Electric Potential

1. Electric potential energy U
2. Electric potential V
3. Electric potential V and field E

Review math

Addition and subtraction: if A + B = C ➔ B = C − A

Multiplication and division: if AB = C ➔ B = C/A

Differentiate and Integrate: if $z(x) = \frac{dy(x)}{dx}$ ➔ $y = \int dy = \int z(x)\,dx$

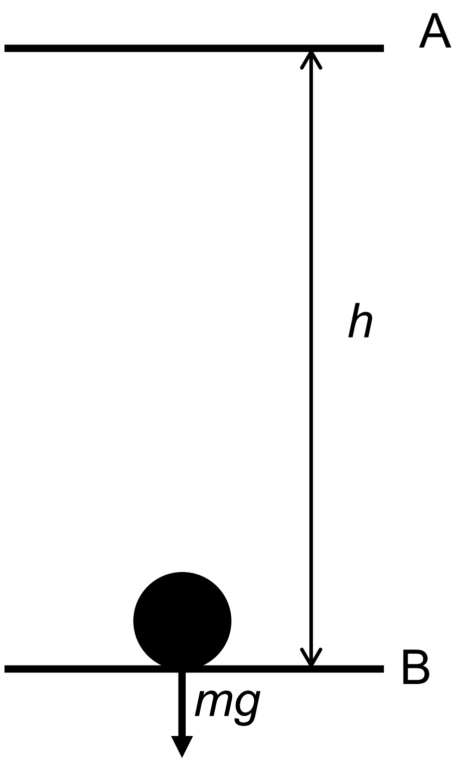
$$\int_A^B \frac{1}{x^2} \cdot dx = -\frac{1}{x}\bigg|_A^B = \frac{1}{A} - \frac{1}{B}$$

If $\vec{u}(\vec{r}) = \hat{x}\frac{\partial v(\vec{r})}{\partial x} + \hat{y}\frac{\partial v(\vec{r})}{\partial y} + \hat{z}\frac{\partial v(\vec{r})}{\partial z} \equiv \nabla u(\hat{r})$, here $\nabla \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

➔ $v(\vec{r}) = \underset{\text{line integral}}{\int} \vec{u}(\vec{r})\,d\vec{s}$

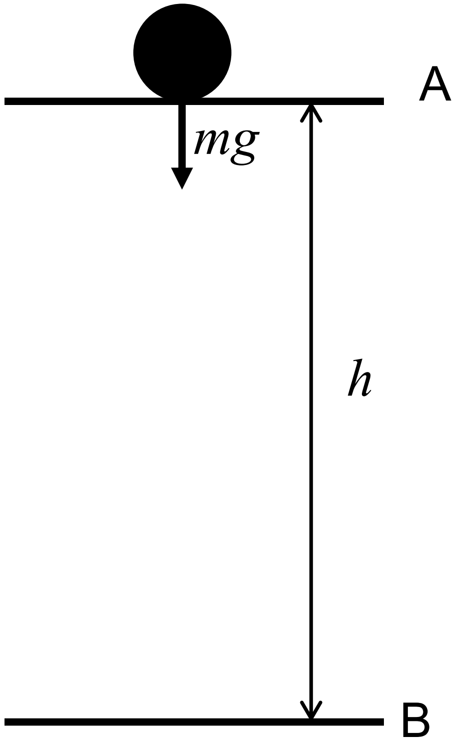
In a Cartesian coordinate system

A review of gravitational potential



When object of mass *m* is on the ground level B, we define that it has zero gravitational potential energy. When we let go of this object, if will stay in place.

A review of gravitational potential



Move the object to elevation A, it has now has gravitational potential energy mgh. When we let go of this object, if will fall back to level B, converting the potential energy to kinetic.

The object has gravitational potential energy U_A-U_B= mgh at elevation A. U_A is the potential energy at point A with reference to point B. When the object falls from level A to level B, the potential energy change is: $\Delta U = U_B - U_A$ The gravitational force does work to causes the potential energy change:

$$W = mgh = U_A - U_B = -\Delta U$$

Gravitational force is conservative.

Electric potential energy, a special case: the electric field is constant

When a charge q_0 is placed inside an electric field, it experiences a force from the field:

$$\vec{F} = q_0\vec{E}$$

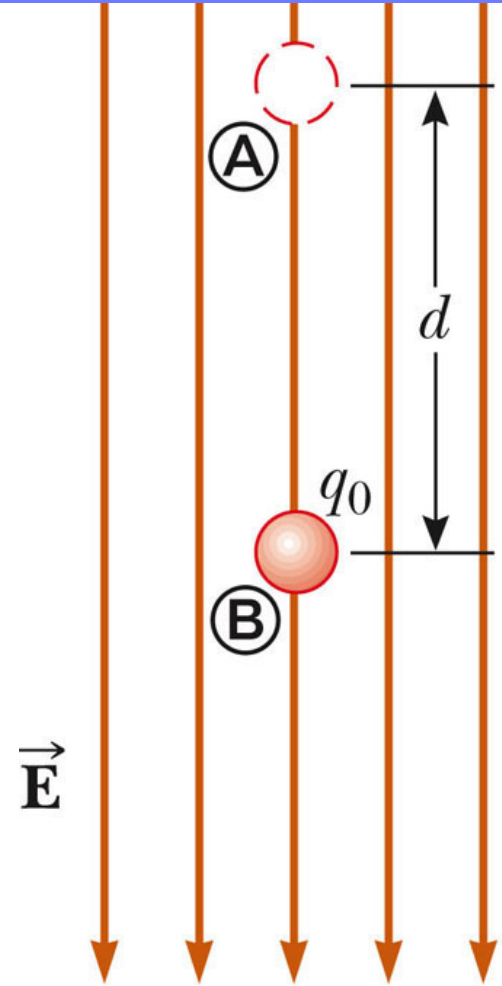
When the charge is released, the field moves it from A to B, doing work:

$$W = \vec{F}\cdot\vec{d} = q_0\vec{E}\cdot\vec{d} = q_0 Ed$$

If we define the electric potential energy of the charge at point A as U_A and at point B as U_B, then:

$$W = q_0 Ed = U_A - U_B = -\Delta U$$

If we define $U_B = 0$, then $U_A = q_0 Ed$ is the electric potential energy the charge has at point A. We can also say that the electric field has an electric potential at point A. When a charge is placed there, the charge acquires an electric potential energy that is the charge times this potential.



Electric Potential Energy, the general case

When a charge is moved from point A to point B in an electric field, the charge's electric potential energy inside this field is changed from U_A to U_B: $\Delta U = U_B - U_A$

When the motion is caused by the electric field force on the charge, this force does work to the charge and causes a change of its electric potential energy: $W = -\Delta U$

The force on the charge is: $\vec{F} = q_0 \vec{E}$

So we have this final formula for electric potential energy and the work the field force does to the charge:

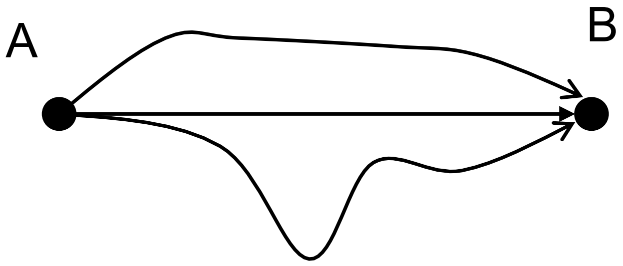
$$-\Delta U = U_A - U_B = W(\text{of the field force}) = \int_A^B q_0 \vec{E} \cdot d\vec{s}$$

Electric Potential Energy

Electric force is conservative. The line integral does not depend on the path from A to B; it only depends on the locations of A and B.

$$-\Delta U = U_A - U_B = \int_A^B q_0 \vec{E} \cdot d\vec{s}$$

Line integral paths



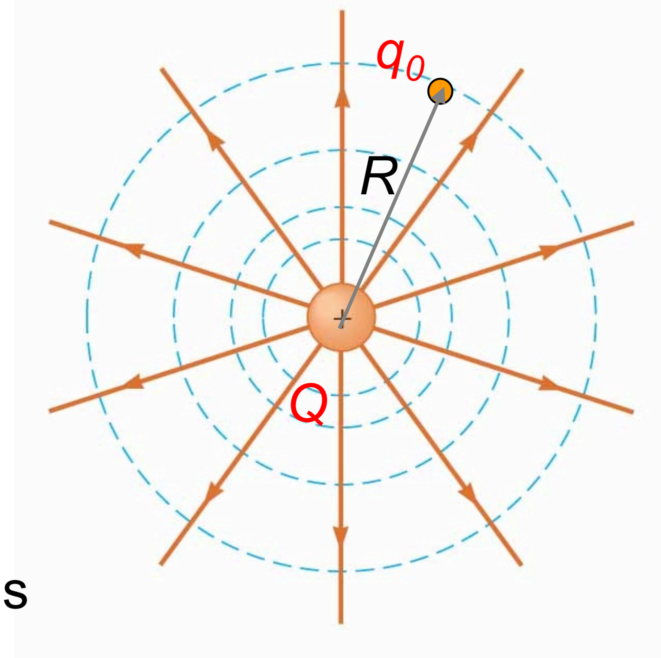
The path is from A to B, and

$$\Delta U \equiv U_B - U_A$$

The electric potential energy of charge q_0 in the field of charge Q?

Reference point:

We normally define the electric potential of a point charge to be zero (reference) at a point that is infinitely far away from this point charge.



Applying this formula:

$$-\Delta U = U_A - U_B = \int_A^B q_0 \vec{E} \cdot d\vec{s}$$

Where point A is where the charge q_0 is, point B is infinitely far away.

$$\vec{E} = k_e \frac{Q}{r^2}\hat{r}, \quad d\vec{s} = d\vec{r}$$

so $$\vec{E} \cdot d\vec{s} = \vec{E} \cdot d\vec{r} = k_e \frac{Q}{r^2} dr$$

and $$\int_R^\infty k_e \frac{Q}{r^2} dr = k_e \frac{Q}{R}$$

So the final answer is

$$U(R) = k_e \frac{q_0 Q}{R}$$

And the result is a scalar!

Electric Potential, the definition

- The potential energy per unit charge, U/q_0, is the **electric potential**
  - The potential is a characteristic of the field only
  - The potential energy is a characteristic of the charge-field system
  - The potential is independent of the value of q_o
- The electric potential is $V = \dfrac{U}{q_0}$
- As in the potential energy case, electric potential also needs a reference. So it is the potential difference ΔV that matters, not the potential itself, unless a reference is specified (then it is again ΔV).

Electric Potential and electric field

- The potential is a scalar quantity
  - Since energy is a scalar
- Potential difference between V_A and V_B is calculated using:

$$-\Delta V = V_A - V_B (\text{often the reference}) = \int_A^B \vec{E} \cdot d\vec{s}$$

Remember that path is from A to B and the potential difference is defined to be:

$$\Delta V \equiv V_B - V_A$$

Potential Difference in a Uniform Field

The equations for electric potential can be simplified if the electric field is uniform:

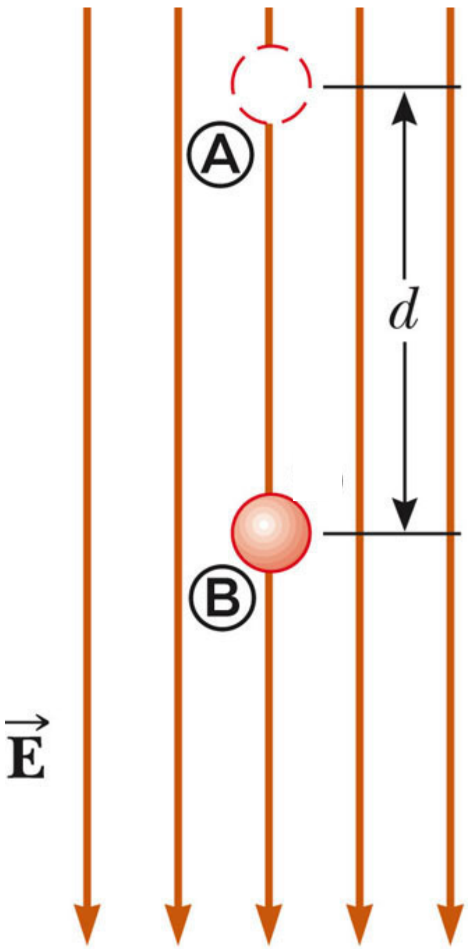
$$-\Delta V = V_A - V_B = \int_A^B \vec{E} \cdot d\vec{s} = \vec{E} \int_A^B d\vec{s} = \vec{E} \cdot \vec{d}$$

When:

$$\vec{E} \cdot \vec{d} > 0, \text{ i.e., } \vec{E} \text{ and } \vec{d} \text{ the same direction,}$$

$$-\Delta V = V_A - V_B > 0, \text{ or } V_A > V_B$$

This is to say that electric field lines always point in the direction of decreasing electric potential. Electric potential decreases down the field line.



Electric Potential, Electric Potential Energy and Work

When there is electric field, there is electric potential V.

When a charge q_0 is in an electric field, this charge has an electric potential energy U in this electric field:

$$U = q_0 V.$$

When this charge q_0 is moved by the electric field force from point A to point B, the work this field force does to this charge equals the negative potential energy change $-\Delta U = -(U_B - U_A)$:

$$W = -\Delta U = -q_0 \Delta V.$$

Understand the Units

- The unit for electric potential energy is the unit for energy joule (J).
- The unit for electric potential is volt (V):

  1 Volt = 1 Joule/Coulomb or 1 V = 1 J/C

  - This unit comes from $U = q_0 V$ (here U is electric potential energy, V is electric potential, not the unit volt)
  - It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt

- But from $-\Delta V = \int_A^B \vec{E} \cdot d\vec{s}$

  We also have the unit for electric potential as 1 V = 1 (N/C)m

  So we have that 1 N/C (the unit of $\vec{E}$) = 1 V/m

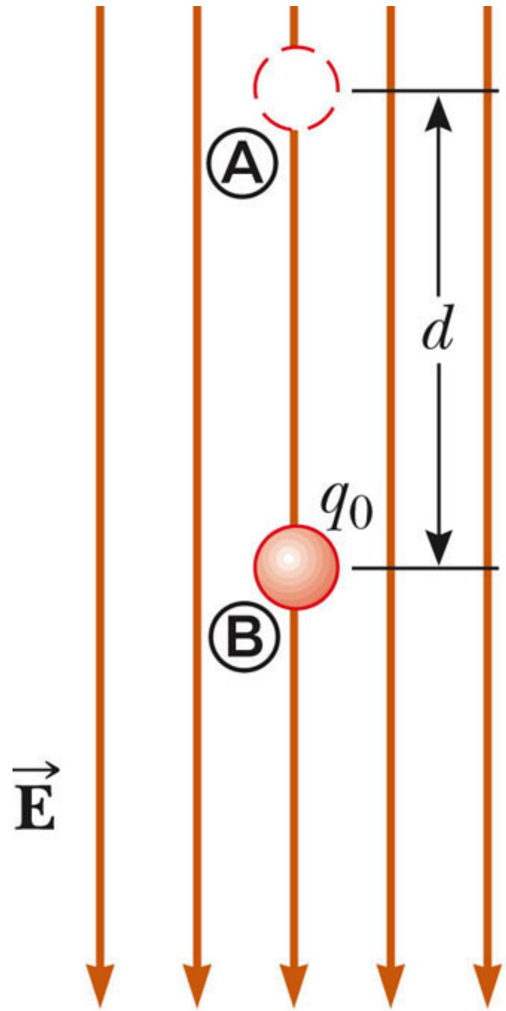
  - This indicates that we can interpret the electric field as a measure of the rate of change with position of the electric potential

Electron-Volts, another unit often used in nuclear and particle physics

- Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt
- One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt
  - 1 eV = 1.60×10^{-19} J

Direction of Electric Field, energy conservation

- As pointed out before, electric field lines always point in the direction of decreasing electric potential
- So when the electric field is directed downward, point *B* is at a lower potential than point *A*
- When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy through doing work to this charge
- Where does this energy go?



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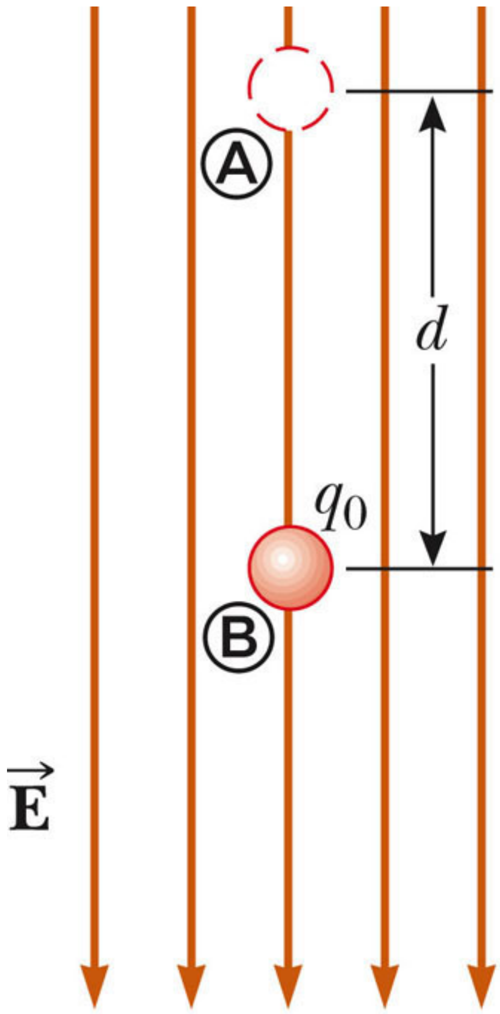
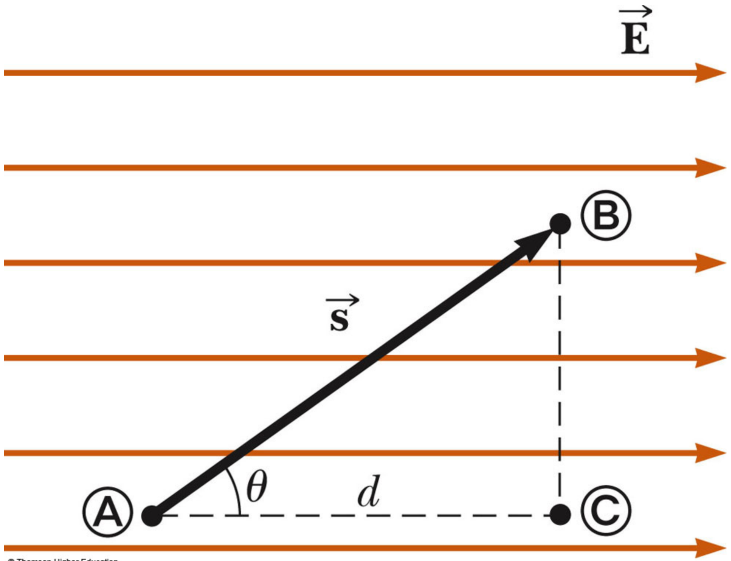
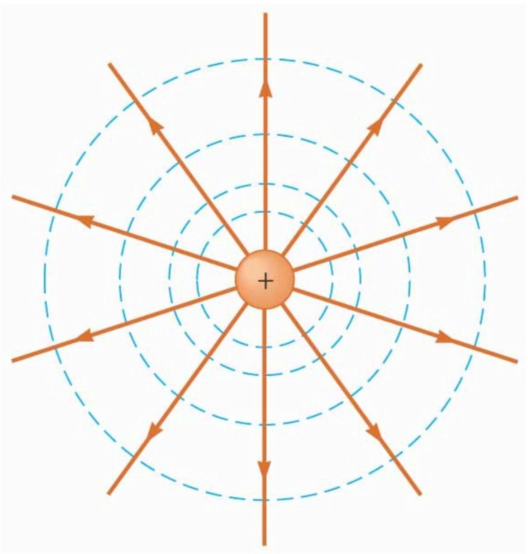


It turns into the kinetic energy of the object (with a mass) that carries the charge q_0.

Equipotentials = equal potentials

- Points *B* and *C* are at a lower potential than point *A*
- Points *B* and *C* are at the same potential
  - All points in a plane perpendicular to a uniform electric field are at the same electric potential
- The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential





Charged Particle in a Uniform Field, Example

Question: a positive charge q (mass m) is released from rest and moves in the direction of the electric field. Find its speed at point B.

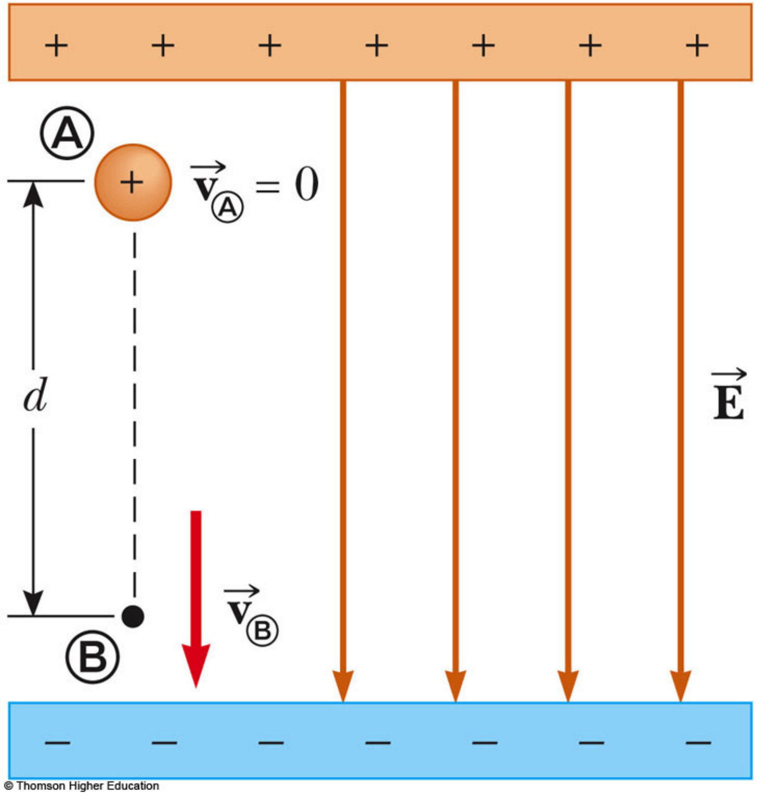
Solution: The system loses potential energy: $-\Delta U = U_A - U_B = qEd$

The force and acceleration are in the direction of the field

Use energy conservation to find its speed:

$$\frac{1}{2}mv^2 = qEd$$

$$v = \sqrt{\frac{2qEd}{m}}$$



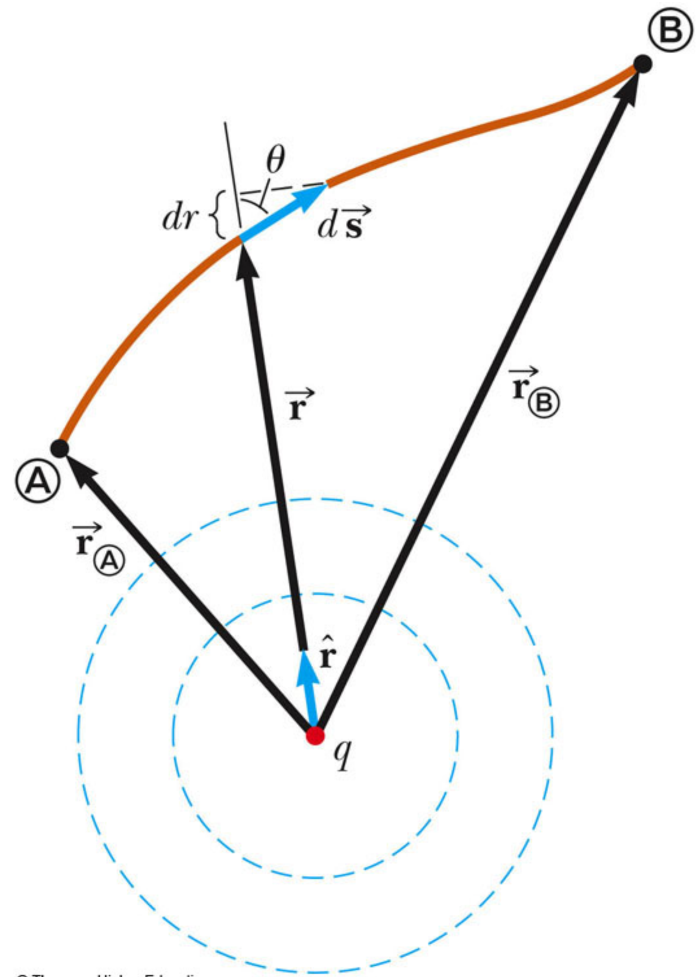
Potential and Point Charges

- A positive point charge q produces a field directed radially outward
- The potential difference between points *A* and *B* will be

$$\Delta V \equiv V_B - V_A = k_e q \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

No line integral needed!

And this is because?



Potential and Point Charges

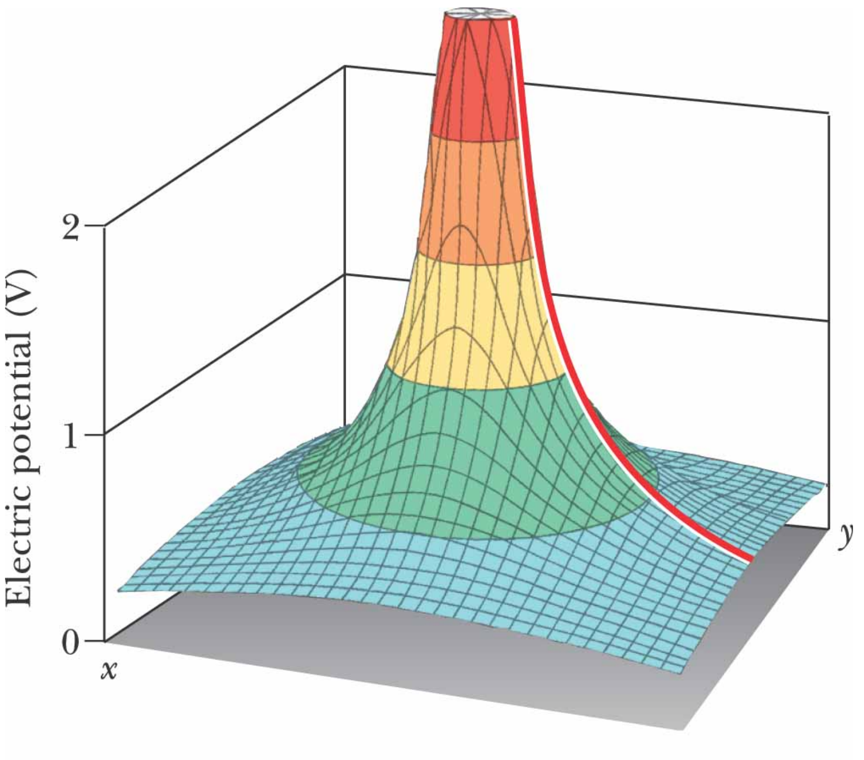
- The electric potential is independent of the path between points *A* and *B*
- It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$
- Then the potential at some point r is

$$V(r) = k_e \frac{q}{r}$$

What happens to the potential when $r \rightarrow 0$?

Electric Potential of a Point Charge

- The electric potential in the plane around a single point charge is shown
- The red line shows the $1/r$ nature of the potential



Electric Potential with Multiple Charges

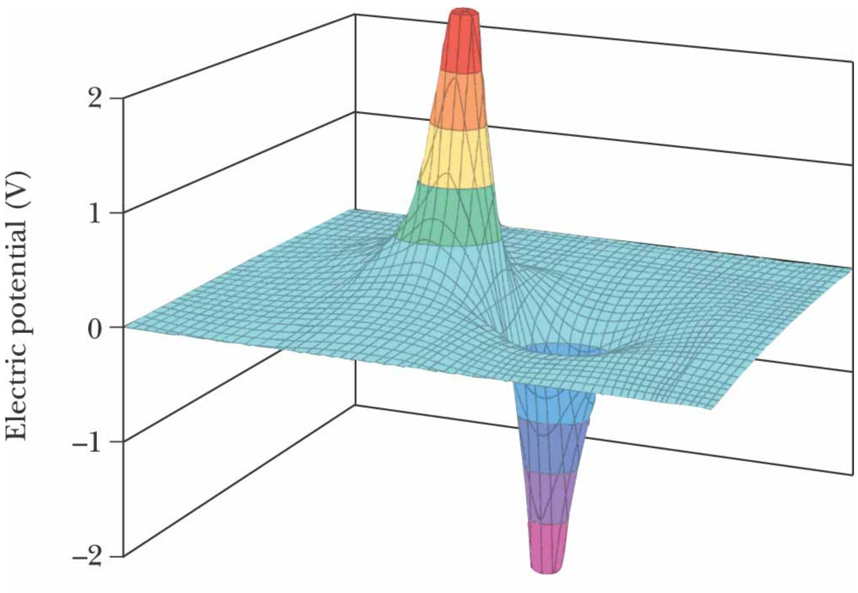
- The electric potential due to several point charges is the sum of the potentials due to each individual charge
  - This is another example of the superposition principle
  - The sum is the algebraic sum

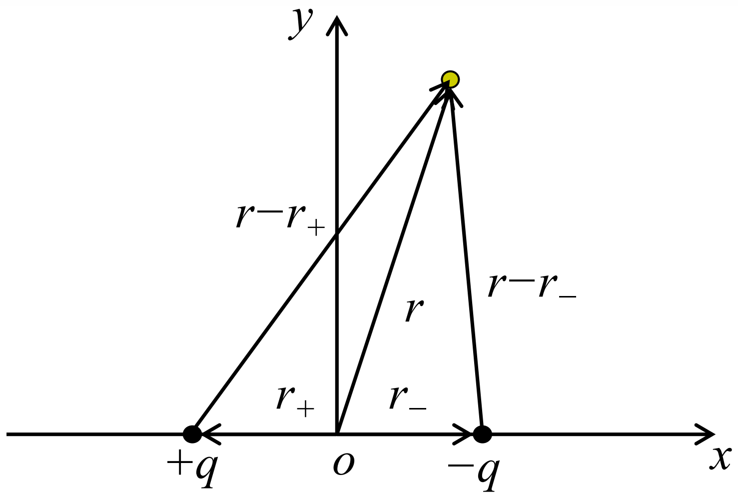
$$V = k_e \sum_i \frac{q_i}{r_i}$$

Immediate application: Electric Potential of a Dipole

$$V = V_+ + V_- = k_e q\left(\frac{1}{|\vec{r} - \vec{r}_+|} - \frac{1}{|\vec{r} - \vec{r}_-|}\right)$$

Work on the board to prove it.

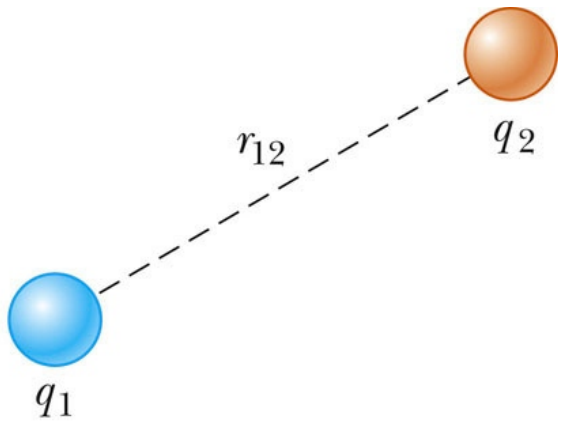




- The graph shows the potential (y-axis) of an electric dipole
- The steep slope between the charges represents the strong electric field in this region

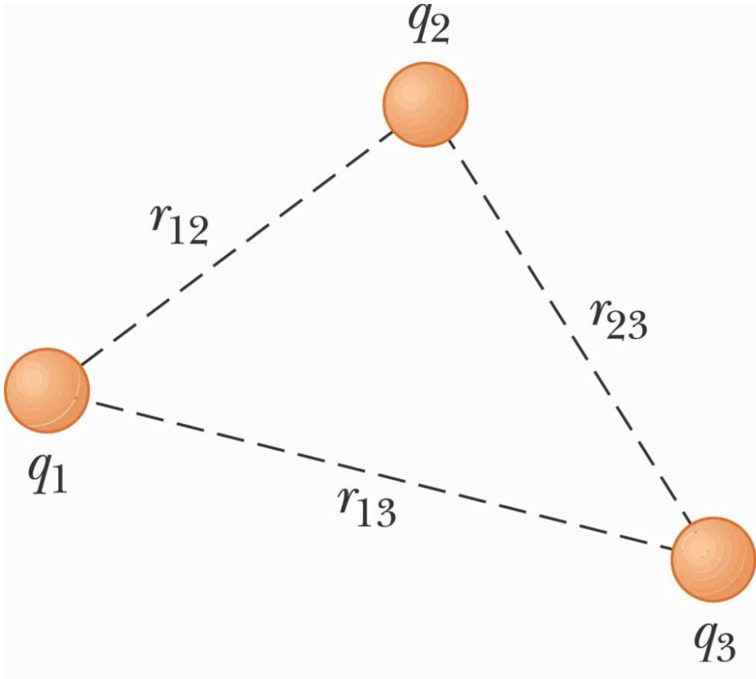
Potential Energy of Multiple Charges

- The potential energy of two system is $U = k_e \frac{q_1 q_2}{r_{12}}$ because $V(r) = k_e \frac{q}{r}$ and $U = qV$



- For a three charge system:

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} \right)$$



For an N charge system?

$$U = \frac{k_e}{2} \sum_{i,j \text{ and } i \neq j}^{N} \frac{q_i q_j}{r_{ij}}$$

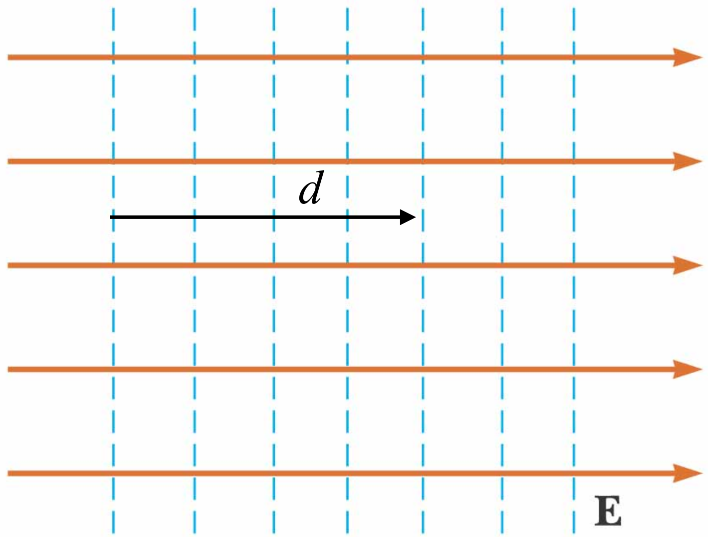
Work and Potential Energy in a two charge system

- If the two charges are the same sign, U is positive and external work (not the one from the field force) must be done to bring the charges together
- If the two charges have opposite signs, U is negative and external work is needed to separate the charges

Find V for an Infinite Sheet of Charge

- We know that $E = \dfrac{\sigma}{2\varepsilon_0}$, a constant
- From $V = \int \vec{E} \cdot d\vec{s}$

  We have $V = Ed$
- The equipotential lines are the dashed blue lines
- The electric field lines are the brown lines
- The equipotential lines are everywhere perpendicular to the field lines



Finding $\vec{E}$ From V

This is straight forward (if you are good in math):

From $-\Delta V = \int \vec{E} \cdot d\vec{s}$

we have $\vec{E} = -\nabla V \equiv \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right) V$

If $\vec{E}$ is one dimensional (say along the x-axis) $E_x = -\frac{dV}{dx}$

If $\vec{E}$ is only a function of $\vec{r}$ (the point charge case):

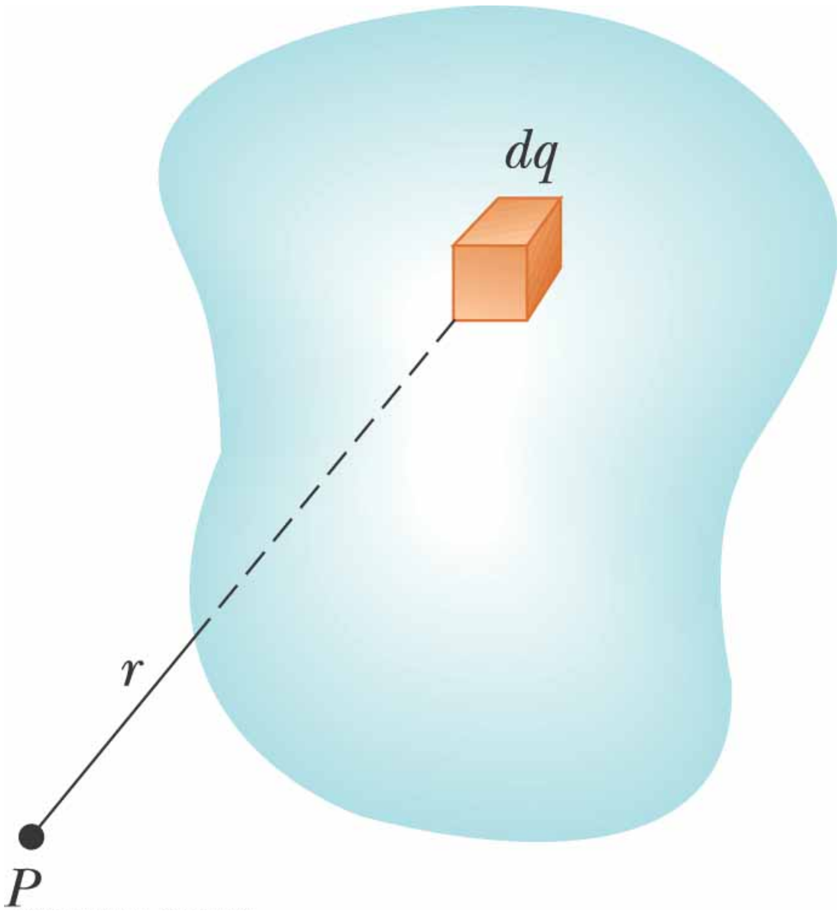
$$\vec{E}(\vec{r}) = E(r)\hat{r} \quad \text{with} \quad E(r) = -\frac{dV}{dr}$$

When you use a computer (program) to calculate electric Potential for a Continuous Charge Distribution:

- Consider a small charge element dq
  - Treat it as a point charge
- The potential at some point due to this charge element is

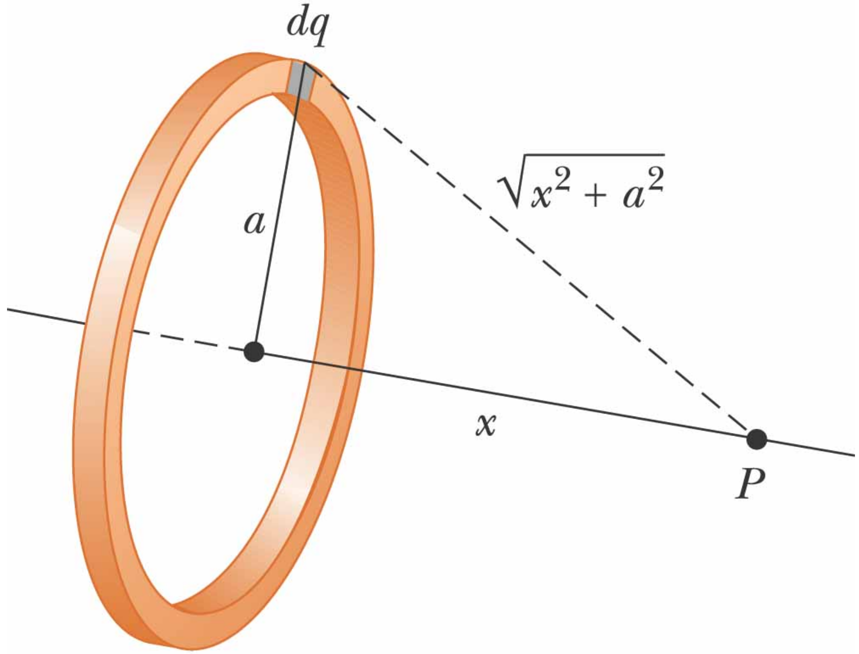
$$dV = k_e \frac{dq}{r}$$

$$V = \int dV$$



V for a Uniformly Charged Ring

- *P* is located on the perpendicular central axis of the uniformly charged ring
  - The ring has a radius a and a total charge Q



$$V = k_e \int \frac{dq}{r} = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

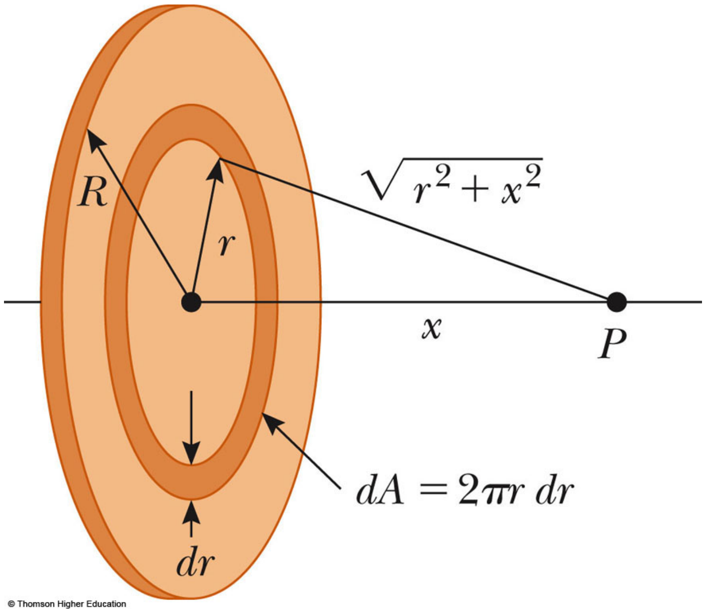
  - A function of x, so

$$E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{k_e Q}{\sqrt{a^2 + x^2}}\right) = k_e \frac{Qx}{\left(x^2 + a^2\right)^{\frac{3}{2}}}$$

Compare the calculation in Chapter 22

V for a Uniformly Charged Disk

- The ring has a radius R and surface charge density of σ
- P is along the perpendicular central axis of the disk



$$dV = k_e \frac{dq}{\sqrt{r^2+x^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi r)dr}{\sqrt{r^2+x^2}} = \frac{\sigma}{4\varepsilon_0}\frac{d\left(r^2+x^2\right)}{\sqrt{r^2+x^2}}$$

$$V = \int_0^R \frac{\sigma}{4\varepsilon_0}\frac{d\left(r^2+x^2\right)}{\sqrt{r^2+x^2}} = \frac{\sigma}{4\varepsilon_0}\left.\frac{\left(r^2+x^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{\left(R^2+x^2\right)}-x\right]$$

Compare this with the same problem in Chapter 22. And when $R \to \infty$

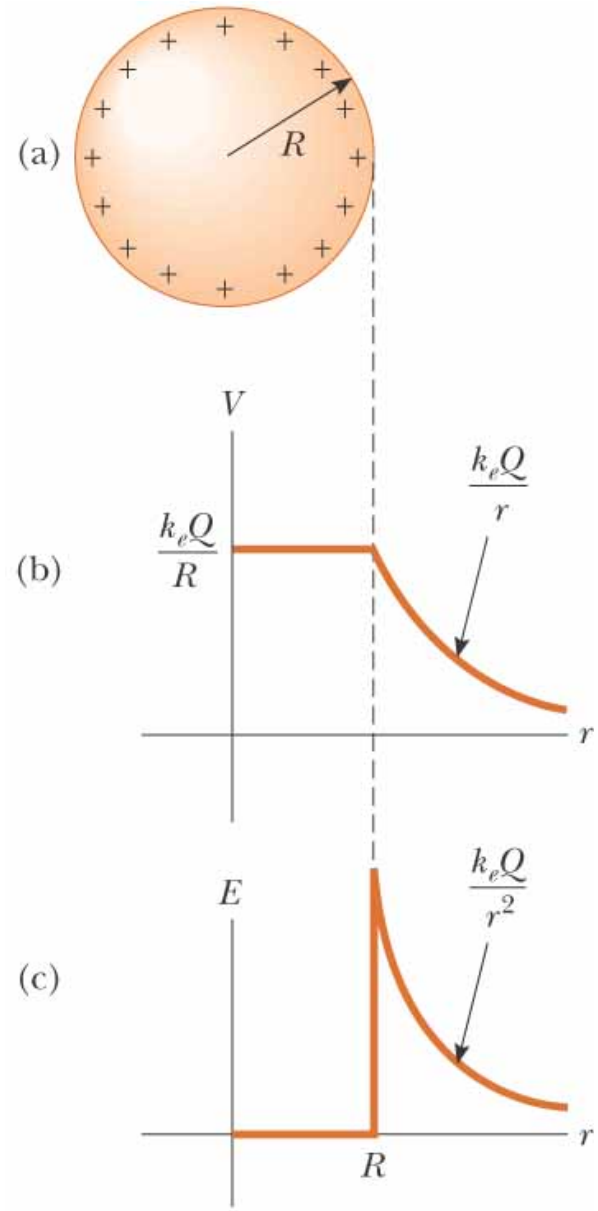
$$E_x = -\frac{dV}{dx} = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{R^2+x^2}}\right)$$

$$E_x \to \frac{\sigma}{2\varepsilon_0}$$

Compare with Gauss Law type 3

E Compared to V

- The electric potential is a function of r
- The electric field is a function of r^2
- The effect of a charge on the space surrounding it:
  - The charge sets up a vector electric field which is related to the force
  - The charge sets up a scalar potential which is related to the energy



Reading material and Homework assignment

Please watch this video (about 50 minutes):
http://videolectures.net/mit802s02_lewin_lec04/

Please check wileyplus webpage for homework assignment.