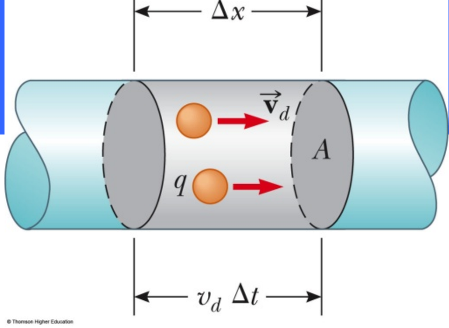
(DC) Circuits

1. Resistors in a circuit (serial and parallel connections).
2. The Direct Current (DC) and the Alternating Current (AC) circuits.
3. How to model a battery in a circuit.
4. Circuits with 2 or more batteries – Kirchhoff's Rules
5. The RC circuit (your low- or high- pass filter in EE, and the drive in MOS transistors): when we have R and C together.

Review



The current is defined as: $I = \dfrac{dQ}{dt}$

Its unit is ampere (A), a base unit in the SI system.

Its relationship with the current density J is: $I = \int \vec{J} \cdot d\vec{A}$ or: $J \equiv dI/dA$

Ohm's Law: $\vec{J} = \sigma \vec{E}$

Here σ is the conductivity of the material.

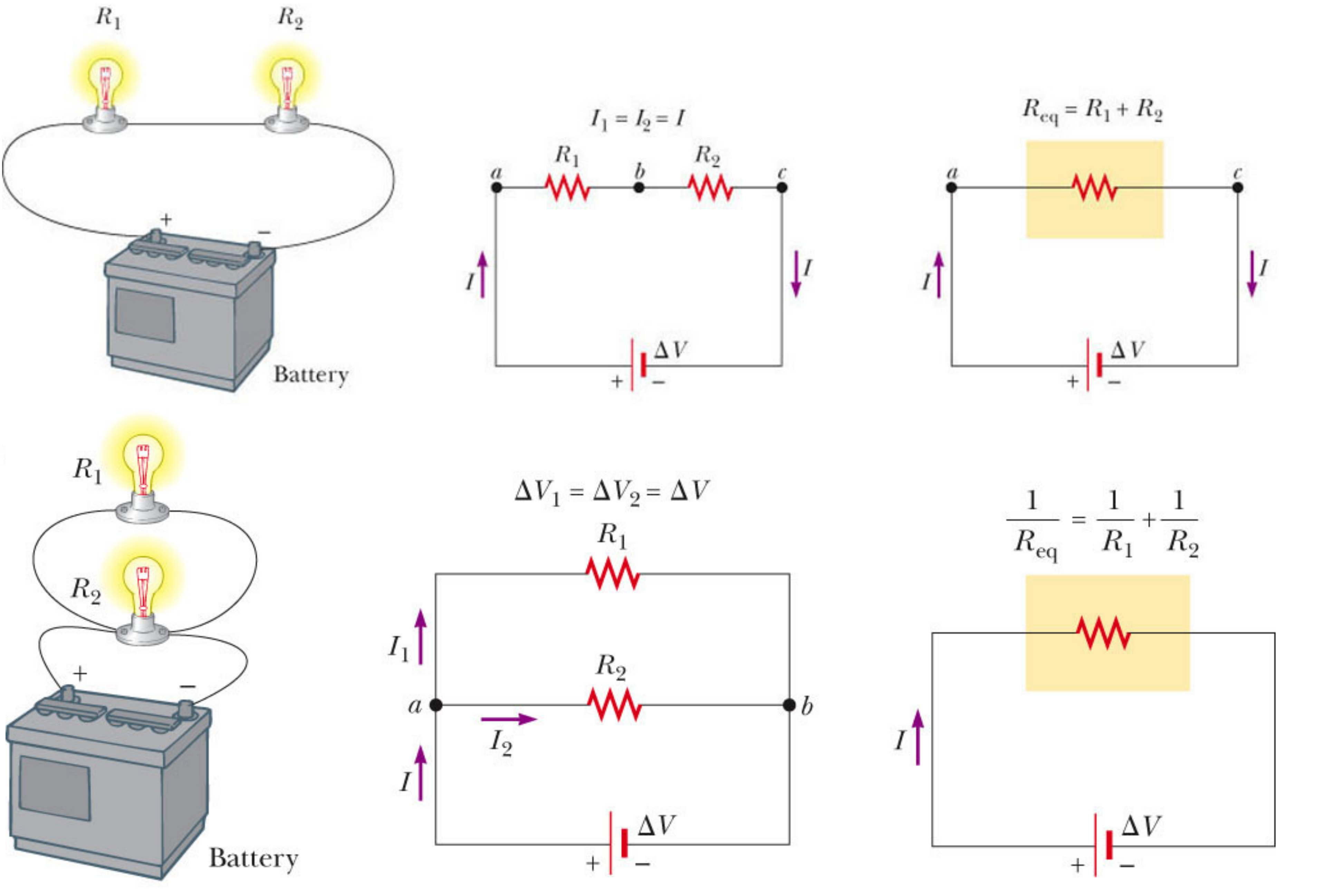
The resistivity is defined as $\rho \equiv 1/\sigma$, and is a more commonly used material parameter which linearly depends on temperature:

$$\rho = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$$

The definition of resistance (Ohm's Law): $R \equiv \dfrac{\Delta V}{I}$

Its relationship with material and shape: $R \equiv \rho \dfrac{l}{A}$

Resistors in Series and in Parallel

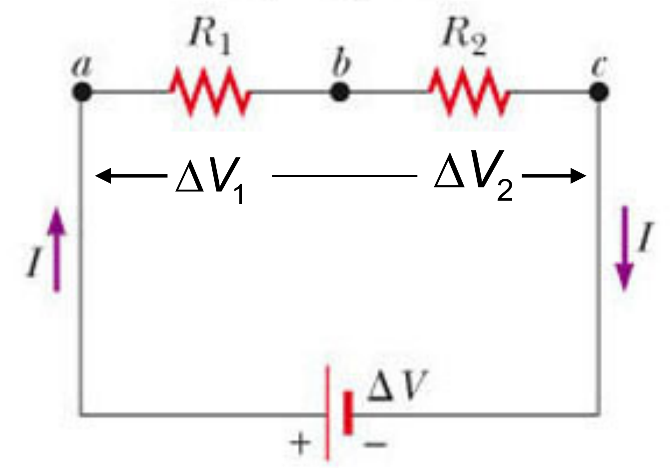


Resistor connections

In series.
Condition: ➡
Result:

$$I = I_1 = I_2$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

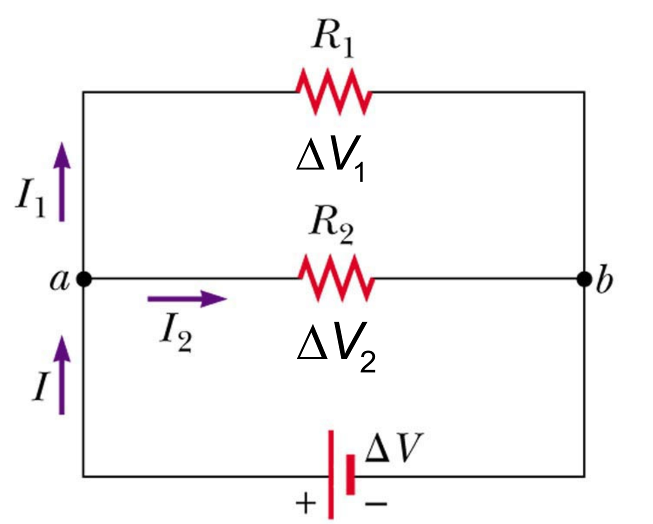


$$R_{eq} = \frac{\Delta V}{I} = \frac{\Delta V_1 + \Delta V_2}{I} = \frac{\Delta V_1}{I_1} + \frac{\Delta V_2}{I_2} = R_1 + R_2$$

In parallel.
Condition: ➡
Result:

$$I = I_1 + I_2$$

$$\Delta V = \Delta V_1 = \Delta V_2$$



$$\frac{1}{R_{eq}} = \frac{I}{\Delta V} = \frac{I_1 + I_2}{\Delta V} = \frac{I_1}{\Delta V_1} + \frac{I_2}{\Delta V_2} = \frac{1}{R_1} + \frac{1}{R_2}$$

Resistor connections

In series,

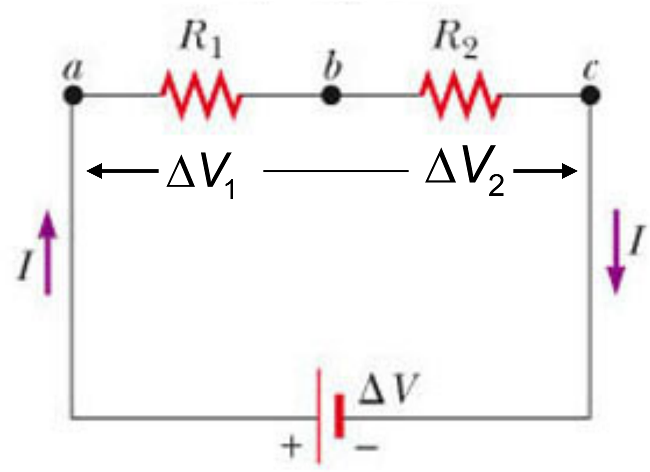
$$\because I = I_1 = I_2$$

voltage sharing

$$\frac{\Delta V_1}{\Delta V_2} = \frac{R_1}{R_2}$$

power sharing

$$\frac{P_1}{P_2} = \frac{R_1}{R_2}$$



In parallel,

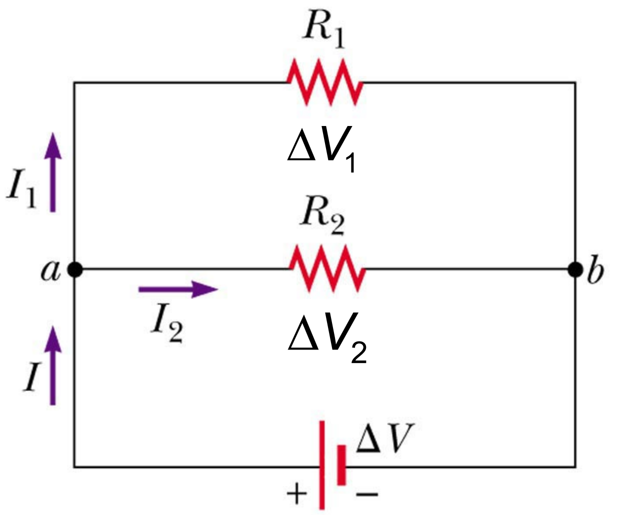
$$\because \Delta V = \Delta V_1 = \Delta V_2$$

current sharing

$$\frac{I_2}{I_1} = \frac{R_1}{R_2}$$

power sharing

$$\frac{P_2}{P_1} = \frac{R_1}{R_2}$$



Resistors connections, summary

- In series

$$I = I_1 = I_2 = I_3 = \ldots$$

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

$$\Delta V_1 : \Delta V_2 : \Delta V_3 : \ldots = R_1 : R_2 : R_3 : \ldots$$

$$P_1 : P_2 : P_3 : \ldots = R_1 : R_2 : R_3 : \ldots$$

- In parallel

$$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots$$

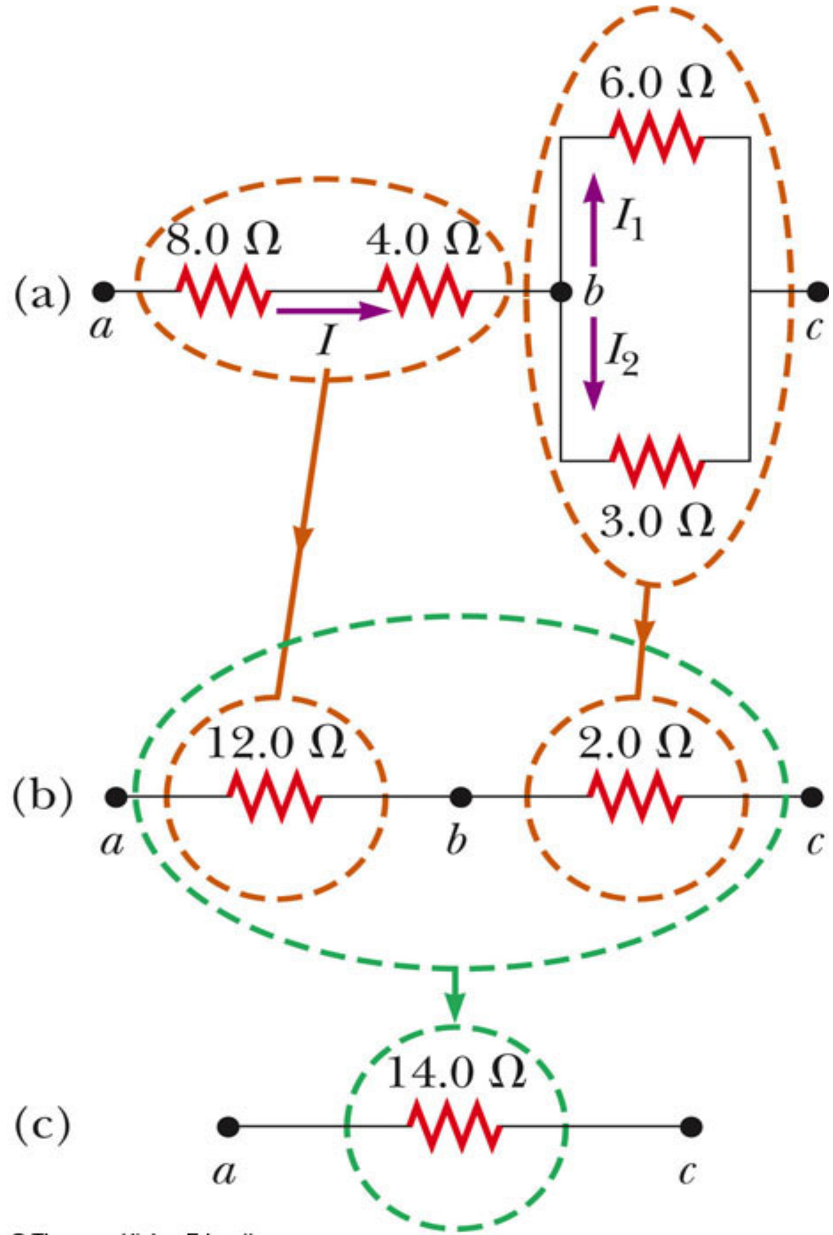
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$I_1 R_1 = I_2 R_2 = I_3 R_3 = \ldots$$

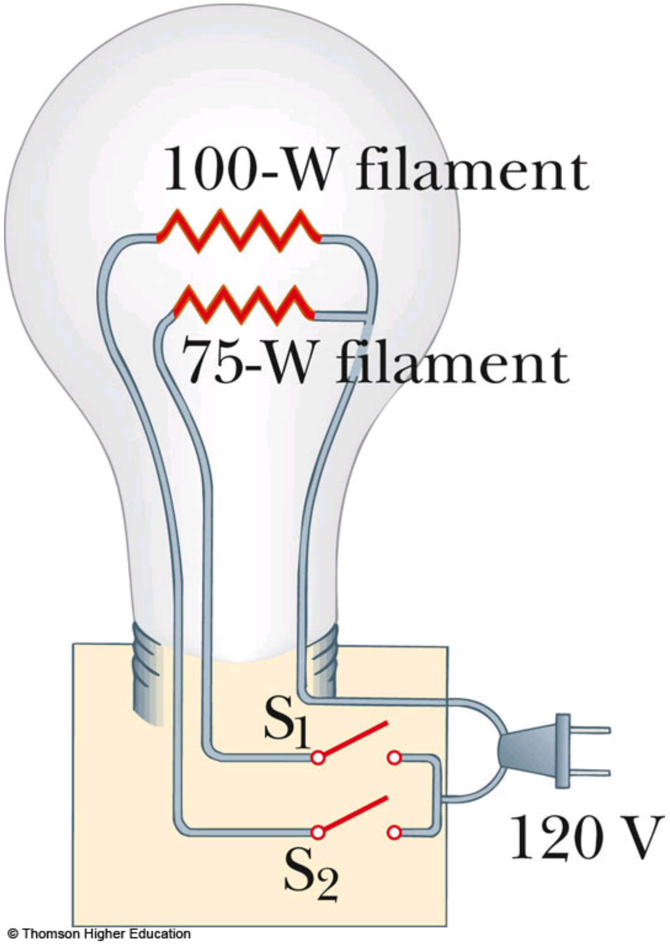
$$P_1 R_1 = P_2 R_2 = P_3 R_3 = \ldots$$

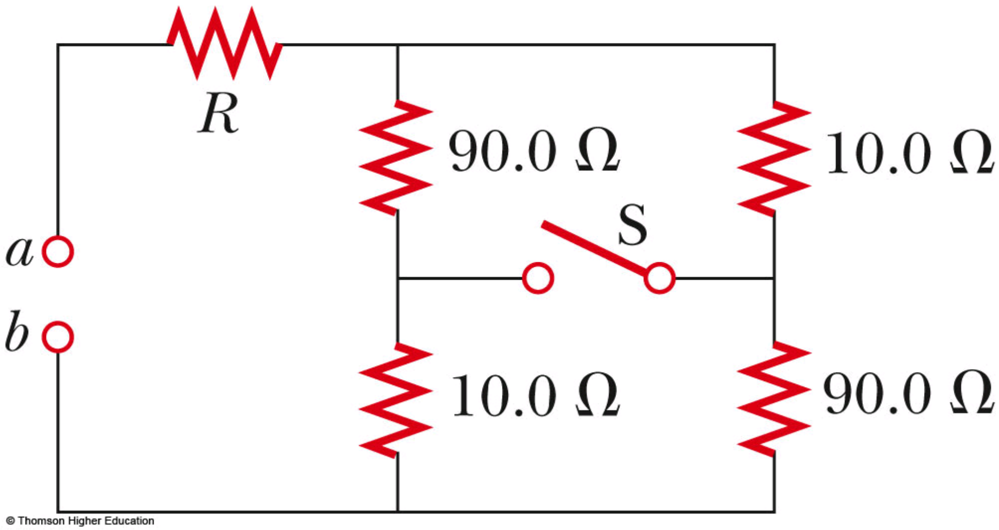
Combinations of Resistors

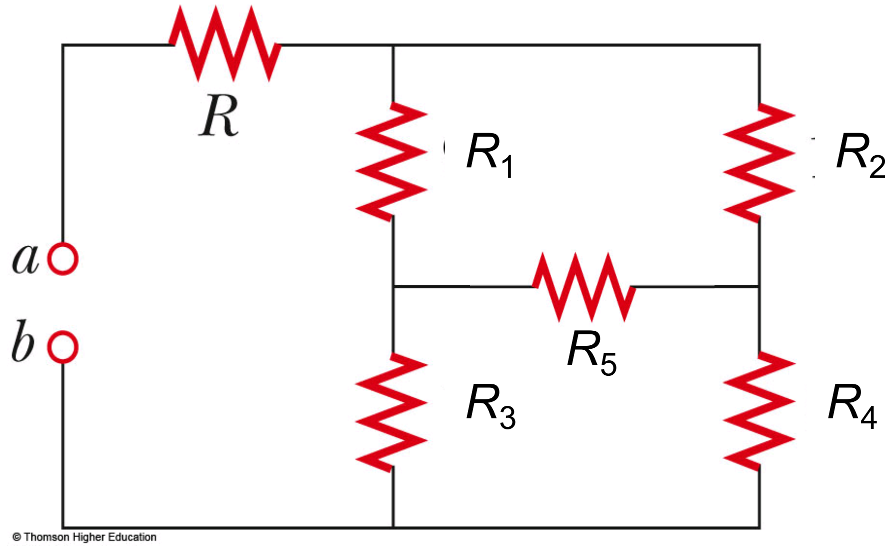
- The 8.0-Ω and 4.0-Ω resistors are in series and can be replaced with their equivalent, 12.0 Ω
- The 6.0-Ω and 3.0-Ω resistors are in parallel and can be replaced with their equivalent, 2.0 Ω
- These equivalent resistances are in series and can be replaced with their equivalent resistance, 14.0 Ω



More examples







Direct Current and Alternating Current

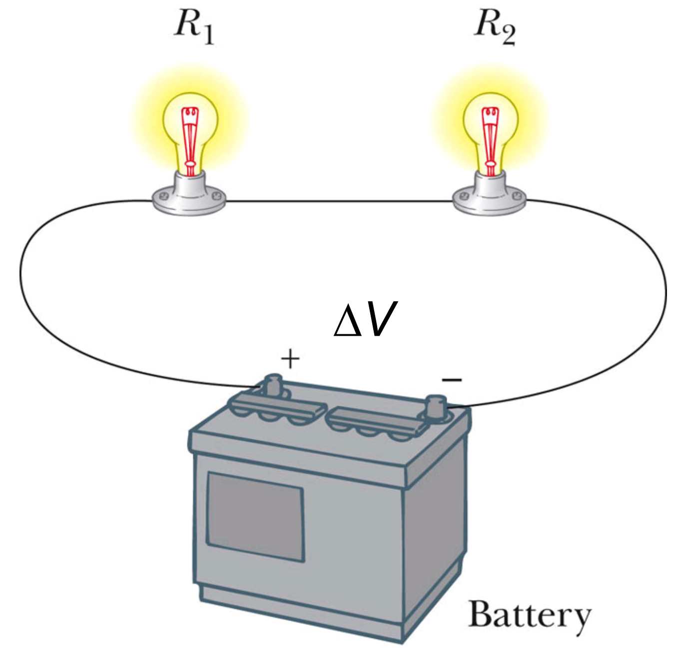
- When the current direction (not magnitude) in a circuit does not change with time, the current is called a direct current (DC).
  - constant current magnitude, like the one powered through a battery, is a common, but special case of DC.
- When the current direction (often also the magnitude) in a circuit changes with time, the current is called an alternating current (AC).
  - The current from the wall outlet is AC.
  - The current from your car's alternator is AC. Question: how to charge the battery with the alternator? Question: are you even interested in the first question?

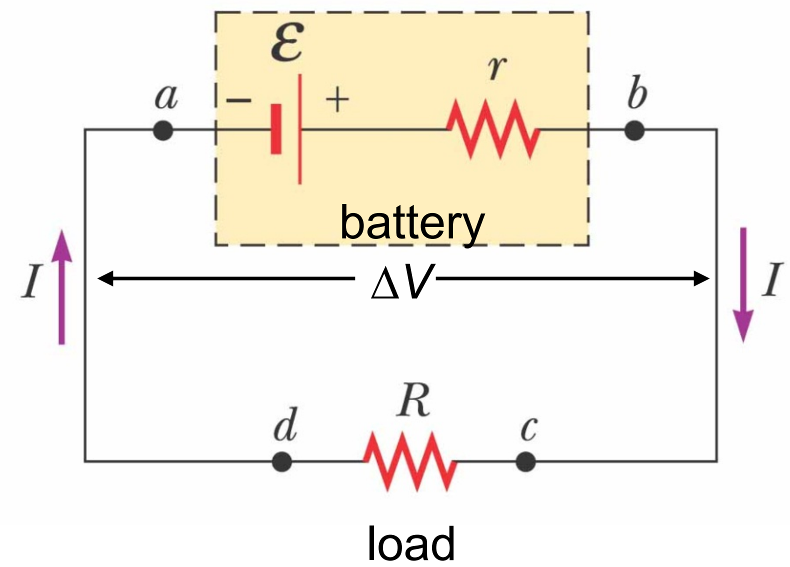
Model of a battery

- Two parameters, electromotive force (emf), ε, and the internal resistance r, are used to model a battery.
- When a battery is connected in a circuit, the electric potential measured at its + and – terminals are called The terminal voltage ΔV, with

$$\Delta V = \varepsilon - Ir$$

- If the internal resistance is zero (an ideal battery), the terminal voltage equals the emf ε.
- The internal resistance, r, does not change with external load resistance R, and this provides the way to measure the internal resistance.





Battery power figure

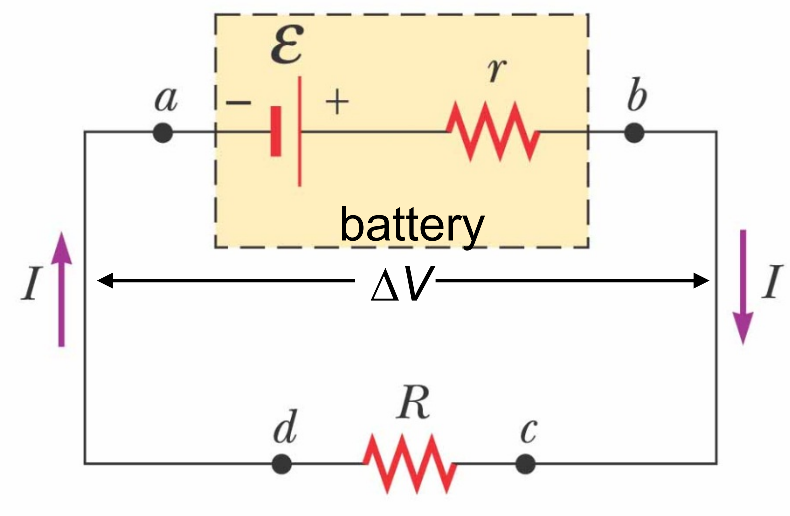
The power a battery generates (ex. thrgh chemical reactions):

$$P = \varepsilon \cdot I = (R + r) \cdot I^2$$

The power the battery delivers to the load, hence efficiency:

$$P_{\text{load}} = \Delta V \cdot I = R \cdot I^2$$

$$\text{efficiency} = \frac{P_{\text{load}}}{P} = \frac{R}{R + r}$$



The maximum power the battery can deliver to a load

From $P_{\text{load}} = R \cdot I^2$ and $\varepsilon = (R + r) \cdot I$ We have $P_{\text{load}} = \frac{R}{(R + r)^2}\varepsilon^2$

Where the emf ε is a constant once the battery is chosen.

From $\frac{dP_{\text{load}}}{dR} = \left(\frac{1}{(R + r)^2} - \frac{2R}{(R + r)^3}\right)\varepsilon^2 = 0$ We get $R = r$ to be the condition for maximum P_{load}, or power delivered to the load.

Battery power figure

One can also obtain this result from the plot of

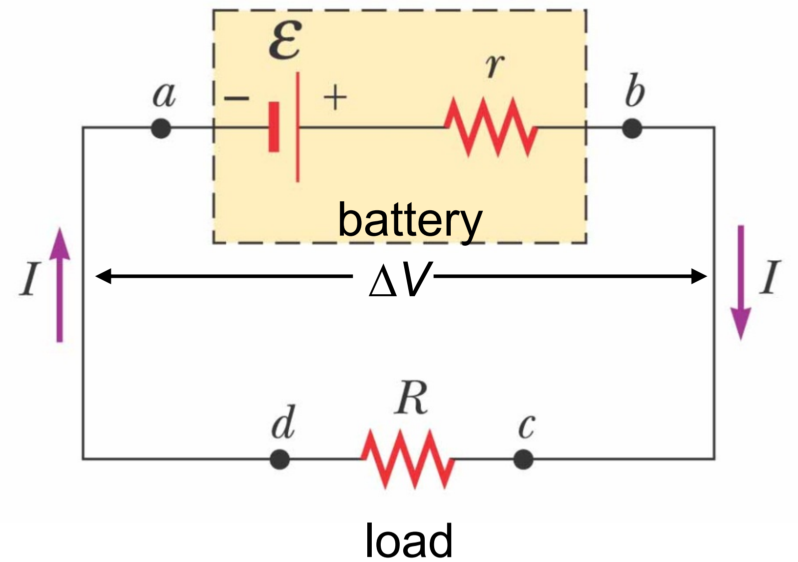
$$P_{\text{load}} = \frac{R}{R+r}\varepsilon^2$$

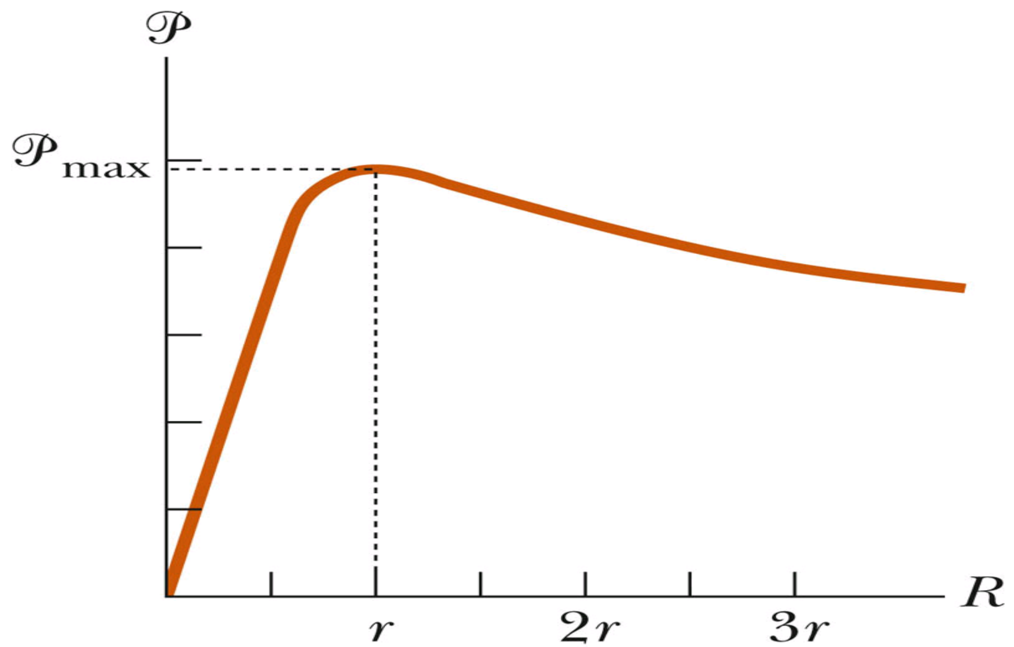
Where when $R = r$

P_{load} reaches the maximum value

The efficiency of the battery at this point is 50% because

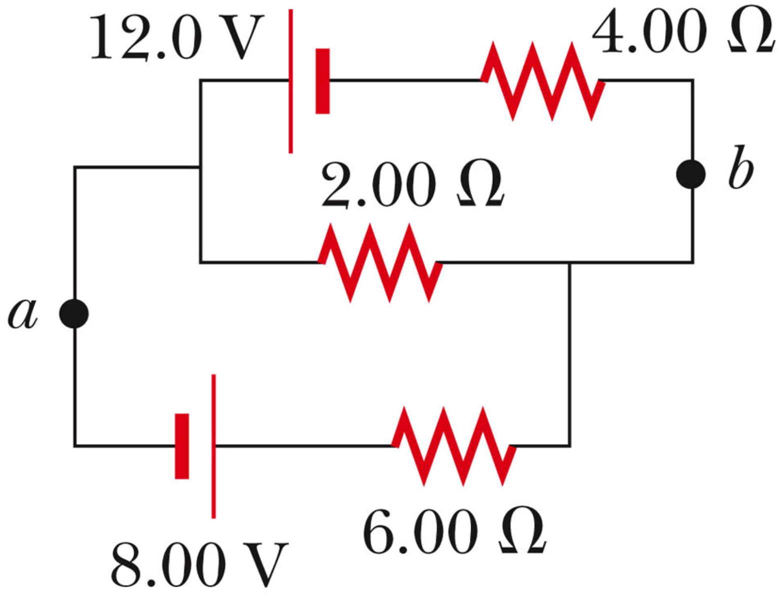
$$\text{efficiency} = \frac{P_{\text{load}}}{P} = \frac{R}{R+r}$$





circuits with 2+ batteries: Kirchhoff's Rules

- A typical circuit that goes beyond simplifications with the parallel and series formulas: ask for the current in the diagram.
- **Kirchhoff's rules** can be used to solve problems like this.



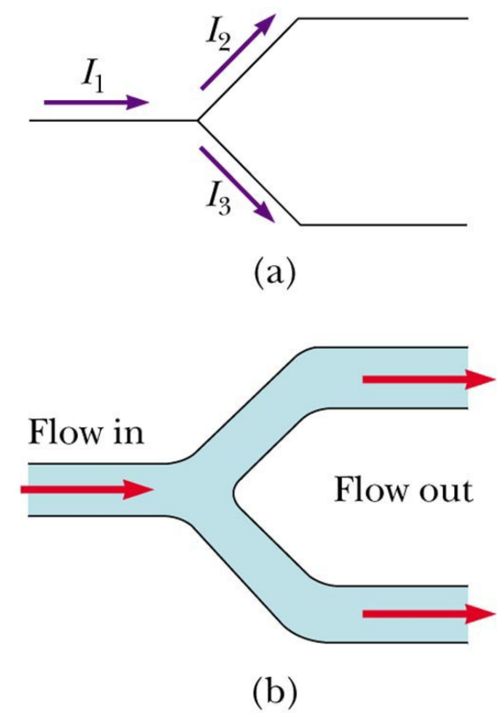
Rule 1: Kirchhoff's Junction Rule

- **Junction Rule**, from charge conservation:
  - The sum of the currents at any junction must equal zero
  - Mathematically:

$$\sum_{\text{junction}} I = 0$$

- The example on the left figure:

$$I_1 - I_2 - I_3 = 0$$

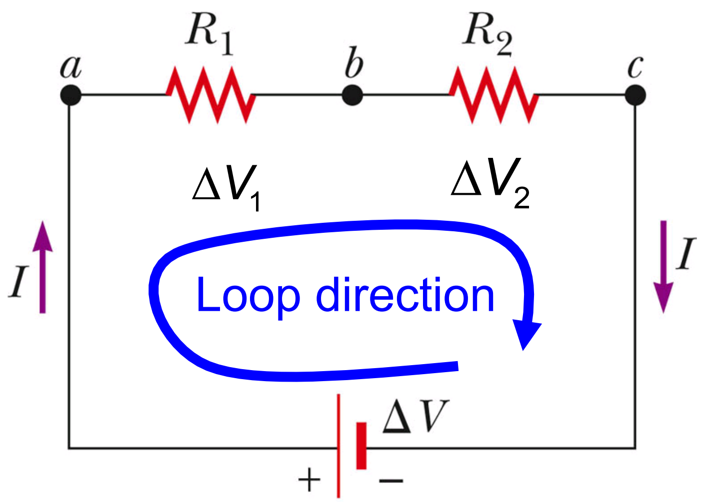


Rule 2: Kirchhoff's Loop Rule

- Choose your loop
- **Loop Rule**, from energy conservation:
  - The sum of the potential differences across all elements around any closed circuit loop must be zero
  - Mathematically:

$$\sum_{\text{closed loop}} \Delta V = 0$$

- One needs to pay attention the sign (+ or -) of these potential changes, following the chosen loop direction.



Remember two things:

1. A battery supplies power. Potential rises from the "–" terminal to "+" terminal.
2. Current follows the direction of electric field, hence the decrease of potential.

Kirchhoff's rules
Strict steps in solving a problem

Step 1: choose and mark the loop.

Step 2: choose and mark current directions. Mark the potential change on resistors.

Step 3: apply junction rule:

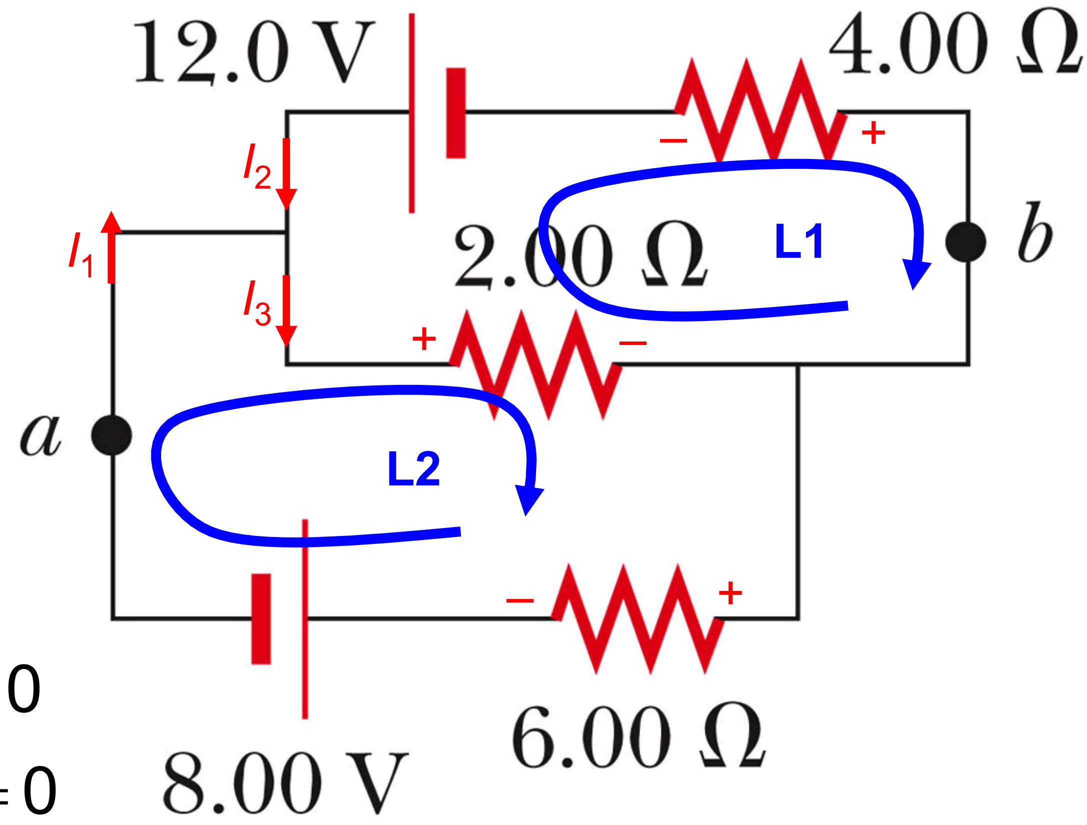
$$I_1 + I_2 - I_3 = 0$$

Step 4: apply loop rule:

$$\text{L1:} \quad +2.00 I_3 - 12.0 + 4.00 I_2 = 0$$

$$\text{L2:} \quad -8.00 - 2.00 I_3 - 6.00 I_1 = 0$$

Step 5: solve the three equations for the three variables.



One more example

Step 1: choose and mark the loop.

Step 2: choose and mark current directions. Mark the potential change on resistors.

Step 3: apply junction rule:

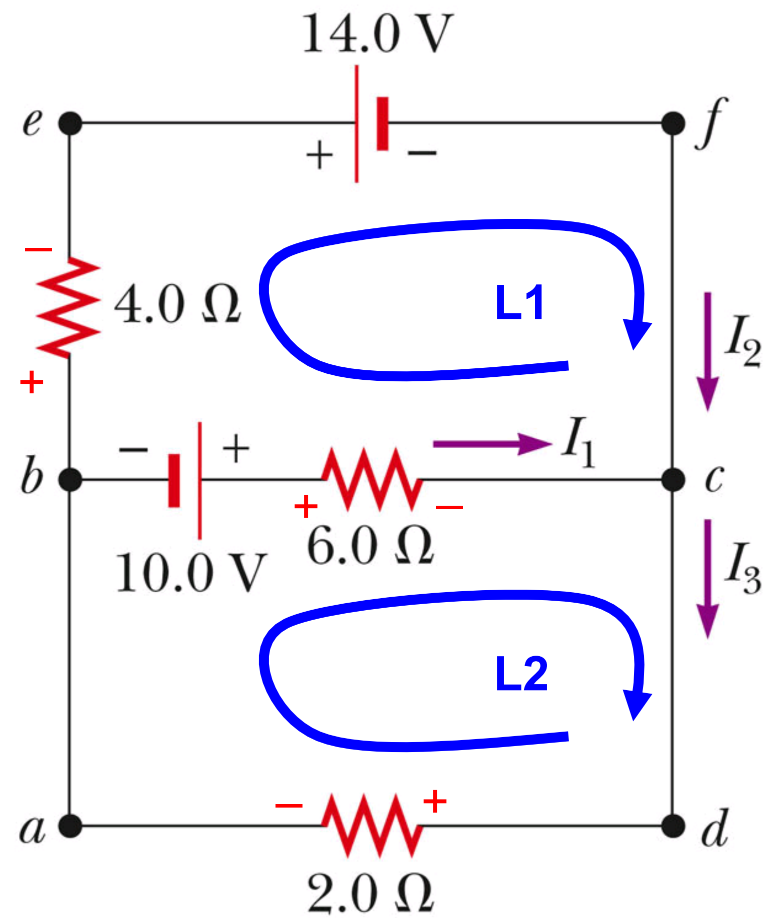
$$I_1 + I_2 - I_3 = 0$$

Step 4: apply loop rule:

$$\text{L1:} \quad +6.0I_1 - 10.0 - 4.0I_2 - 14.0 = 0$$

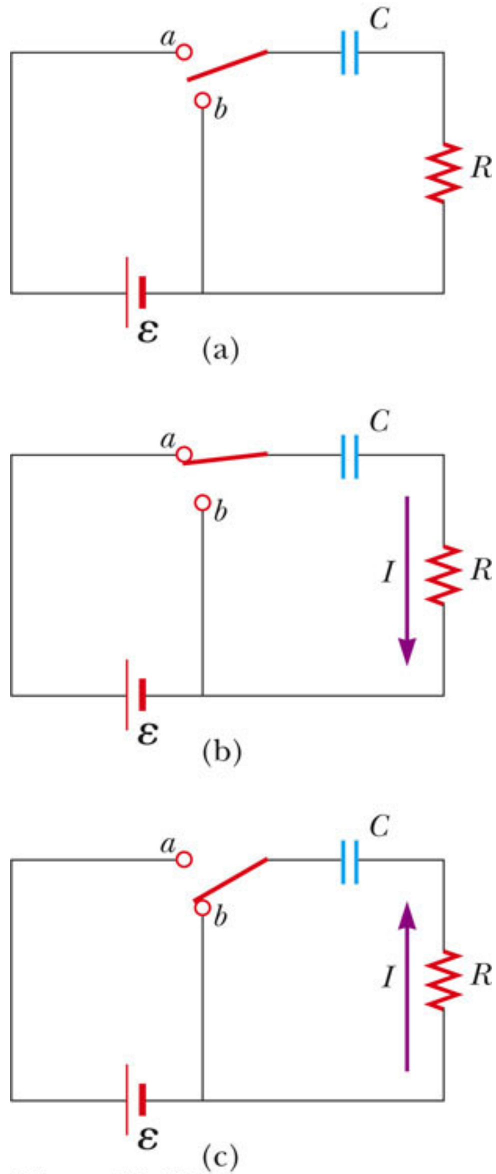
$$\text{L2:} \quad -2.0I_3 + 10.0 - 6.0I_1 = 0$$

Step 5: solve the three equations for the three variables.



RC Circuits, solve with Kirchhoff's rules

- When a circuit contains a resistor and a capacitor connected in series, the circuit is called a RC circuit.
- Current in RC circuit is DC, but the current magnitude changes with time.
- There are two cases: charging (b) and discharging (c).



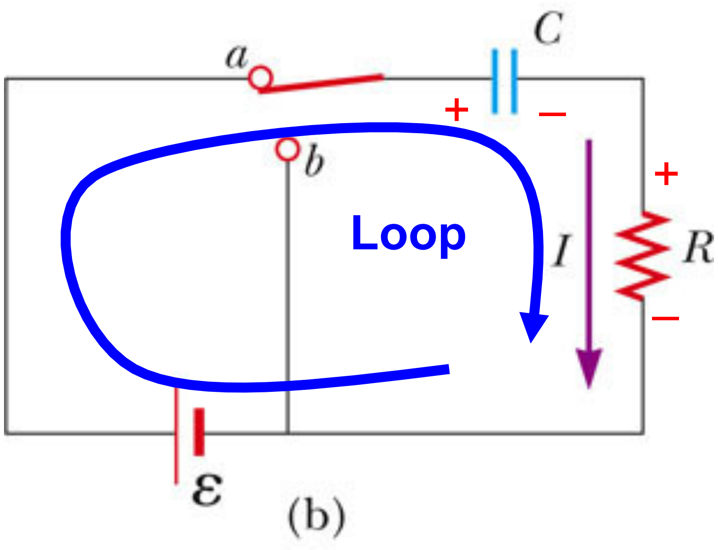
Not a circuit

charging

Dis-charging

Charging a Capacitor

When the switch turns to position a, current starts to flow and the capacitor starts to charge.



Kirchhoff's rule says:

$$\varepsilon - \Delta V_c - \Delta V_R = 0$$

Re-write the equation in terms of the charge q in C and the current I, and then only the variable q:

$$\varepsilon - \frac{q}{C} - RI = 0 \text{ and then } \varepsilon - \frac{q}{C} - R\frac{dq}{dt} = 0$$

Solve for q:

$$q(t) = C\varepsilon\left(1 - e^{\frac{-t}{RC}}\right)$$

The current I is

$$I(t) = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{\frac{-t}{RC}}$$

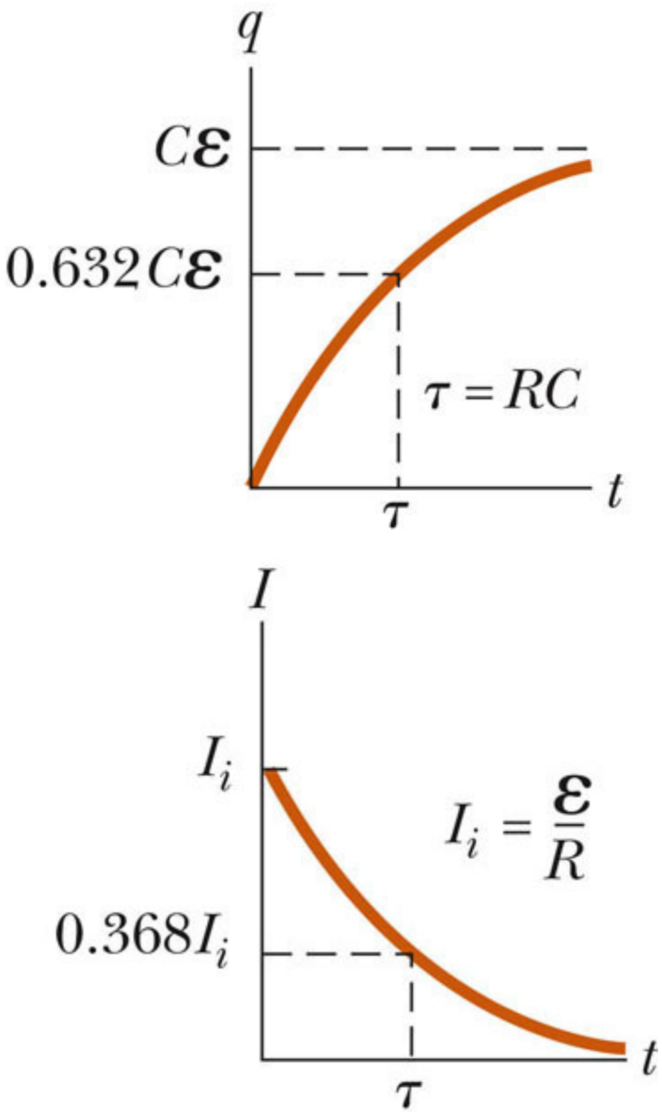
Here RC has the unit of time t, and is called the time constant.

Charging a Capacitor, graphic presentation

- The charge on the capacitor varies with time
  - $q(t) = C\varepsilon\,(1 - e^{-t/RC})$
    $= Q(1 - e^{-t/RC})$
- The current decrease with time

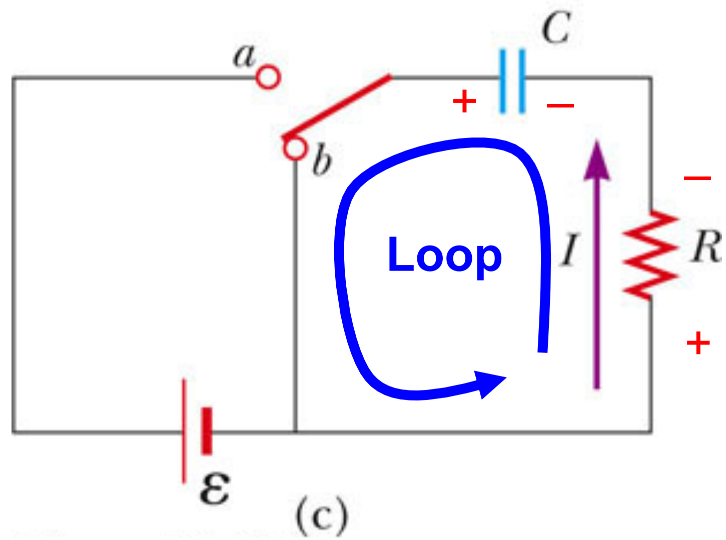
$$I(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

  - τ is the *time constant*
    - $\tau = RC$



Discharging a Capacitor

When the switch turns to position b, after the capacitor is fully charged to Q, current starts to flow and the capacitor starts to discharge.

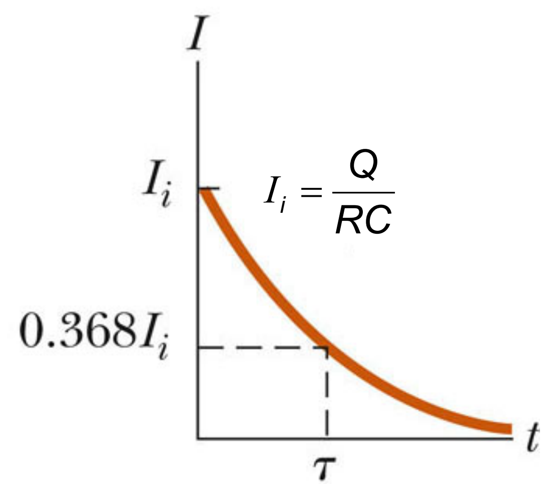


Kirchhoff's rule says:

$$\Delta V_c - \Delta V_R = 0$$

Re-write the equation in terms of the charge *q* in C and the current *I*, and then only the variable *q*:

$$\frac{q}{C} - RI = 0 \text{ and then } \frac{q}{C} - R\frac{-dq}{dt} = 0$$



Solve for *q*:

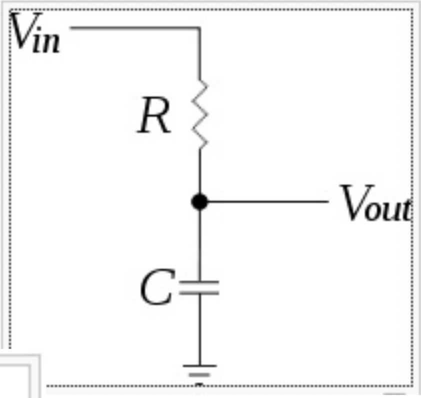
$$q(t) = Qe^{\frac{-t}{RC}}$$

The current *I* is

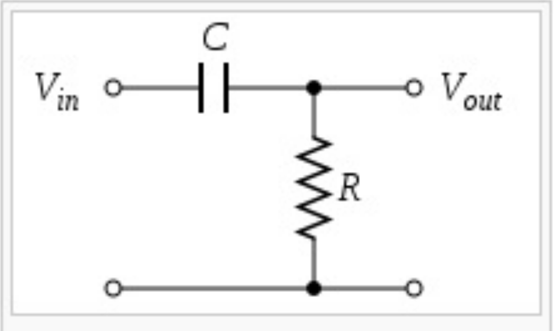
$$I(t) = -\frac{dq}{dt} = \frac{Q}{RC}e^{\frac{-t}{RC}}$$

Connections to EE

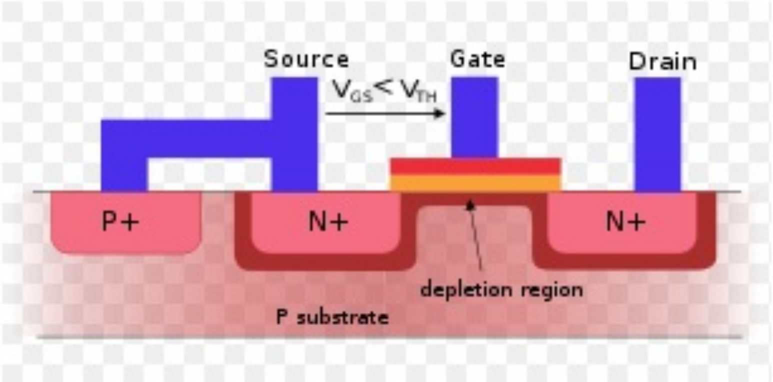
A low-pass filter



A high-pass filter



A MOSFET:



Reading material and Homework assignment

Please watch this video (about 50 minutes each):
http://videolectures.net/mit802s02_lewin_lec10/

Please check wileyplus webpage for homework assignment.