Jeffrey S. Dunham, *Editor*
Department of Physics, Middlebury College, Middlebury, Vermont 05753

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A compact apparatus for muon lifetime measurement and time dilation demonstration in the undergraduate laboratory

Thomas Coan,[a] Tiankuan Liu,[b] and Jingbo Ye[c]
Physics Department, Southern Methodist University, Dallas, Texas 75275

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We describe a compact apparatus for measuring the charge-averaged lifetime of atmospheric muons in plastic scintillator using low-cost, low-power electronics. We present measurements of the stopping rate of atmospheric muons as a function of altitude to demonstrate relativistic time dilation. The apparatus is designed for the advanced undergraduate physics laboratory and is suitable for field measurements.
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I. INTRODUCTION

Measurement of the mean lifetime of muons produced in the Earth's atmosphere from collisions is a common experiment in advanced undergraduate physics laboratories.[1–3] Typically, a single scintillating medium, massive enough to stop some useful fraction of the muons impinging on it, is viewed by one or two photomultiplier tubes (PMTs) that detect the pair of scintillation light flashes associated with the entry of a muon and the subsequent decay of a stopped muon. The time interval between the two flashes may be histogrammed and the time distribution fitted with an exponential function to yield the mean muon lifetime. Various PMT readout and histogramming techniques have been implemented to produce the decay-time histogram. Such techniques tend to rely on relatively expensive and bulky electronic instrumentation (e.g., NIM and CAMAC-standard modules) and stand-alone multi-channel analyzers to generate the decay-time histogram. We have developed equivalent readout instrumentation, using a complex programmable logic device[4] (CPLD), that is compact ($20\times25\times5$ cm^3), low cost, and low power (<25 W). The readout instrumentation is easily interfaced to a laptop computer which can be used to display and fit the decay-time histogram.

II. DETECTOR AND READOUT ELECTRONICS

We use a standard detector configuration: no attempt is made to select only vertically traveling muons. A plastic scintillator (NE 102) in the shape of a right circular cylinder (15.2 cm diameter and 12.7 cm tall) is viewed by a single 10-stage, 51 mm diameter, bialkali photocathode PMT biased for nominal gain of 3×10^5 that is attached to one end. Both scintillator and PMT are wrapped carefully with aluminum foil and electrical tape to prevent light leaks. A compact, commercially available direct current (dc)-dc converter[5] is used to supply an adjustable negative high voltage (HV) of nominal value -1150 V to the PMT photocathode. To mimic events where a muon enters, stops and then decays inside the scintillator, a blue light emitting diode (LED) is permanently inserted into a small hole drilled in one end of the scintillator. The LED can be driven by a pulser circuit that produces pairs of pulses at a nominal repetition rate of 100 Hz with an adjustable interpulse separation of 300 ns to 20 μs. For robustness and portability, and so that no HV electrodes are exposed to students, the scintillator, PMT, HV circuit, and pulser are all enclosed inside a black anodized aluminum tube 36 cm tall with a 15.2 cm inner diameter. The cylinder is capped at both ends. Power to the HV supply and pulser circuitry is provided by a single multi-connector cable, and the PMT signal is sent to the readout electronics module by a coaxial cable. Potentiometers installed in the cylinder cap allow adjustment of the PMT HV and the LED interpulse time-separation. Provision is made to monitor the HV with a conventional voltmeter, and the output of the pulser circuitry is accessible by a coaxial connector on the top cap. The mass of the overall detector is 5 kg. Figure 1 shows the detector with its associated readout electronics.

The electronic circuitry to process PMT signals, perform timing, communicate with the computer, and provide power to the detector is mounted on one printed circuit board (PCB) located inside a single 20 cm$\times$25 cm$\times$5 cm enclosure. A block diagram of this circuitry is shown in Fig. 2. Signals from the PMT anode are coupled by the coaxial cable to the input of a two-stage amplifier constructed from a fast current

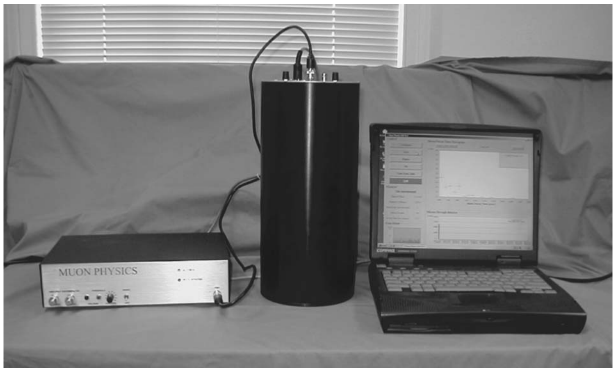

Fig. 1. Photograph of the detector between its readout electronics box and a laptop running data analysis software.

feedback amplifier.[6] A typical raw PMT signal amplitude for muon decay events is 100 mV into 50 Ω impedance. The amplifier output feeds a discriminator with an adjustable threshold and *TTL*-logic output. Students can monitor the amplifier and discriminator outputs at BNC connectors that are mounted on the front of the electronics enclosure.

The discriminator output signal is processed by a CPLD that is mounted on a single PCB. A CPLD is a single, flexible integrated circuit (IC) that comprises thousands of programmable logic gates that can implement a variety of digital logic functions. The chip includes programmable interconnections and multiple input/output (I/O) pins, and can be clocked at rates up to ~100 MHz. Its behavior can be simulated before its controlling program is downloaded to its electrically-erasable and programmable read-only memory. Such ICs are reprogrammable and relatively cheap (typically costing a few tens of dollars), making the implementation of a wide variety of digital logic signal-processing circuits practical and inexpensive.

The CPLD is programmed in a digital hardware-description language to function primarily as a timer. A logical "yes" output from the discriminator, corresponding to an amplified PMT signal above threshold, causes the CPLD to either start recording clock cycles from the 50 MHz crystal oscillator that clocks it, or, if the CPLD had already started counting clock cycles, to stop counting. In this way, any amplified PMT signal above threshold can serve as a stop or start timing signal. The CPLD is also programmed to reset itself if a second yes does not occur within 20 μs of the first one. This simple logic scheme corresponds to our desired event sequence: a flash of scintillator light when a muon enters and stops in the scintillator, and a second flash when the stopped muon decays.

The CPLD formats data in a simple fashion. For successive PMT signals above threshold and within the 20 μs timing window, the time difference, in integral units of the 50 MHz clock period, between these two signals is recorded. Data of this type are ultimately histogrammed and fitted to an exponential curve by the laptop software to extract the muon lifetime. For cases when there are no signal pairs within the timing window, the CPLD merely records the integer 1000, corresponding to the number of clock periods in the 20 μs timing window. All data are subsequently sent to the laptop through either a serial or USB port.

The CPLD I/O circuitry shown in Fig. 2 has two ports to simplify interfacing to laptop computers. One port is a standard serial port, which follows the RS-232 protocol and relies on a dedicated RS-232 transceiver chip to shift RS-232 standard voltage levels to low-voltage *TTL* levels so that a laptop can communicate with the RS-232 interface module resident in the CPLD. The data transmission rate between the CPLD and a laptop is 115 kbits/s. The other port adheres to the USB 1.1 protocol and relies on a USB-FIFO ("first in, first out") translator chip to communicate with the FIFO interface module within the CPLD. Data transmission rates in this case are 2.4 Mbits/s.

Overall power consumption of the electronics module is less than 25 W, sufficiently low that it can be powered in the field from a 12 V automobile cigarette lighter.

III. DATA DISPLAY SOFTWARE

The laptop-resident software that displays and curve-fits the decay-time histogram is written in the Tcl/Tk scripting language, a non-proprietary language that permits easy implementation of graphical user interfaces and is compatible with Unix, Microsoft Windows and Apple Macintosh operating systems. The laptop continuously examines its own I/O-port buffers for the presence of any data that the CPLD has sent it. Any data consistent with PMT pulse pairs within the timing window have the corresponding pulse separation time entered into a decay-time histogram. Data not corresponding to pulse pairs are used to update various rate-meters monitoring the frequency of PMT signals above threshold. All data are then slightly reformatted to include the absolute time in seconds when it was examined by the laptop before being written to disk in ASCII format for easy human interpretation and exporting to student written data analysis software routines. The histogram and rate meters are displayed in real time for student observation.

The laptop software has a provision for simulating muon decay by randomly generating times according to an exponential distribution with a user-selectable lifetime. This permits students to test their curve-fitting and lifetime-extracting software routines on large simulated data sets.

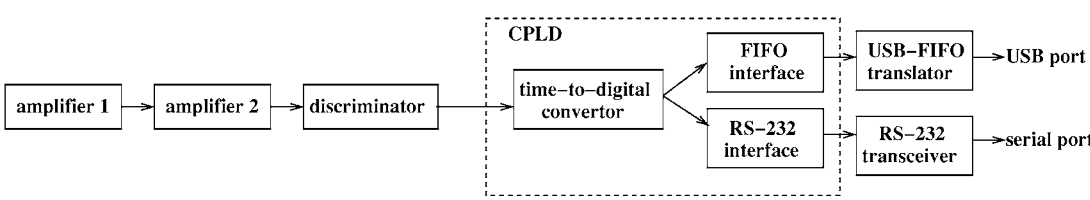

Fig. 2. Block diagram of the readout electronics showing the two-stage amplifier, discriminator, CPLD, and I/O communications circuitry. Signals are processed from left to right.

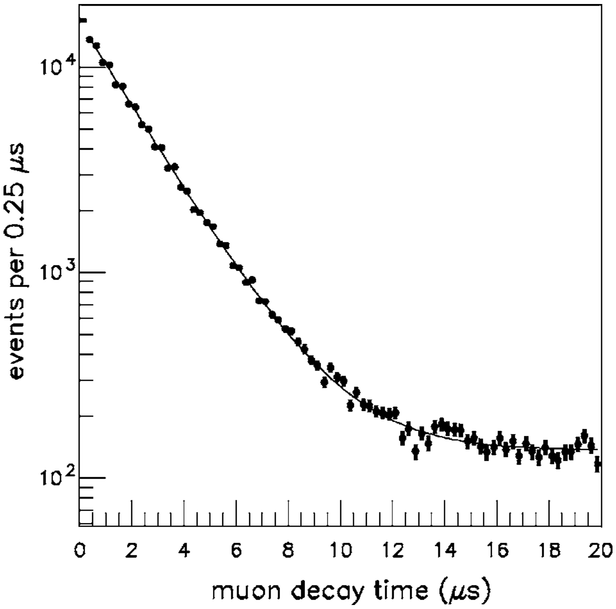

Fig. 3. Decay time histogram for 28 963 events collected over 480 h. The dots with error bars are data and the line is the three-parameter function $f(t)=P_1P_2\exp(-P_2t)+P_3$ fitted to the data. The fitted values are $P_1 = 9341\pm69$, $P_2=0.472\pm0.004$, and $P_3=18.44\pm0.97$. The χ^2 per degree-of-freedom is 1.3.

IV. MEAN MUON LIFETIME

A decay-time histogram for muons stopping in our detector is shown in Fig. 3. Dots with crosses are data, and the line is an exponential fitting of the data. This histogram contains 28 963 events collected over 480 h of running and contains μ^+ and μ^- decays as well as background. The data are fitted to the functional form $\dot{N}(t)=P_1P_2\exp(-P_2t)+P_3$ characteristic of radioactive decay with background. Here $\dot{N}(t)$ represents the observed number of decays per unit time at time t; the quantities P_1, P_2^{-1}, and P_3 are constants extracted from the fit that represent an overall normalization constant, the muon lifetime, and the background level, respectively. The quality of the fitted curve is indicated by the low χ^2 per degree-of-freedom of 1.3. The fit was done with PAW,[7] a fitting and plotting freeware software package. Other fitting packages return similar results.

The extracted value of the mean muon lifetime, $\tau=P_2^{-1} =2.12\pm0.02$ μs (statistical error only), is less than the free-space value $\tau_\mu=2.197\,03\pm0.000\,04$ μs due to the non-negligible probability that a μ^-, but not a μ^+, will be captured into the K-shell of a scintillator carbon atom and then be absorbed by its nucleus[8] without producing a high energy decay electron. (The probability that a stopped μ^- will be absorbed by a target atom of atomic number Z is proportional to Z.[4])

The extracted background rate of false muon decays inferred from the value of P_3 and the 480 h running time is 0.6 mHz. This rate for two unrelated PMT signals to accidentally coincide within 20 μs is consistent with the observed rate of single PMT signals above threshold ($\sim$6 Hz) and our 20 μs timing window. For comparison, the fitted rate of valid muon decays in the scintillator is 17 mHz ($\sim$1 min^{-1}).

From the charge-averaged lifetime of muons in what is essentially a carbon target (the ratio of hydrogen to carbon in plastic scintillator is 1:1), and the lifetime of μ^- in carbon,[9] it is straightforward[10] to measure the charge-ratio $\rho =N(\mu^+)/N(\mu^-)$ of low-energy ($E\lesssim200$ MeV) muons at sea-level. For example, from our measured lifetime, we find $\rho =1.08\pm0.01$ (statistical error only), averaged over the angular acceptance of the detector, a value consistent with the diminishing trend[11] for ρ as the muon momentum approaches zero.

V. DEMONSTRATION OF RELATIVISTIC TIME DILATION

The stopping rate of muons in the detector as a function of elevation above sea level can be used to demonstrate relativistic time dilation. Although the detector design is not optimal for this demonstration, since the detector is sensitive to muons with a range of velocities as well as non-vertical trajectories, our design has the advantage that no bulky velocity-selecting absorbers or additional trajectory defining scintillators[12] are required. The technique for demonstrating time dilation is simple. The total number of stopped muons in the detector in some fixed time-interval and at some fixed altitude above sea level is measured from the decay-time histogram. A lower altitude is then selected and predictions are made for the new stopping rate that do and do not include the time-dilation effect of special relativity. Measurement then discriminates between the two predictions.

To make the comparison between the competing assumptions meaningful, one should include the effects of the energy loss of a muon as it descends in the atmosphere as well as the shape of the sub-GeV/c muon momentum spectrum[13] near sea-level. The first effect tends to increase the transit time of the muon from one altitude to another, and the second effect tends to overemphasize the effects of time dilation.

The transit time t' measured in the muon's rest frame as it descends vertically in the atmosphere from a height H down to sea-level is given by

$$t' = \int_H^0 \frac{dh}{c\beta(h)\gamma(h)}, \tag{1}$$

where β and γ have their normal relativistic meanings, dh is a differential element of pathlength, and c is the speed of light. All quantities on the right-hand side of Eq. (1) are measured in the detector rest frame. As the muon descends it loses energy in a manner described by the Bethe-Bloch equation,[14] so that the integral in Eq. (1) can be evaluated numerically if great precision is desired. Instead, we use the common approximation that a singly-charged relativistic particle loses energy by ionization and excitation of the medium it traverses with a magnitude $dE/ds=2$ MeV g^{-1} cm$^2\equiv S_0$. (We use distance units s normalized to the mass density ρ of the traversed material so that $s=\rho x$, with ρ and x measured in standard cgs units.) Equation (1) then becomes

$$t' \simeq \frac{mc}{\rho S_0}\int_{\gamma_2}^{\gamma_1} \frac{d\gamma}{\sqrt{\gamma^2-1}}, \tag{2}$$

where γ_1 is the muon's Lorentz factor at height H, γ_2 is its Lorentz factor just before it enters the sea-level scintillator, m is the muon Lorentz invariant mass and ρ denotes the pathlength-averaged mass density of the atmosphere. We take $\gamma_2\simeq1.5$ since we want muons that stop in the scintilla-

Table I. Muon stopping rate measurements and calculated proper transit times.

Elevation (meters above sea-level)	Observed stopping rate (min^{-1})	Proper transit time (τ_μ)
146 (Dallas, TX)	1.24 ± 0.01	⋯
2133 (Los Alamos, NM)	2.21 ± 0.05	1.06
3154 (Taos, NM)	3.00 ± 0.34	1.32

tor and assume, consistent with our detector geometry, that stopped muons travel an average distance $s = 10\ \mathrm{g/cm^2}$ in the scintillator. (The muon range-momentum graphs from the Particle Data Group[15] correlate a muon's range with its momentum.) The appropriate value of γ_1 depends on the height H where the upper measurement is made and is computed from the energy E_1 a muon has at that height if it is to arrive at the sea-level detector with $\gamma_2 = 1.5$, which corresponds to a total muon energy $E_2 = 160$ MeV. Clearly, if a muon loses energy ΔE in traversing a vertical distance H, then $E_1 = \Delta E + 160$ MeV. The quantity ΔE can be computed from the Bethe-Bloch equation or estimated from the above rule-of-thumb for minimum ionizing particles and properties of the standard atmosphere.

Since the time dilation demonstration relies on stopping muons in the detector, we must account for the fact that muons that eventually stop in the lower detector have, at the position of the upper detector, an energy that is greater than those muons that would be stopped in the upper detector. Since the momentum spectrum of sub-GeV muons near sea-level is not flat, but peaks at muon momentum $p_\mu \sim 500$ MeV/c, we correct for this effect so that the effective flux of incident muons is appropriately normalized. (This is easy to see if you assume muons do not decay at all and only lose energy in the atmosphere as they descend.) We do this by measuring the ratio of stopping rates at a pair of altitudes to determine a single scaling factor that we can apply to other pairs of altitudes.

To illustrate the procedure, we measured the muon stopping rate at two different elevations ($\Delta h = 3000$ m between Taos, NM and Dallas, TX) and computed the ratio R_{obs} of observed stopping rates to be $R_{\mathrm{obs}}(\mathrm{Dallas/Taos}) = 0.41 \pm 0.05$. The transit time t' in the muon's rest frame for vertical trajectories between the two elevations is computed using Eq. (2) yielding $t' = 1.32\ \tau_\mu$. This corresponds to a naive theoretical stopping rate ratio $R = \exp(-t'/\tau_\mu) = 0.267$. The double ratio $R_0 = R_{\mathrm{obs}}/R = 1.5 \pm 0.2$ is then interpreted as a correction factor for the shape of the muon momentum spectrum. Note that this same correction tends to account for muons with non-vertical trajectories that stop in the detector. These slanting muons with a projection onto the vertical axis of distance H travel further in the atmosphere and, hence, start with more energy than their purely vertical counterparts.

To verify that the procedure is sensible, we chose a new elevation ($H = 2133$ m above sea-level at Los Alamos, NM) and computed the transit time $t' = 1.06\ \tau_\mu$ and the expected stopping-rate ratio (Dallas/Los Alamos) $R_{\mathrm{thy}} = R_0 \exp(-t'/\tau_\mu) = 0.52 \pm 0.06$. The observed ratio was $R_{\mathrm{obs}} = 0.56 \pm 0.01$, showing good agreement with the expectation. Table I summarizes relevant measurements; the third column provides calculated proper transit times for vertical muon trajectories, in units of the proper muon lifetime τ_μ and relative to the elevation of Dallas.

To compare the stopping-rate measurements with the competing assumption that there is no time dilation effect ("ntd"), we proceed as before except we calculate the transit time as simply the change in elevation between two sites divided by the speed of light. For the case of transit between the elevations of Los Alamos and Dallas, the transit time t_{ntd} in the detector rest frame is $t_{\mathrm{ntd}} = 6.62\ \mu$s, implying an expected stopping rate ratio $R_{\mathrm{ntd}} = R_0 \exp(-t/\tau) = 0.08 \pm 0.01$, a result strongly disfavored by observation.

VI. SUMMARY

We have designed a compact and low-cost apparatus for measuring the mean muon lifetime and for demonstrating relativistic time dilation that is suitable for undergraduate teaching. An electronics schematic and the Tcl/Tk data-acquisition/display software are available upon request.

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[a] Electronic mail: coan@mail.physics.smu.edu
[b] Electronic mail: liu@mail.physics.smu.edu
[c] Electronic mail: yejb@mail.physics.smu.edu

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