model, parametrization form, statistical analysis**, ...)

The shape of PDFs is optimized w.r.t. hundreds of **nuisance** parameters

2018-09-17

Classes of PDFs

General-purpose

For (N)NLO calculations with $N_f \leq 5$ active quark flavors

```
From several groups:
ABMP'16
CTEQ-Jlab (CJ'2015)
HERA2.0
CT14 (\rightarrow 18p)
MMHT'14 (\rightarrow 16)
NNPDF3.1
```

 Specialized

 For instance, for CT14:

 CT14 LO

 CT14 $N_f = 3, 4, 6$

 CT14 HERA2

 [arXiv:1609.07968]

 CT14 Intrinsic charm

 CT14 QCD+QED

 [1509.02905]

 CT14 Monte-Carlo

ATLAS & CMS exploratory

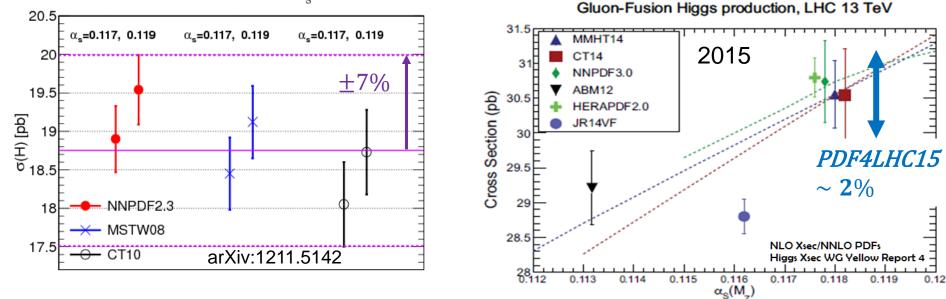
SMU theory contributions Combined [1509.03865]
PDF4LHC'15=CT14+MMHT'14+NNPDF3.0

Toward a new generation of PDFs ["CT18preliminary" PDFs]

What can the LHC do to constrain the PDFs?

Example: $gg \rightarrow H_{SM}^0$ at the LHC

LHC 8 TeV - iHixs 1.3 NNLO - PDF+a, uncertainties

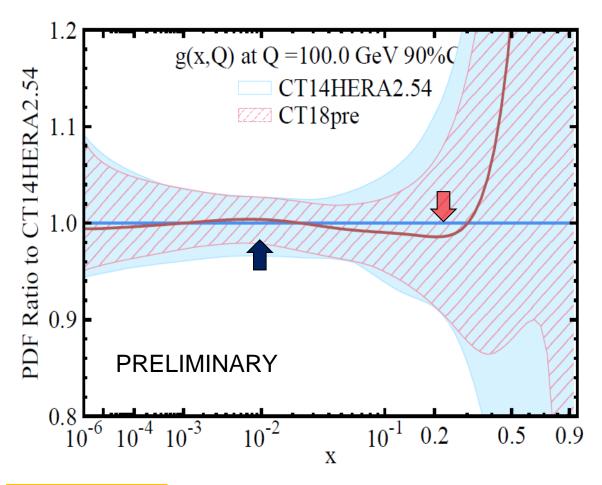


2012->2015: Uncertainty δ_{PDF} on Higgs cross sections based on 3 global fits has reduced from 7% to within 3%, i.e., the PDF uncertainty is now of order of N3LO QCD scale uncertainty

This improvement is due to benchmarking of general-mass factorization schemes and development of new PDF-averaging methods with active participation of SMU's Jun Gao (2018 Altarelli Award).

Can we further improve on this?

Gluon PDF before and after including the LHC data [CT14HERA2 vs. CT18pre NNLO]



 $x \approx 0.01$: g(x, Q) mildly increases within the uncertainty

⇒ slightly larger Higgs production rates at 14 TeV

Minor reduction in the gluon PDF uncertainty

 $0.05 \leq x \leq 0.3$: g(x,Q)mildly decreases; lower gg luminosities for $M_X > 700$ GeV

After the fit

CT18p: preliminary PDFs with new LHC data

Issues:

- Experimental, theoretical, and procedural systematic uncertainties dominate the PDF uncertainty in many cases
 - Tensions between some experimental data sets
 - Large QCD uncertainties in some kinematic regions (e.g., large y)

The CT18pre analysis examines how the PDFs depends on... ... settings of NNLO calculations

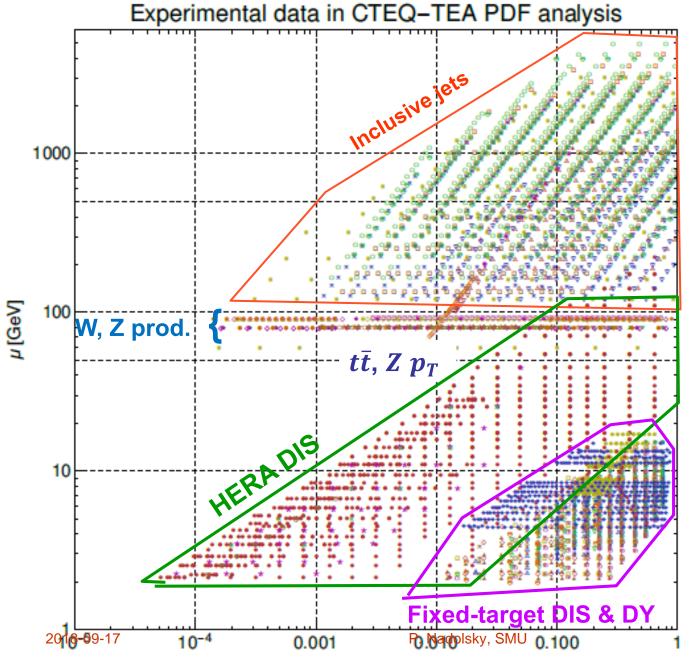
(SACOT- χ heavy-quark scheme, QCD scales, m_c , numerical codes,...)

... selection of experiments and kinematic cuts

For instance, g(x, Q) at x > 0.05 is already constrained in CT14/MMHT14 by D0 Run-2 incl. jet data which is not in NNPDF3.1. Disagreements exist within available ATLAS/CMS experiments and between some LHC and non-LHC experiments

...the fitting procedure

Definition of PDF uncertainties Parametrization forms PDF error analysis (Hessian vs. Monte-Carlo)



х

CT18pre analysis includes new LHC experiments on W/Z, high- $p_T Z$, jet, $t\bar{t}$ production

(25% more data)

Selected using fast statistical tools **PDFSense** and **ePump**

12

Experiments in the CT14 HERA2 fit

	Experimental dataset		N_d
101	BCDMS F_2^p	[47]	337
102	BCDMS F_2^d	[48]	250
104	$\begin{array}{c} \text{NMC } F_2^d / F_2^p \\ \text{CDHSW } F_2^p \end{array}$	[49]	123
108	CDHSW $F_2^{\overline{p}}$	[50]	85
109	CDHSW F_3^p	[50]	96
110	$CCFR F_2^p$	[51]	69
111	$CCFR xF_3^p$	[52]	86
124	NuTeV $\nu\mu\mu$ SIDIS	[40]	38
125	NuTeV $\bar{\nu}\mu\mu$ SIDIS	[40]	33
126	$CCFR \nu \mu \mu$ SIDIS	[41]	40
127	$\begin{array}{c} \text{CCFR } \bar{\nu}\mu\mu \text{ SIDIS} \\ \text{III} h (57.4 \pm 1.5) \end{array}$	[41]	38
145		[54]	10
147	Combined HERA charm production (1.504 fb^{-1})	[39]	47
160 169	HERA1+2 Combined NC and CC DIS (1 fb^{-1})	L	$\frac{1120}{9}$
169	H1 F_L (121.6 pb ⁻¹)	[55]	9
ID#	Experimental dataset		N_d
201	E605 DY	[56]	119
203	E866 DY, $\sigma_{pd}/(2\sigma_{pp})$	[57]	15
204	E866 DY, $Q^3 d^2 \sigma_{pp} / (dQ dx_F)$	[58]	184
225	$(ODDDD 1 + \langle \beta \rangle (110 + 1))$		
	CDF Run-1 $A_e(\eta^e)$ (110 pb ⁻¹)	[59]	11
227	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹)	[59] [60]	11 11
227 234	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹)	[59] [60] [61]	11 11 9
227 234 240	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹)	[59] [60] [61] [62]	11 11 9 14
227 234 240 241	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹)	[59] [60] [61] [62] [62]	11 11 9 14 5
227 234 240 241 260	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹)	[59] [60] [61] [62] [62] [63]	11 11 9 14 5 28
227 234 240 241 260 266	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹) CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹)	[59] [60] [61] [62] [62] [63] [64]	$ \begin{array}{r} 11 \\ 11 \\ 9 \\ 14 \\ 5 \\ 28 \\ 11 \\ \end{array} $
227 234 240 241 260 266 267	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹) CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹) CMS 7 TeV $A_e(\eta)$ (0.840 fb ⁻¹)	[59] [60] [61] [62] [62] [63] [63] [64] [65]	11 11 9 14 5 28 11 11
$\begin{array}{r} 227\\ 234\\ 240\\ 241\\ 260\\ 266\\ 267\\ 268\\ \end{array}$	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹) CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹) CMS 7 TeV $A_e(\eta)$ (0.840 fb ⁻¹) ATLAS 7 TeV W/Z Xsec, $A_\mu(\eta)$ (35 pb ⁻¹)	[59] [60] [61] [62] [62] [63] [64] [65] [66]	11 11 9 14 5 28 11 11 41
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$\begin{array}{r} 227\\ 234\\ 240\\ 241\\ 260\\ 266\\ 267\\ 268\\ \end{array}$	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹) CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹) CMS 7 TeV $A_e(\eta)$ (0.840 fb ⁻¹) ATLAS 7 TeV W/Z Xsec, $A_\mu(\eta)$ (35 pb ⁻¹) DØ Run-2 $A_e(\eta)$ (9.7 fb ⁻¹) CDF Run-2 incl. jet $(d^2\sigma/dp_T^j dy_j)$ (1.13 fb ⁻¹)	[59] [60] [61] [62] [62] [63] [64] [65] [66]	11 11 9 14 5 28 11 11 41
$\begin{array}{r} 227\\ 234\\ 240\\ 241\\ 260\\ 266\\ 266\\ 267\\ 268\\ 281\\ \end{array}$	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹) CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹) CMS 7 TeV $A_e(\eta)$ (0.840 fb ⁻¹) ATLAS 7 TeV W/Z Xsec, $A_\mu(\eta)$ (35 pb ⁻¹) DØ Run-2 $A_e(\eta)$ (9.7 fb ⁻¹)	[59] [60] [61] [62] [62] [63] [63] [64] [65] [66] [66] [67]	$ \begin{array}{r} 11 \\ 11 \\ 9 \\ 14 \\ 5 \\ 28 \\ 11 \\ 11 \\ 41 \\ 13 \\ \end{array} $
$\begin{array}{r} 227\\ 234\\ 240\\ 241\\ 260\\ 266\\ 267\\ 268\\ 281\\ 504\\ \end{array}$	CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) LHCb 7 TeV $W A_\mu(\eta^\mu)$ (35 pb ⁻¹) DØ Run-2 $Z \ d\sigma/dy_Z$ (0.4 fb ⁻¹) CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹) CMS 7 TeV $A_e(\eta)$ (0.840 fb ⁻¹) ATLAS 7 TeV W/Z Xsec, $A_\mu(\eta)$ (35 pb ⁻¹) DØ Run-2 $A_e(\eta)$ (9.7 fb ⁻¹) CDF Run-2 incl. jet $(d^2\sigma/dp_T^j dy_j)$ (1.13 fb ⁻¹)	[59] [60] [61] [62] [62] [63] [64] [65] [66] [66] [67] [36]	$ \begin{array}{r} 11 \\ 11 \\ 9 \\ 14 \\ 5 \\ 28 \\ 11 \\ 11 \\ 41 \\ 13 \\ 72 \\ \end{array} $

New experiments in the CT17pre fit

- 1. LHCb 7 TeV Z/W muon rapidity 1505.07024
- 2. LHCb 8 TeV Z rapidity 1503.00963
- 3. CMS 8 TeV W lept. asymmetry 1603.01803
- 4. LHCb 8 TeV Z/W muon rapidity 1511.08039
- 5. ATLAS 7 TeV Z p_T 1512.02192
- 6. CMS incl. jet 7 TeV, R=0.7 1406.0324
- 7. ATLAS incl. jet at 7 TeV, R=0.6 1410.8857
- 8. CMS incl. jet at 8 TeV, R=0.7 1609.05331
- 9. ATLAS 8 TeV $t\bar{t} p_T$ 1511.04716
- 10. ATLAS 8 TeV $t\bar{t}$ $m_{t\bar{t}}$ 1511.04716
- 11. CMS 8 TeV $t\bar{t} d^2\sigma/dp_{Tt} dy_t$ 1703.01630

 N_d is the number of data points

How sensitive is an experiment to a PDF? Can we estimate it **before** doing the global fit?

Yes, using new statistical tools:

Generalized correlations

 (sensitivities S_f) comparing
 experimental and PDF
 uncertainties for fitted data
 points



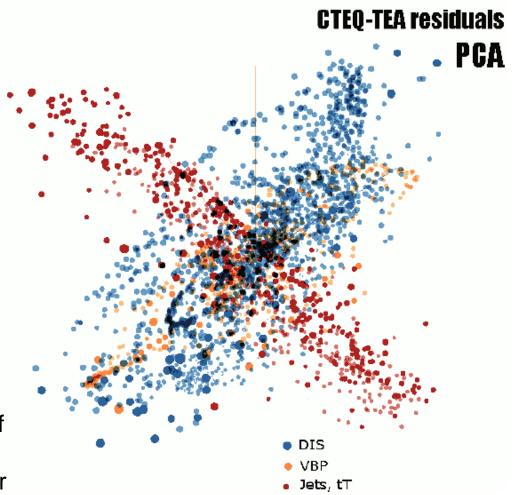
- 2. PDF reweighting
- 3. Hessian profiling

Manifolds of data residuals [TensorFlow]

Analysis flow:

- We give you a table of N_{pt} normalized data point residuals $\vec{r}_i(\vec{a})$ for every CT14HERA2 error PDF [on the PDFSense website]
- You examine the 56-dim. distribution of $\vec{r}_i(\vec{a})$ in PDFSense or another data analysis software

Right: a sample 3-dim. projection of the 56-dim. manifold obtained with the TensorFlow Embedding Projector (<u>http://projector.tensorflow.org</u>)

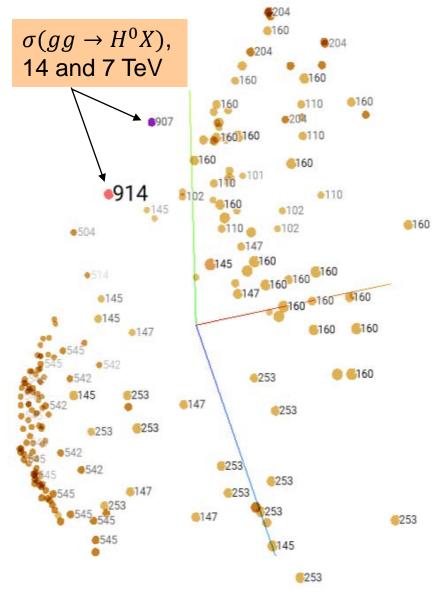


A PDF-dependent quantity f, such as the Higgs cross section at 7 or 14 TeV (ID=907, 914), defines a direction $\vec{\delta}_f$ in the (2)N-dim space.

The net constraint of the *i*-th point on $\sigma(H)$, including systematic errors, is quantified by the projection of \vec{r}_i on $\vec{\delta}_f[\sigma(H)]$, called the sensitivity $S_{f,i}$.

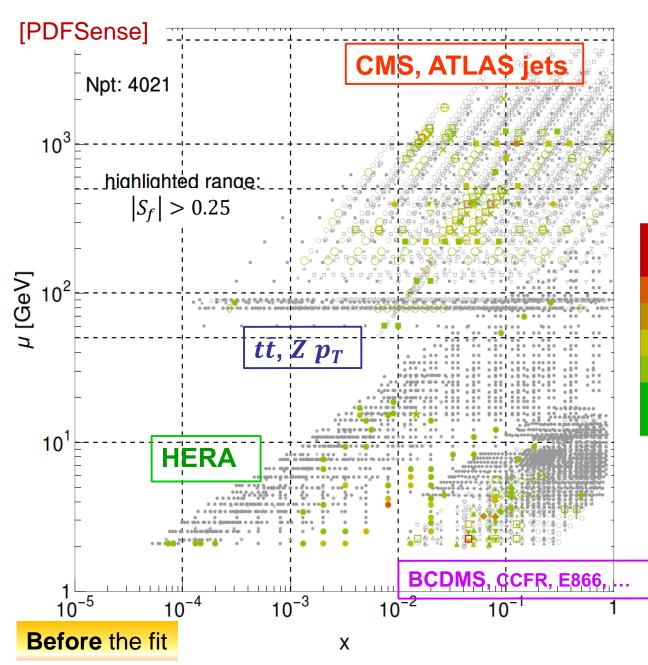
Right: 300 vectors \vec{r}_i of the CT14HERA2 global set whose directions are closest to $\vec{\delta}_f(\sigma(H^0))$. These vectors are given by the experiments:

160=HERA I+II; 101, 102=BCDMS; 110=CCFR F2p; 147, 145=HERA I+II *c*, *b*; 204=E866 σ_{pp} ; 253=Z p_T 8 TeV; 542, 545=CMS jets 7, 8 TeV; 504, 514=Tevatron jets



Sensitivity of expt E = sum of $S_{f,i}$ over data points in E

$|S_f|$ for sig(H0), 14 TeV, CT14HERA2NNLO



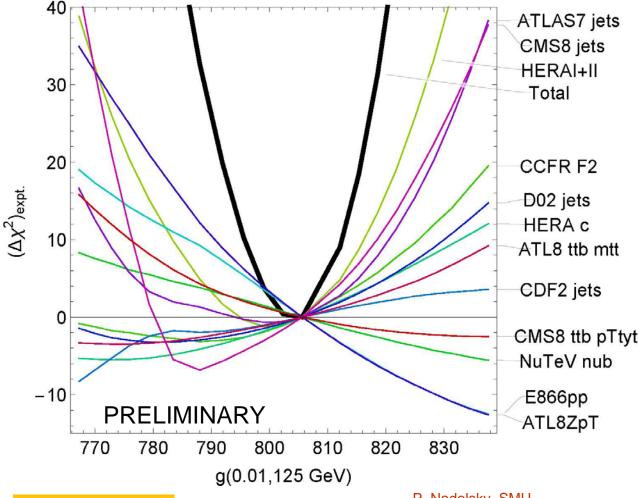
Higgs boson production

HERA DIS still has the dominant sensitivity!

CMS 8 TeV jets is the next expt. after HERA

- 1.2 sensitive to
- 1.0 σ_H (14 TeV); jet scale
- 0.8 uncertainty dampens
- ${}^{0.6}_{0.4}$ |*S*_{*f*}| for jets
- $^{0.2}$ Good correlations C_f
 - with some points in E866, BCDMS, CCFR, CMS WASY, $Z p_T$ and $t\bar{t}$ production; but not as many points with high $|S_f|$ in these processes

Which experiments constrain the gluon? x = 0.01, Q = 125 GeV [Higgs region]



A Lagrange multiplier scan [Stump et al., hep-ph/0101151] of

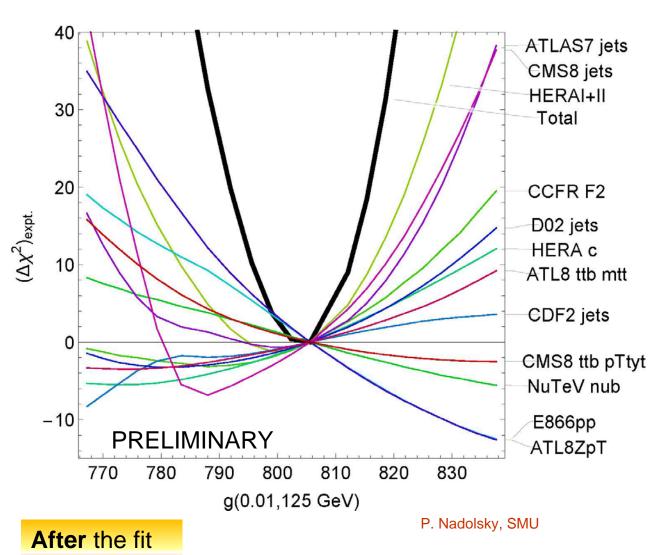
 $\Delta\,\chi^2 = \chi^2(g) - \chi^2_{best-fit}$

for all (black line) and individual (colored lines) experiments

Best-fit *g*(0.01,125GeV)=806

After the fit

Which experiments constrain the gluon? x = 0.01, Q = 125 GeV [Higgs region]

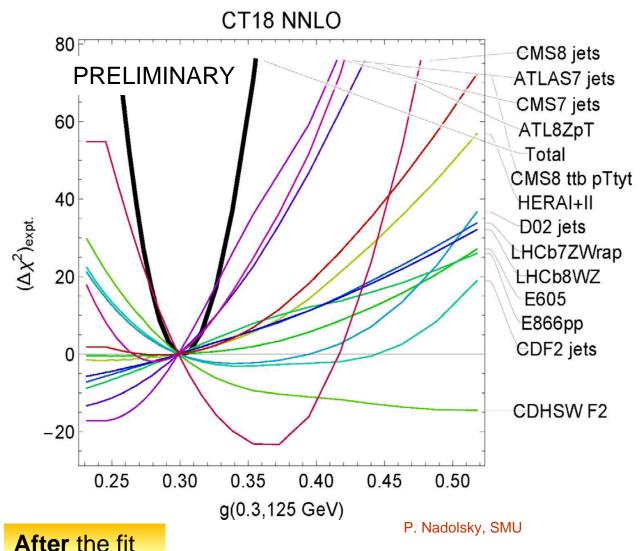


The LM scans broadly confirm S_f estimates

HERAI+II, ATLAS7 jets, CMS8 jets impose the tightest constraints; are in agreement

E866, ATLAS 8 Z p_T prefer higher gluon

Which experiments constrain the gluon? x = 0.3, Q = 125 GeV

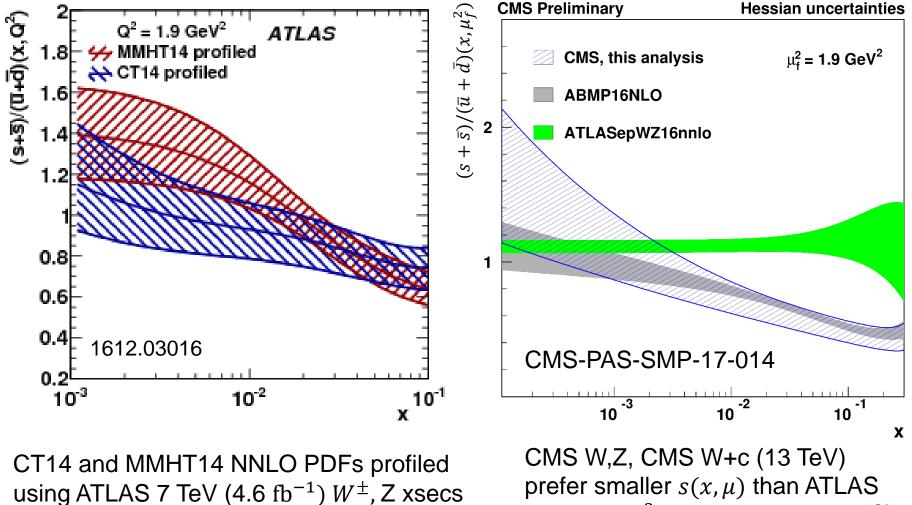


ATLAS 7 and CMS 7 TeV jets, ATLAS 8 Z p_T disagree with CMS8 jets

Weaker constraints from HERAI+II, E866, LHCb, Tevatron jets, CDHSW F2, $t\bar{t}$ production

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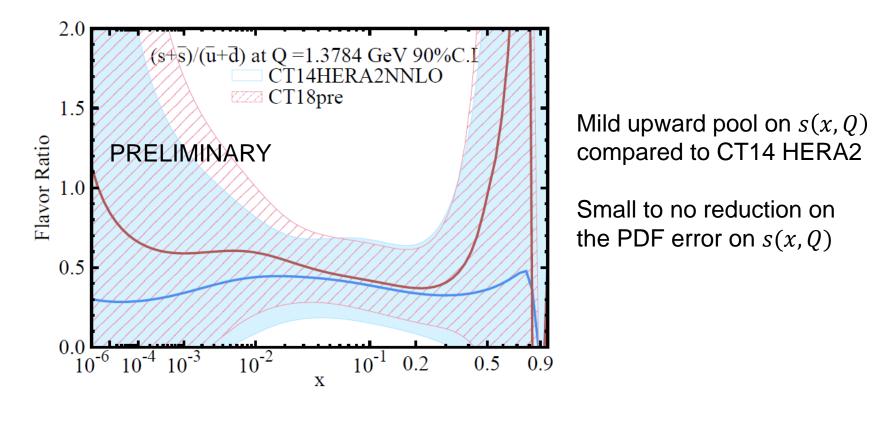
Inconsistent conclusions in literature about strangeness preferred by the LHC data



prefer $s(x, 1.9 \text{ GeV}) \sim 1 @ x = 0.023$

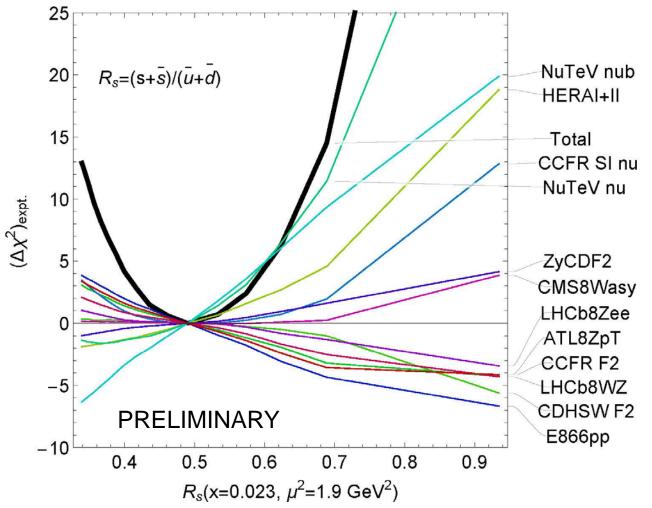
prefer smaller $s(x, \mu)$ than ATLAS for $x \gtrsim 10^{-3}$ 21

Effect of LHC data on strangeness: the actual CT18pre fit



$$R_s(CT18pre) = \frac{s+\bar{s}}{\bar{u}+\bar{d}} = 0.53 \pm 0.16$$
 (90% c.l.) at $x = 0.023$, $Q^2 = 1.9 \text{ GeV}^2$

Effect of LHC data on strangeness: the actual CT18pre fit



Some tension between NuTeV, CCFR dimuon production, HERAI+II (preferring $R_s < 0.6$);

and vector boson production at the LHC and Tevatron (preferring $R_s > 0.6$)

However, still large uncertainties

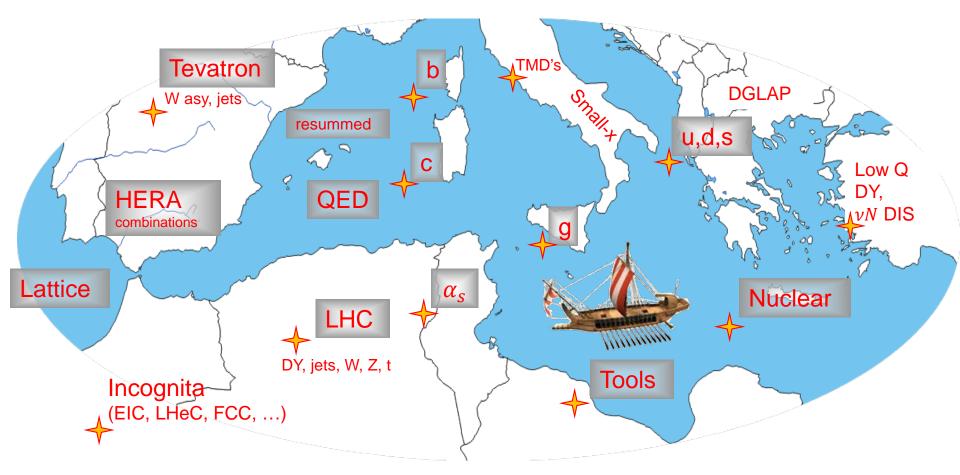
Outlook for CTEQ-TEA PDFs

Ongoing CTEQ-TEA PDF analysis

Detailed investigation of the LHC 7 and 8 TeV vector boson, jet, $t\bar{t}$ production data suggests mild changes in the central fits, PDF uncertainties, and precision EW observables, as compared to the CT14HERA2 NNLO data set. We notice the potential of the future ATLAS/CMS jet data, together with other LHC processes, for strengthening the constraints on the g, s, \overline{u} , and \overline{d} PDFs with modest improvements in experimental systematics and full implementation of NNLO jet cross sections

- **CT14 PDFs** with photon PDFs **[1509.02905]**, intrinsic/fitted charm [1706.00657], and Monte-Carlo error PDFs [1607.06066]
- NLO calculation for c, b production at LHCb, ATLAS in the S-ACOT- χ •
- scheme using MCFM/Applgrid [Campbell, P. N., Xie, in pre-publication]
- Further development of programs for fast survey [PDFSense] and Hessian reweighting of the data [ePump] 2018-09-17

Oekumene of the PDF universe



+ Recent SMU contributions

New lands for charting, new tools for exploring



Fast estimation of sensitivity S_f of experimental data to theory cross sections [PDFSense]

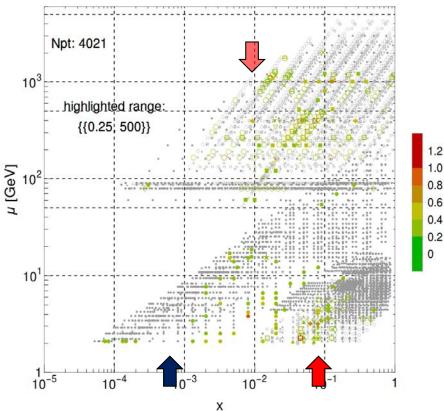
[B.T. Wang, T.J. Hobbs, et al., arXiv:1803.02777]

The sensitivity S_f compares full experimental and PDF uncertainties

 S_f is not affected by limitations of PDF reweighting

If a **data point** is sensitive to a given PDF f, then the sensitivity $|S_f|$ of this data point to f is much larger than 0 [say, $|S_f| > 0.25$]. S_f is defined on the next slide.

Sensitivity to the PDF error on $\sigma(pp \rightarrow H^0X)$ at 14 TeV





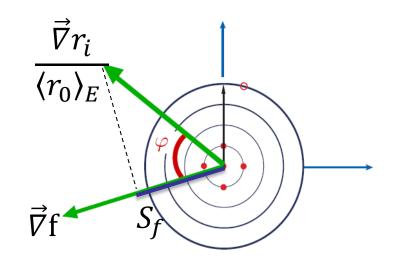
P. Nadolsky, SMU

Correlation C_f and sensitivity S_f

The relation of data point i on the PDF dependence of f can be estimated by:

• $C_f \equiv \operatorname{Corr}[\rho_i(\vec{a})), f(\vec{a})] = \cos\varphi$ $\vec{\rho}_i \equiv \vec{\nabla} r_i / \langle r_0 \rangle_E$ -- gradient of r_i normalized to the r.m.s. average residual in expt E; $(\vec{r}_{ij}) = (\alpha_i(\vec{r}_{ij}))/2$

 $\left(\vec{\nabla}r_i\right)_k = \left(r_i(\vec{a}_k^+) - r_i(\vec{a}_k^-)\right)/2$



 C_f is **independent** of the experimental and PDF uncertainties. In the figures, take $|C_f| \ge 0.7$ to indicate a large correlation.

•
$$S_f \equiv |\vec{\rho}_i| \cos \varphi = C_f \frac{\Delta r_i}{\langle r_0 \rangle_E}$$
 -- projection of $\vec{\rho}_i(\vec{a})$ on $\vec{\nabla} f$

 S_f is proportional to $\cos\varphi$ and the ratio of the PDF uncertainty to the experimental uncertainty. We can sum $|S_f|$. In the figures, take $|S_f| > 0.25$ to be significant.