### The axial anomaly and neutrinophoton interactions

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#### A new class of standard model interactions



### Overview

- **Theory toolbox:** effective field theory, chiral lagrangians, anomalies
- Baryons, backgrounds, and vector mesons
- **Phenomenology:** anomaly mediated neutrino-photon interactions

## Theory toolbox

These interactions are part of the low-energy "effective field theory" of the standard model

The hadronic sector of this effective description is a "chiral lagrangian"

The new interactions are a necessary consequence of enforcing that the chiral lagrangian has the correct "*anomaly*" structure

- ⇒ What's an effective field theory ?!
- ⇒ What's a chiral lagrangian ?!

 $\Rightarrow$  What's an anomaly ?!

## Effective field theory

#### Two sides to effective field theory

At low energies,

- physics is dictated by field content and symmetry;

- operators are ordered by an expansion in small parameters

Can make use of effective field theory in two ways:

The DON'T KNOW side.

The DON'T CARE side.

#### The DON'T KNOW side of effective field theory

Sometimes we don't know the "fundamental theory".

This has always been the case, and perhaps always will be, as we probe to ever shorter distances

With a complete accounting of fields and symmetries at the relevant energy scales, can still provide a rigorous description

#### Examples

- The standard model ( *but*, we don't know that the SM Higgs sector is the correct effective description ! )

- Fermi theory of weak interactions ( before we knew about W,Z ! )

- Nonrelativistic quantum mechanics (before we knew about QED !)

#### The DON'T CARE side of effective field theory

Sometimes we do know the "fundamental" theory.

But often the theory has far too much information, or is not easy to calculate

Useful to "Taylor expand" about the kinematics we're interested in, usually low energy

Examples

- Fermi theory of weak interactions (after we know about W,Z !)

 nonrelativistic quantum mechanics (NRQED, after we know about QED !)

- heavy quark physics (HQET, NRQCD, SCET)
- low energy QCD  $\Rightarrow$  chiral lagrangians

# Chiral lagrangians

### A special case of low-energy effective theory

Spontaneously broken symmetries give rise to massless fields (pions)

At low energies, these fields are what survive

Our effective theory is constructed out of these fields, under the constraint that the theory respects the original symmetry (even though the vacuum breaks it)

Know the fields. Know the symmetries.  $\Rightarrow$  Construct the theory !

#### Spontaneous symmetry breaking and Goldstone bosons

Example: scalar field theory in "mexican hat" potential - vacuum breaks rotational symmetry

- perturbing around this vacuum, there is a massless excitation along the bottom of the well, corresponding to rotations into equivalent vacuums



$$\phi_1 + i\phi_2 = r \, e^{i\theta} - \operatorname{light}$$
 heavy

- "states" of the low-energy theory correspond to elements of the rotational symmetry group. The states are created by the corresponding field.

'chiral field" → 
$$O(x) = e^{i\theta(x)} \in SO(2) = S^1$$
 ← field space  
"pion"

Unlike most "regular" field theories, here the field space is curved

Example: three-dimensional potential well

 vacuum state breaks some symmetry, but some symmetry remains



$$\exp\left[\theta_{1}\left(\begin{array}{ccc} \cdot & \cdot & -1\\ \cdot & \cdot & \cdot\\ 1 & \cdot & \cdot\end{array}\right) + \theta_{2}\left(\begin{array}{ccc} \cdot & \cdot & \cdot\\ \cdot & \cdot & 1\\ \cdot & -1 & \cdot\end{array}\right)\right]\left(\begin{array}{c} 0\\ 0\\ r\end{array}\right)$$
  
broken generators

"chiral field"  $\longrightarrow O(x) \in SO(3)/SO(2) = S^2$  field space

#### Low energy QCD

The QCD lagrangian for massless (u,d,s) quarks is invariant under unitary SU(3)<sub>L</sub> x SU(3)<sub>R</sub> flavor transformations:  $Q_L = \begin{pmatrix} u_L \\ d_L \\ s_T \end{pmatrix}$ 

$$\mathcal{L} \sim \bar{Q} \, i \partial \!\!\!/ Q = \bar{Q}_L \, i \partial \!\!\!/ Q_L + \bar{Q}_R \, i \partial \!\!\!/ Q_R$$

 $Q_R = \left(\begin{array}{c} u_R \\ d_R \\ s_P \end{array}\right)$ 

But a condensate forms in the QCD vacuum:

 $\left< \bar{Q}_R Q_L \right> \neq 0$ 

For each broken generator, a massless "Nambu Goldstone boson"

Low energy QCD described by unitary matrix of "pions"

$$U(x) = \exp\left[i \begin{pmatrix} \pi^{0} + \eta/\sqrt{3} & \sqrt{2}\pi^{+} & K^{0} \\ \sqrt{2}\pi^{-} & -\pi^{0} + \eta/\sqrt{3} & K^{+} \\ \frac{1}{K^{0}} & K^{-} & -2\eta/\sqrt{3} \end{pmatrix}\right]$$

We know the fields:

a unitary matrix of pions

$$U(x) = e^{i\pi(x)}$$

We know the symmetry: global  $U(n)_L \times U(n)_R$ 

$$U \to e^{i\epsilon_L} U e^{-i\epsilon_R}$$

#### What interactions can we build? Rule: to every U a $U^{\dagger}$ , and then trace:

$$\mathcal{L} = \operatorname{Tr}(\partial_{\mu}U^{\dagger}\partial_{\mu}U) + \dots$$

Rule: for gauge fields, use covariant derivative:

$$\partial_{\mu}U \to D_{\mu}U = \partial_{\mu}U - iA_{L\mu}U + iUA_{R\mu}$$

#### Successes

Description of many low-energy hadronic processes in terms of a small number of parameters

$$\pi\pi o \pi\pi$$
  
 $\eta o 3\pi$   
Failures

Too much symmetry!

By the naive rules,  $U \leftrightarrow U^{\dagger}$  is an exact symmetry. Forbids observed processes:  $\pi \rightarrow 2\gamma$ 

$$\pi \to 2\gamma$$
 $K\bar{K} \to 3\pi$ 

Not enough anomaly!

Shouldn't be possible to couple gauge fields to *all* of the flavor symmetries: at the quark level - this leads to anomalies

Maybe we have the wrong effective theory ?

Or maybe we've just left out an operator ?

#### **Resolution to the "paradox"**

An operator was left out

Consider a toy example: QM of a particle on the sphere

#### fields:

surface of the sphere

$$x(t) = \begin{pmatrix} x^{1}(t) \\ x^{2}(t) \\ x^{3}(t) \end{pmatrix}$$

$$(x^1)^2 + (x^2)^2 + (x^3)^2 = R^2$$

#### symmetry:

rotational invariance

#### What operators can we build ?

Rule: to every x an  $x^{T}$ :

$$L \sim \frac{1}{2} \frac{dx}{dt}^T \frac{dx}{dt} + \dots$$

Is there anything else ?

In particular, is the parity  $x \leftrightarrow -x$  a necessary symmetry ?



#### Topological interactions: Wess-Zumino-Witten terms (in one dimension)



We've found another way of taking our fields x(t), and building an action that is rotationally invariant ! It breaks the  $x \leftrightarrow -x$  symmetry !



Two choices: for consistency, exp(i action) should not depend on this choice:



#### Move from quantum mechanics to quantum field theory

Topological interaction is particularly simple when the field space is the (d+1)-sphere

- S<sup>2</sup> for d=1 (quantum mechanics)
- S<sup>5</sup> for d=4 (four-dimensional field theory)

Consider the symmetry breaking pattern  $U(3)/U(2) = S^5$ "the simplest WZW term"

- relevant for SM Higgs sector, by reducing to U(2)/U(1), or to axion by reducing to U(1)/e
- occurs in extensions of SM: little higgs models

#### What operators can we build ? Rule: to every $\varphi$ a $\varphi^{\dagger}$ :

$$(x) = \left(\begin{array}{c} \phi^1(x) \\ \phi^2(x) \\ \phi^3(x) \end{array}\right)$$

Φ

$$\mathcal{L} = \partial^{\mu} \Phi^{\dagger} \partial_{\mu} \Phi + \dots$$

Is there anything else ?



U(3) symmetry acts as subgroup of rotations on the sphere: we've found another way of taking our fields  $\varphi(x)$ , and building an action that is invariant under the original symmetry !



Two choices: for consistency, exp(i action) should not depend on this choice:



The simplest Wess-Zumino-Witten term:

- topological derivation of SM Higgs WZW term
- prevalent in Little Higgs/ composite EWSB models
- mathematically interesting: not a "symmetric space"

[RJH, 2007]

These constructions rely on the trivial topological properties of spheres:

$$\pi^1(S^2) = 0$$
 - given a circle, can make a disc

 $\pi^2(S^2) = \mathbb{Z}$  ----- difference of two discs wraps sphere nontrivially (quantization)

The same thing, just three dimensions up:

$$\pi^4(S^5) = 0$$
$$\pi^5(S^5) = \mathbf{Z}$$

The same thing, just with the QCD field space:

$$\pi^4(SU(n)) = 0$$
$$\pi^5(SU(n)) = \mathbf{Z}$$

#### Wess-Zumino-Witten terms (in four dimensions)



This action breaks the spurious parity in the naive QCD chiral lagrangian, solving the first paradox (too much symmetry)

Will see that it also solves the second paradox (not enough anomaly)

### Anomalies

#### Two parts to the QCD chiral lagrangian:

"regular" part: the kinetic term

- manifestly invariant under the global chiral symmetry

- can couple to gauge fields just by replacing partial derivatives by covariant derivatives

$$\partial_{\mu} \to \partial_{\mu} - iA_{\mu}$$

"anomalous" part: the topological or WZW term

- simple prescription doesn't work: would require a fivedimensional gauge field

$$\Gamma(U) = \int d^4x \, \mathcal{L}(x) = \# \times \text{ "area"}$$



- so coupling to gauge fields is more intricate, and in general,

#### Cannot gauge all of the symmetries simultaneously !

#### Who ordered the fermions ?

If we try to gauge too many symmetries, find an anomaly:

$$\delta\Gamma_{\rm meson} = \frac{-p}{24\pi^2} \int \operatorname{Tr}\left\{\epsilon_L \left[ (dA_L)^2 - \frac{i}{2}d(A_L^3) \right] \right\} - (L \leftrightarrow R)$$

Recall the fermion anomaly:

$$\delta\Gamma_{\text{quark}} = \frac{-N_c}{24\pi^2} \int \text{Tr}\left\{\epsilon_L \left[ (dA_L)^2 - \frac{i}{2}d(A_L^3) \right] \right\} - (L \leftrightarrow R)$$

⇒ We started with a boson theory, and realize that it is secretly remembering the properties of underlying fermions

#### This solves the second paradox (not enough anomaly)

# Recap so far

#### We're building the low-energy standard model

We have the basic toolkit for anomaly mediated interactions

- effective field theory
- chiral lagrangian
- anomalies

We now want to explore interactions of the weak force with baryons. An essential new aspect is the introduction of background vector fields.

#### The standard model coupled to general background fields

We need background vector fields to:

- I) find baryons in the chiral lagrangian
- 2) represent vector mesons (that transmit strong force)
- 3) represent physical backgrounds (e.g. finite baryon density)

But adding these background fields into the theory with gauge fields leads to a new and interesting complication

# Baryons, backgrounds and vector mesons

#### **Baryons from the quark side**

Conserved quantities are fundamental to understanding a field theory - electric charge, baryon number, etc.

At the quark level:

$$\mathcal{L} = \bar{q} \, i \partial \!\!\!/ q$$

Notice that there is an invariance of the Lagrangian:

$$q \to e^{i\epsilon} q \qquad \Rightarrow \qquad \mathcal{L} \to \mathcal{L}$$

Noether's theorem: for every invariance, a conservation law!

$$q \to e^{i\epsilon(x)} q \qquad \Longrightarrow \qquad \int \mathcal{L}(x) \to \int (\partial_{\mu}\epsilon) J^{\mu} = -\int \epsilon \,\partial_{\mu} J^{\mu}$$

Can't create or destroy "baryon number"

$$J_B^{\mu} = \sum_q \bar{q} \gamma^{\mu} q \quad \Rightarrow \quad \partial_{\mu} J_B^{\mu} = 0$$

#### **Baryons from the meson side**

Can we find baryons in the chiral Lagrangian ? <u>At the meson level:</u>

$$\mathcal{L} = \mathrm{Tr}(\partial^{\mu} U^{\dagger} \partial_{\mu} U)$$

Recall the transformation law for U:

 $q_L \to e^{i\epsilon_L} q_L, \quad q_R \to e^{i\epsilon_R} q_R \qquad \Longleftrightarrow \qquad U \to e^{i\epsilon_L} U e^{-i\epsilon_R}$ 

But our transformation is trivial when  $\epsilon_L = \epsilon_R$ :

$$U \to U$$

We get nothing ! Where are the baryons ?

#### Putting a baryon "handle" in the quark theory

<u>Idea</u>: Introduce "probes": vector fields coupled to the various flavor symmetries

$$\mathcal{L} \to \bar{q}(i\partial\!\!\!/ + B\!\!\!/)q = \mathcal{L}_0 + B_\mu J_B^\mu$$

With the handle in place, can forget about the quarks:

$$\frac{\delta}{\delta B_{\mu}} \int \mathcal{L} = J_{B}^{\mu}$$
$$\delta B_{\mu} = \partial_{\mu} \epsilon \quad \Longrightarrow \quad \delta \int \mathcal{L} = \int (\partial_{\mu} \epsilon) J^{\mu} = -\int \epsilon \partial_{\mu} J_{B}^{\mu}$$

#### Putting a baryon "handle" in the meson theory

<u>Idea</u>: put the same handle in the meson theory !

Find the conserved current corresponding to baryon number in the quark theory

$$J_B^{\mu} = \frac{N_c}{72\pi^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{Tr}[U(\partial_{\nu}U^{\dagger})U(\partial_{\rho}U^{\dagger})U(\partial_{\sigma}U^{\dagger})]$$

$$Skyrme (1962)$$

$$Goldstone and Wilczek (1981)$$

$$= \text{ some configuration of pions } !$$

baryon

$$U(t,x) = e^{if(|x|)\hat{x}^a \tau^a}, \quad f(0) = 0, f(\infty) = \pi$$



the conserved current implies that it's impossible to "unwrap" a baryon !

 $\Rightarrow \qquad \int d^3x \, J_B^0(t,x) = 1$ 

### What happens when we have gauge fields in addition to the backgrounds ?

Suppose we have a consistent gauge theory of "fundamental" gauge fields A  $(A=W,Z,\gamma)$ 

$$\begin{split} &\Gamma\sim\int d^4x\,\sum_{\psi}\bar\psi(i\partial\!\!\!/+A)\psi\\ &\delta\Gamma\sim\int d^4x\sum_{\psi}\mathrm{Tr}\Big\{\epsilon\big[(dA)^2-\frac{i}{2}d(A^3)\big]\Big\}=0 \end{split}$$

A new difficulty when background vector fields are present:

$$\begin{split} &\Gamma\sim\int d^4x\,\sum_{\psi}\bar{\psi}(i\partial\!\!\!/ + A + B)\psi\\ &\delta\Gamma\sim\int d^4x\sum_{\psi}\mathrm{Tr}\bigg\{\epsilon\big[(dA + dB)^2 - \frac{i}{2}d((A + B)^3)\big]\bigg\}\neq 0 \end{split}$$

**Paradox**: we added what appeared to be a gauge-invariant perturbation to a gauge invariant theory, and now we have a new anomaly

**Resolution**: we must add a counterterm at the same time as the perturbation

*Fact*: for a given set of "fundamental" fields A, and a general "background" B, there is a unique counterterm that maintains A gauge invariance

$$\Gamma \sim \int d^4x \sum_{\psi} \bar{\psi} (i\partial \!\!\!/ + A + B)\psi + \Gamma_c(A, B)$$
  
J. Harvey, C. Hilll and RJH, 2007  
$$\delta \Gamma \sim \int d^4x \sum_{\psi} \text{Tr} \left\{ \epsilon \left[ (dA + dB)^2 - \frac{i}{2} d((A + B)^3) \right] \right\} + \delta \Gamma_c(A, B) = 0$$

This counterterm is the missing ingredient for a consistent theory in general backgrounds

generalizes the "Bardeen counterterm" appropriate for pure vector-like gauging Bardeen 1969 Kaymakcalan, Rajeev, Schechter, 1984

# The baryon number anomaly

With the full apparatus in place, can simply turn the crank to recover the baryon number anomaly of the SM at zero background:

$$\partial_{\mu}J^{\mu} = \frac{1}{32\pi^{2}}\epsilon^{\mu\nu\rho\sigma} \left(\frac{1}{2}g_{2}^{2}F_{\mu\nu}^{a}F_{\rho\sigma}^{a} - \frac{1}{2}g_{1}^{2}F_{\mu\nu}^{Y}F_{\rho\sigma}^{Y}\right)$$

Two uses for anomalies:

- make sure gauge theories are consistent

- find nonconservation of naively conserved quantities

This is the second type of anomaly

- a fundamental ingredient of the standard model

- could explain baryogenesis at electroweak phase transition, if a large source of CP violation is present

### what are the experimental consequences of the baryon number anomaly?

<u>Idea</u>: keep the background fields in place: they represent physical fields coupling to baryon number !

## Applications

# QCD vector meson decays

#### "Existence proof" of pseudo-Chern Simons terms in the Standard Model



$$\Gamma_{\text{expt}} = 700 \pm 170 \pm 150 \text{ keV}$$
[Amelin et.al.VES collab, 1994]
$$\Gamma_{\text{theory}} \approx 200 \times \left(\frac{g_{\rho}}{6}\right)^4 \text{ keV}$$
[Harvey, Hill & Hill, 2008]

- To obtain the correct (nonzero!) normalization, it is essential to include the new counterterm

The polarization structure is also interesting:

$$\left(\frac{\Gamma_{\perp}}{\Gamma_{\parallel}}\right)_{\rm expt} = 0.26 \pm 0.06 \pm 0.07$$

$$\left(\frac{\Gamma_{\perp}}{\Gamma_{\parallel}}\right)_{\text{theory}} \approx \frac{m_{\rho}^2}{m_{f_1}^2} = 0.37$$

Normalization and polarization structure contradict previous predictions based on naive vector dominance

[Babcoock & Rosner, 1976]

-  $\rho$  and  $\gamma$  are not the same particle: no violation of Landau-Yang theorem for transverse final state

## Anomaly mediated Neutrinodynamics



- take one as the isoscalar coupling to nucleons
- the other is the Z boson
- most dramatic effects possible in neutrino interactions

### The framework provides slots that vector fields fit into

#### Can interpret these as

- background fields
- vector mesons

The framework provides slots that vector fields fit into



Photon decay to neutrino pair in background baryon density: *neutron star cooling* 

The framework provides slots that vector fields fit into





Hard photon from neutrino-nucleus scattering: *laboratory neutrino detection* 

Detector = hadronic matter

Quarks like to bunch up in threes: our elemental detector component is a nucleon (= twisted pion configuration)

What is the cross section for neutrino scattering on a detector element (nucleon) ?



- idealized nucleon,
- includes simplest form factor (omega propagator)
- includes recoil of the nucleon (treated as a free particle)
- neglects coherence, Fermi motion and other nuclear effects

#### competing processes

Not easy for a neutrino-nucleus scatter to yield a photon

- neutrinos are neutral
- heavy nucleons don't radiate !

#### Other vector-current exchanges:

$$\frac{g_{\rho NN}}{g_{\omega NN}} \sim \frac{1+1-1}{1+1+1} = \frac{1}{3}$$



"coherence over the nucleus"

 $\rightarrow$  in rate,  $\rho$  exchange suppressed by  $\sim (1/3)^4$ : negligible

#### competing processes

Axial-currents:

- pion exchange potentially significant, due to small mass, but subdominant

$$1 - 4\sin^2\theta_W \ll 1$$

$$\frac{1}{f_{\pi}^4} \lesssim \frac{g_{\omega}^4}{m_{\omega}^4}$$

not coherent over adjacent nucleons
could in principle be probed in charged current process





#### competing processes

Bremstrahlung and related contact interactions

- formally suppressed by nucleon mass



- for neutron, dominant effect is magnetic form factor,
- for proton, no other large enhancements

As a rough guide neglect:

- form factor and recoil suppression (valid for E<<1 GeV)
- coherence and other enhancements



The real world is slightly more complicated: **nucleons vs. nucleus** 

Baryons like to clump together into nuclei



Fermi momentum of the nucleons inside the nucleus leads to:

- initial state (smearing over p up to  $p_F$ )
- final state (Pauli blocking for final states below  $p_F$ )
- coherence for some processes (like the omega !)

Analogy: parton distribution functions in a hadron collider

Will ignore these complication in the following: scattering on free nucleons

# Can we observe these effects?

Current and near future neutrino experiments should be sensitive to anomaly mediated interactions

- measure the baryon anomaly of the standard model
- signals and backgrounds for neutrino oscillation searches
- constrain new neutrino interactions for astrophysics A good place to look:

-  $E_v = 100 \text{ MeV}$  to 1000 MeV where process is prominent (coherence can make low energy important too)

- pure beam of  $v_{\mu}$ , unless we can distinguish final state electron from final state photon (otherwise a  $v_{\rho} \rightarrow e$  background)
- $\Rightarrow$  overlap with experiments looking for  $v_{\mu}$  oscillations !

But this is a bonus - we didn't set out to explain existing data

#### Some examples:





#### [J. Monroe, MiniBooNE, hep-ex/0408019]

[ltow et. al., T2K, hep-ex/0106019]

anomaly mediated photon emission is signal or background depending on perspective !



#### Is this process observable ?

For a rough estimation, normalize to charged current interactions, neglecting form factor and recoil:

$$\sigma \approx \frac{1}{480\pi^6} G_F^2 \alpha \frac{g_\omega^4}{m_\omega^4} E_\nu^6$$

E.g. at MiniBooNE, for a flux of 700 MeV v's, for every  $2x10^5$  CCQE events, expect:

$$\sim 120 \left(\frac{g_{\omega}}{10}\right)^4$$

new events.

This normalization is very rough, but several tens to several hundreds of events are expected

More accurate normalization requires complete flux information, acceptance corrections, plus nuclear corrections

What are the expectations independent of the normalization ?



[G. Zeller

hep-ex/0312061]

Characteristic photon energy distribution:

$$\frac{d\sigma}{dE_{\gamma}} \propto E_{\gamma}^3 (E - E_{\gamma})^2$$

And photon angle distribution:

$$\frac{d\sigma}{d\cos\theta} \propto \text{const.}$$

Including simplest form factor, and recoil, for E ~700 MeV neutrino beam:



Is there room for such a contribution at MiniBooNE ?

#### Has this process been seen?

Events that look like  $V_{\rho}$  charged-current scattering



- energy dependence of excess (~100 events) not consistent with 2 neutrino oscillation

- excess of events at low energy appears to be growing ? Is it real ? Is anything else left out ?

- the "reconstructed  $E_{\rm v}$ " assumes 2-body kinematics to find initial-state energy from final state "electron" energy and angle

- if it's a 3-body state,  $E_v$  underestimated

- what does the excess look like in terms of visible (electron or photon) energy ?





[R.Tayloe, MiniBooNE, Lepton Photon 07]







- consistent with expectations for the anomaly-mediated photon process



- for a detailed study, including normalization, require accurate flux, acceptance corrections, accurate coupling, nuclear effects

# Many applications and directions to explore

• High energy neutrinos, e<sup>-</sup>p scattering, diffractive processes A new theoretical regime: Regge physics, pomeron, ...

Axions
 Supernova bounds; Laboratory detection



• Astrophysics: neutron star cooling; supernova energy transfer? SN nucleosynthesis? magnetic field enhancements? neutron star kicks?



- Theory: driving forces and spinoffs
- the simplest WZW term: SU(3)/SU(2)
  - Little Higgs models
  - topological derivation of standard model WZW term
- partial SU(2) multiplets and nonlinear realizations: adding the strange quark
- planar equivalences at the chiral lagrangian level
- AdS CFT: take the fifth dimension seriously

## Summary

 many new applications of anomaly physics both in and beyond the Standard Model

• need to include background vector fields: to define baryons, to represent physical bkgds, or physical mesons

• new structure is required along with the vector fields for consistency. New structure leads to new interactions

• new experimental predictions: Observing the baryon number anomaly of the standard model !

should be observable at present and/or near-future experiments

• many directions to explore !