

Electric field and its work on charges

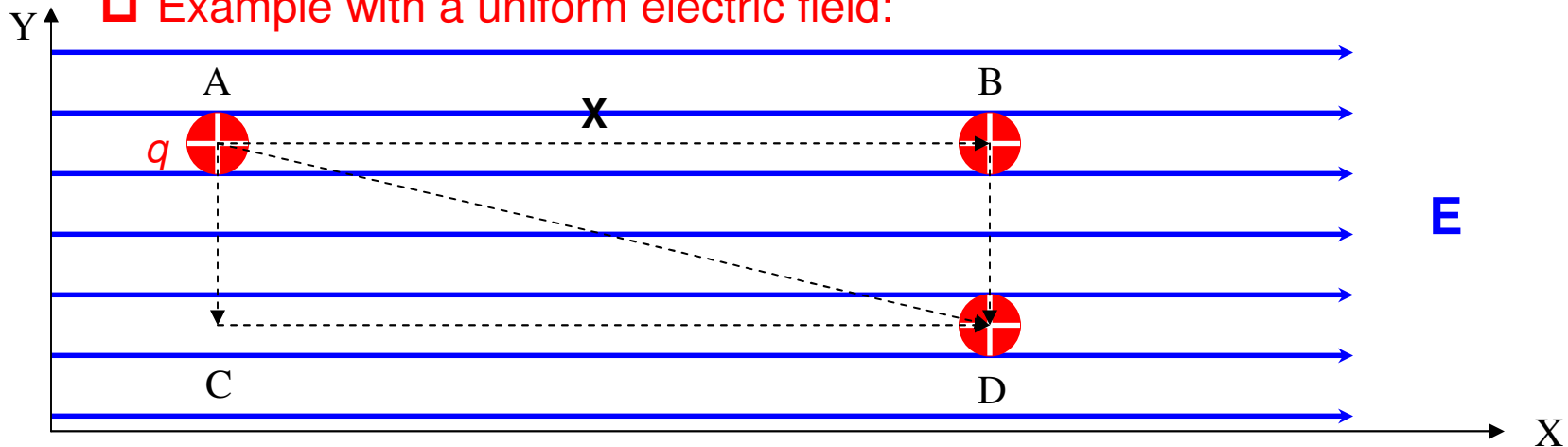
We introduced the concept of electric field and this formula: $\mathbf{F} = q \cdot \mathbf{E}$

- What can you associate force with?

- Motion or acceleration from $\mathbf{F} = m\mathbf{a}$.

- Work, from $W = \mathbf{F} \cdot \mathbf{X}$ (a review on vector dot product here).

- Example with a uniform electric field:



The work to the field \mathbf{E} does to move the charge from A to B is:

$$W_{AB} = \mathbf{F} \cdot \overline{AB} = q\mathbf{E} \cdot \mathbf{X} = qEX, \quad \text{where } \mathbf{X} \text{ is the vector from A to B.}$$

From A to D:

$$W_{AD} = \mathbf{F} \cdot \overline{AD} = q\mathbf{E} \cdot \mathbf{Y} = qEX = W_{AB}, \quad \text{where } \mathbf{Y} \text{ is the vector from A to D. And this answer is true no matter the path is ABD, ACD or AD.}$$

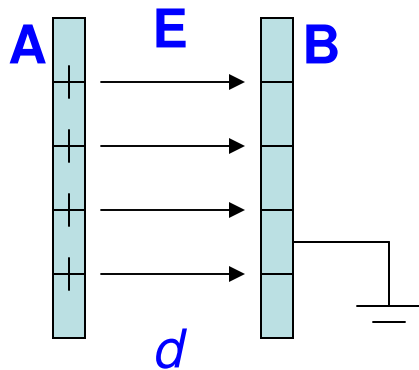
So the electric force $\mathbf{F} = q\mathbf{E}$ is conservative. (a review on conservative force).

- Electric potential of an electric field.

- Electric field exerts force on a charge inside it.
- This force moves the charge and does work to it. The work the electric force does not depend on the path, but only the start and end points of the charge → the electric force is conservative.
- For a charge in a conservative force field, one can define a potential that associates with the charge and the field:

$$W_{AB} = q\mathbf{E}\cdot\mathbf{X} = -\Delta U = -(U_B - U_A) = U_A - U_B$$

- Here the function $U(A)$ is the electric potential of the charge q in the field \mathbf{E} at point A, with a reference to point B.
- Example: what is the electric potential of a unit positive charge on plate A?



The electric field between the plates is E , and the distance is d . Plate B is grounded (potential 0) as shown.

$$U_A = q\mathbf{E}\cdot\mathbf{d} = \mathbf{E}\cdot\mathbf{d}$$

More examples

- Calculate the electric potential of a unit positive charge in an electric field generated by a point charge of $+5.00 \mu\text{C}$.

Diagram illustrating the calculation of electric potential. A point charge $+5.00 \mu\text{C} = Q$ is shown on the left. A path is drawn from point A to point B. The distance from the charge to point A is labeled R . A unit positive charge is shown at point A. The electric field E is indicated by a vector pointing away from the charge. The calculation shows the potential U_A at point A is equal to the work done $\int E \cdot d$, where $d = R$.

$$U_A = \int E \cdot d, \quad d = R$$
$$= \int \frac{kQ}{R^2} \cdot R$$
$$= \int \frac{kQ}{R}$$
$$= \frac{kQ}{R} \quad \text{when } \int = 1, \text{ unit charge.}$$

Or, there is another way:

Example

F_g is the force on q from $Q (= +5.00 \mu\text{C})$.

$$U_A - U_B = W_{AB} = \int_R^\infty F_g \cdot dr = - \int_\infty^R F_g \cdot dr$$
$$= - \int_\infty^R \frac{kQq}{r^2} dr \quad \text{Coulomb's Law}$$

$$= kQq \left(- \int_\infty^R \frac{1}{r^2} dr \right)$$

$$= \frac{kQq}{R} \quad \text{check quiz 0}$$

$$= \frac{kQ}{R} \quad \text{when } q = 1, \text{ unit charge.}$$

Usually $U_B (r = \infty) \equiv 0$, so $U_A = \frac{kQ}{R}$

- Charges can move freely inside a conductor.
- Electric field exerts forces on charges.
- When a conductor is placed inside an electric field:
 - ❑ Charges in the conductor get redistributed until no charge is moving. This new state when no charge is still moved by the electric field is called electrostatic equilibrium.
 - ❑ At this state:
 - there is no electric field inside the conductor, all induced charges must be on the surface.
 - The surface charge density is proportional to the curvature on the surface.
 - The field lines that end at the conductor must be perpendicular to the surface.
 - ❑ Application: the Faraday Cage.
- When a conductor carries charges, they generate electric field that:
 - ❑ The field lines are perpendicular to the conductor's surface.
 - ❑ All charges are distributed on the surface. There is no net charge inside the body of the conductor.
 - ❑ The surface charge density is proportional to the curvature on the surface.

- Electric field is defined as:

$$\mathbf{E} = \frac{\mathbf{F}}{q_{test}}$$

- Electric field is a vector quantity illustrated by field lines.
- The quantity of the field of a point charge is:

$$E = \frac{k|q|}{r^2}$$

- Electric field exerts force on charges inside it:

$$\mathbf{F} = q \cdot \mathbf{E}$$

- The force of an electric field is conservative, hence a potential can be defined.