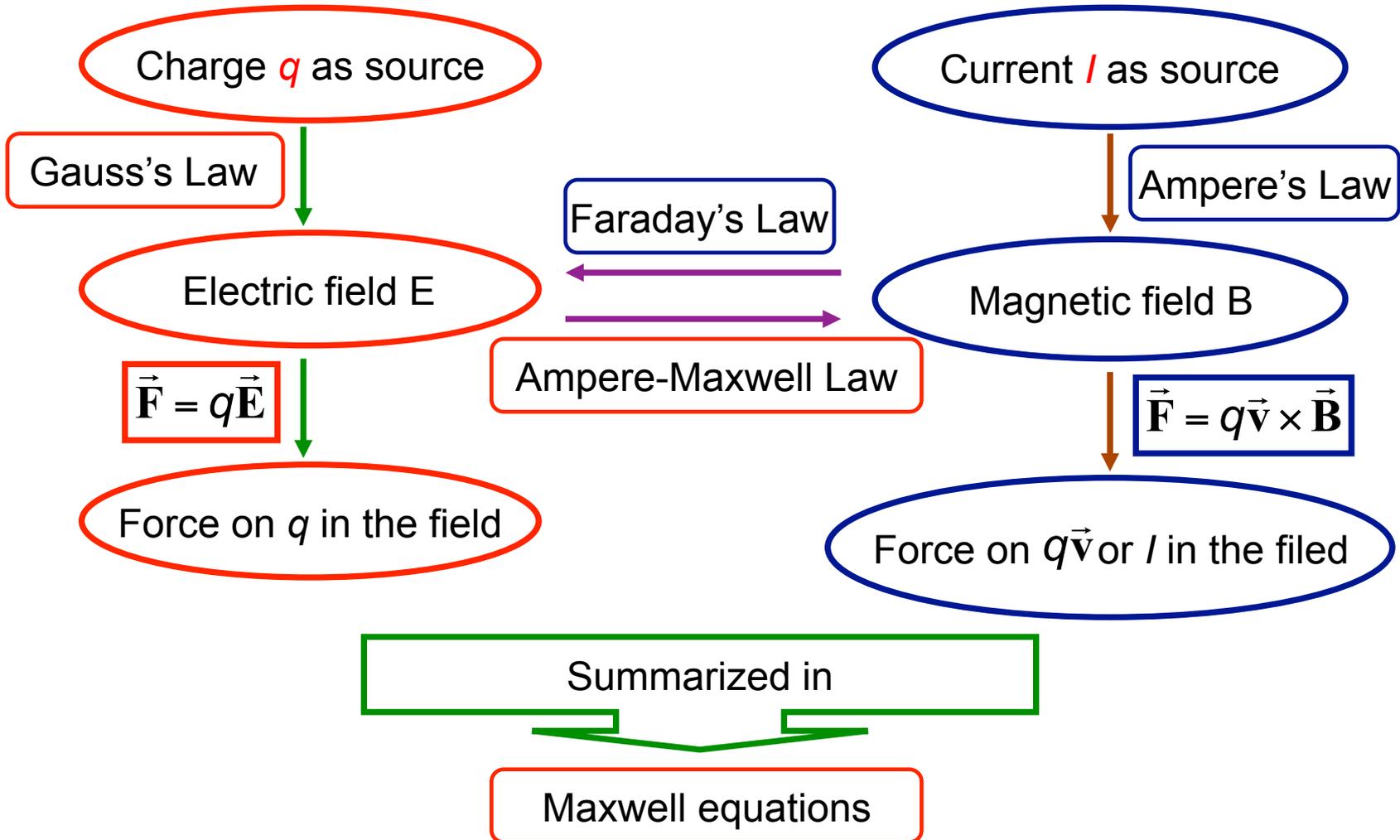


Induction and Inductance

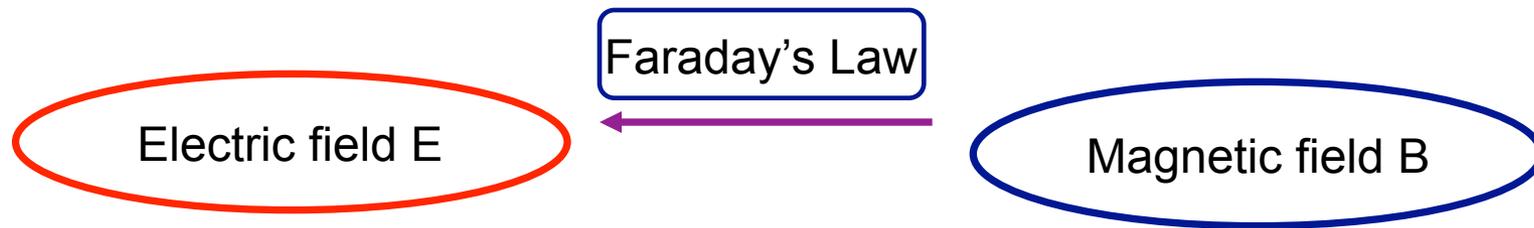
How we generate E by B , and the passive component inductor in a circuit.

1. A review of “emf” and the magnetic flux.
2. Faraday’s Law of Induction
3. Lentz Law
4. Inductance and inductor
5. Self-inductance and L in circuits (RL and LC)
6. Energy considerations.

Electricity and Magnetism

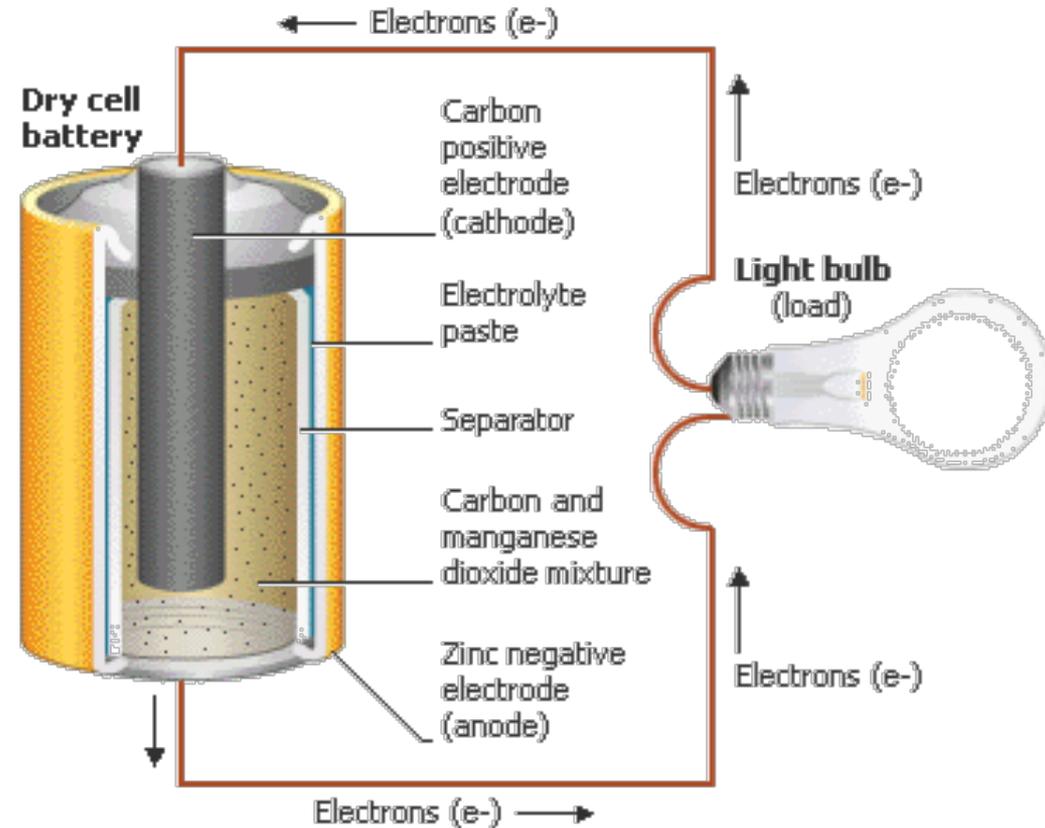


Electricity and Magnetism



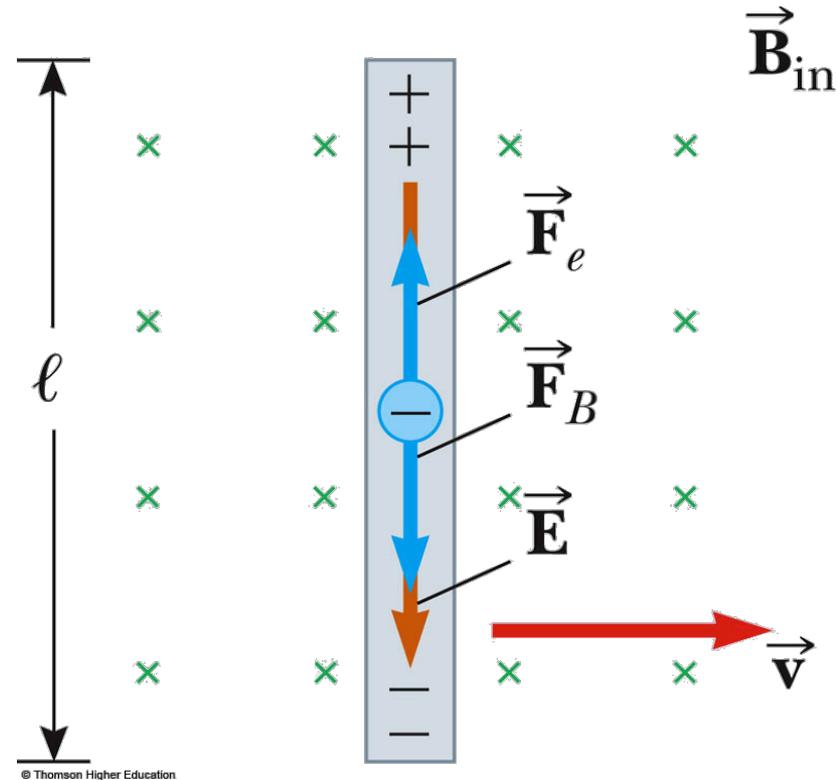
Chemical emf: A dry-cell battery

Chemical reactions in the battery cell transport charge carriers (electrons) from one terminal to the other to create the needed electric potential (emf) which drives the current through the outside load, a light bulb here.



Motional emf, the concept

- With $\vec{F}_B = q\vec{v} \times \vec{B}$
We can group charges by moving them in a magnetic field
→ motional emf.
- A **motional emf** is the emf induced in a conductor moving through a magnetic field
- The electrons in the conductor experience a force, $\vec{F}_B = q\vec{v} \times \vec{B}$ that is directed along ℓ
- Charges are accumulated at the ends of the conductor to create an electric field inside the conductor to stop further charge transportation.



When in equilibrium:

$$F_B = qvB = F_E = qE$$

Motional emf, the calculation

- Start from the equilibrium condition

$$F_B = qvB = F_E = qE$$

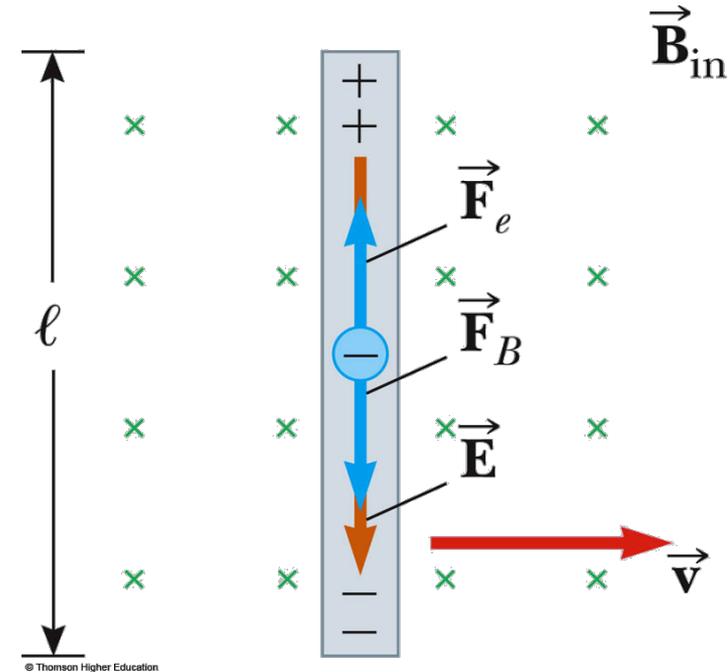
One has

$$E = vB$$

Or the emf, potential difference:

$$\text{emf} = \Delta V = El = vBl$$

- As long as the bar is kept moving with a velocity v , the motional emf is maintained to be vBl .



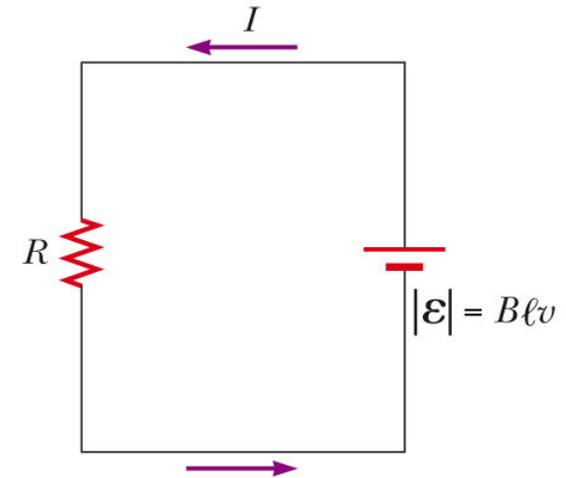
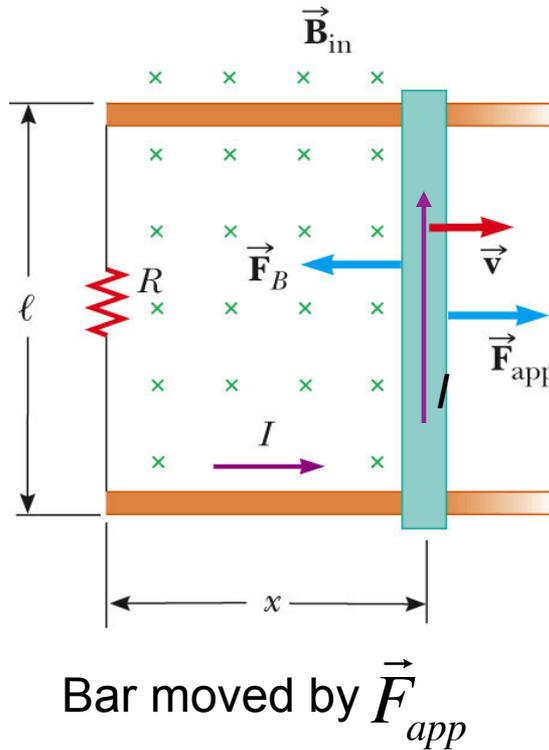
Motional emf, put in use to power a resistor

Condition:

A bar moves on two rails. The bar and the rails have negligible resistance. A resistor of R is connected to the end of the two rails.

Result:

The emf = $vB\ell$, so the current $I = vB\ell/R$



Equivalent circuit diagram

Two issues need attention:

1. The moving bar carrying current I , inside the magnetic field, experiences a force from the field is $F_B = I\ell B$
2. The magnetic flux in the enclosed area (bar, rails and resistor) is $\Phi_B = x\ell B$, and it is changing with time as

$$\frac{d\Phi_b}{dt} = \frac{d}{dt}(x\ell B) = \frac{dx}{dt}\ell B = vB\ell = emf \quad \Rightarrow \quad \frac{d\Phi_b}{dt} = emf$$

Example, what is the terminal velocity?

A bar of mass m slides on two vertical rails. A resistor is connected to the end of the rails. When the bar is released at $t = t_0$, (a) calculate the velocity of the bar at time t , (b) what is the terminal velocity? Assuming that the rails and the magnetic field is long and large enough.

Once the bar starts to move, accelerated by the gravitational force, there is:

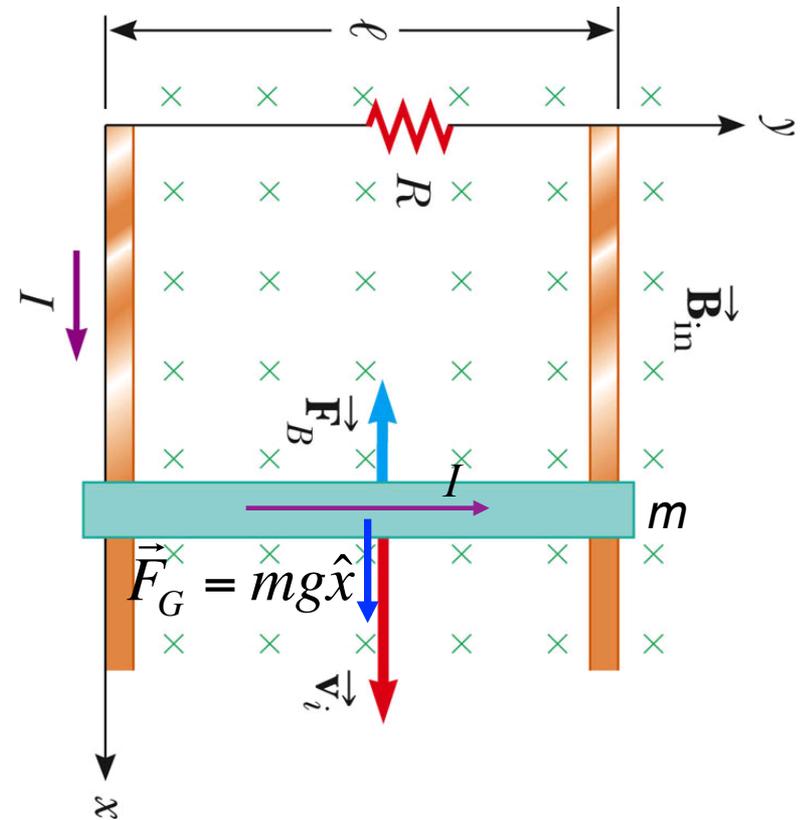
$$\text{emf} = vBl$$

And there is current as well:

$$I = (vBl)/R$$

And there is magnetic force on the bar, pointing opposite to the gravitational force:

$$\vec{F}_B = -\frac{I}{B} \hat{x} = -\frac{v(lB)^2}{R} \hat{x}$$



Example, what is the terminal velocity?

Construct the equation of velocity v :

$$\vec{F}_G + \vec{F}_B = \left(mg - \frac{v(lB)^2}{R} \right) \hat{x} = ma\hat{x}$$

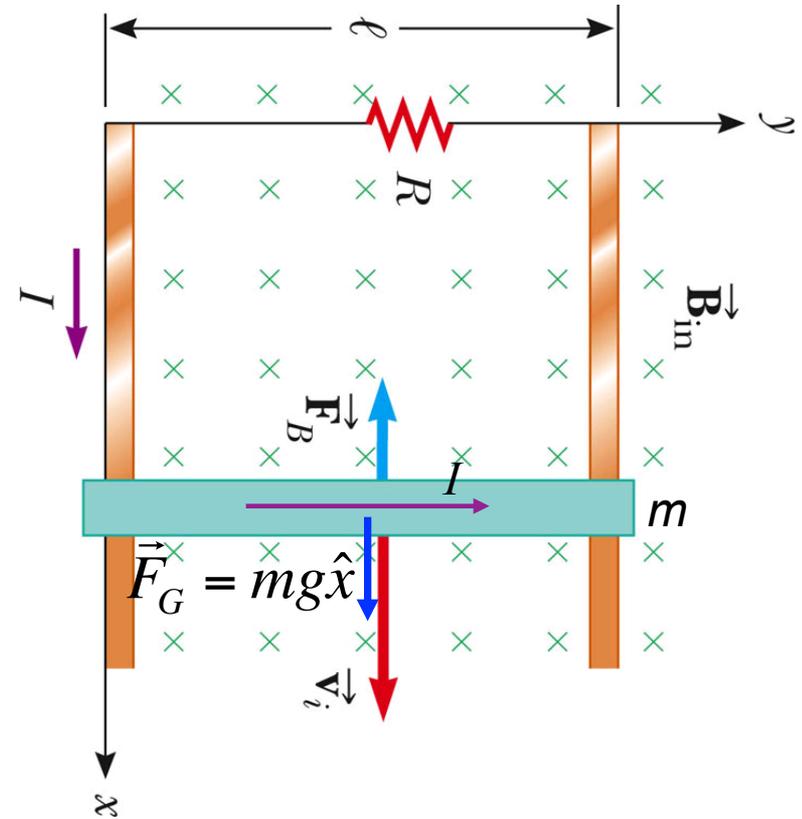
$$mg - \frac{v(lB)^2}{R} = ma = m \frac{dv}{dt}$$

Solve this equation

$$\frac{dv}{v - \tau g} = -\frac{dt}{\tau}, \quad \tau \equiv \frac{mR}{(lB)^2}$$

$$v = \tau g \left(1 - e^{-\frac{t}{\tau}} \right)$$

This is the answer to (a). For (b), the terminal velocity is τg when $t \rightarrow \infty$



Faraday's Law of induction

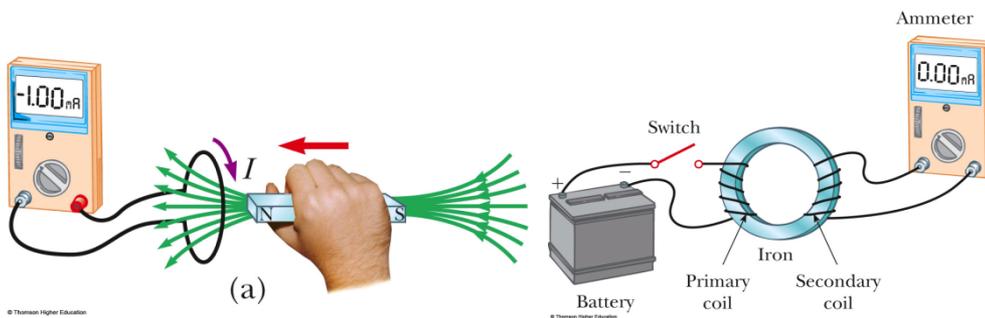
In the sliding bar “experiment”, we proved that:

$$\text{emf} = \frac{d\Phi_B}{dt}$$

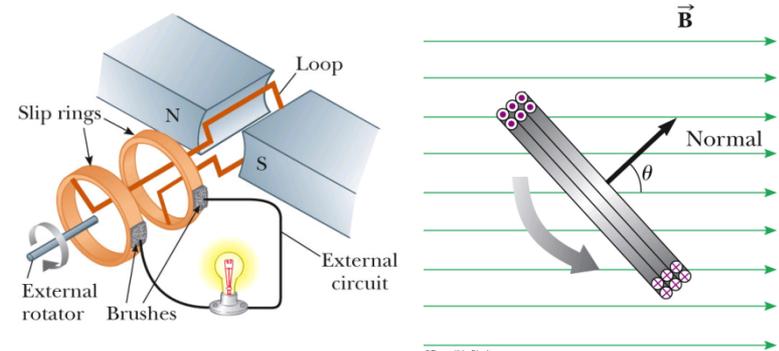
We also know that the magnetic flux is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad \text{or} \quad \Phi_B = BA \cos \theta$$

In the sliding bar experiment, we changed A by moving the bar. More practically people change B or the angle θ to achieve a changing flux.



Changing B



Changing θ

Faraday's Law of induction

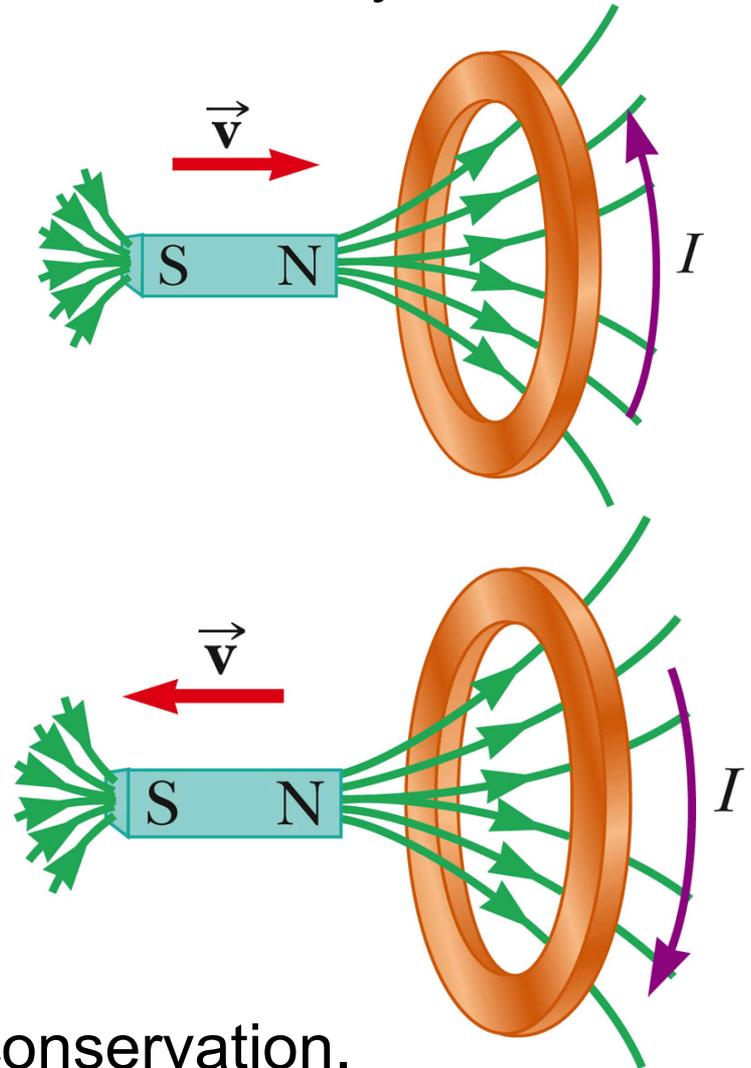
In any case, the induced emf follows the Faraday's Law of induction

$$\text{emf} = -\frac{d\Phi_B}{dt}$$

Yes, I sneaked in the “-”

in front of $\frac{d\Phi_B}{dt}$

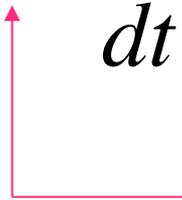
Because Mr. Lenz told me so in order to answer the question that in which direction should the induced current flow. Or more fundamentally because of energy conservation.



Faraday's Law, in English and in Math

- Faraday's law of induction states that “the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit”
- Mathematically,

$$\text{emf} = - \frac{d\Phi_B}{dt}$$

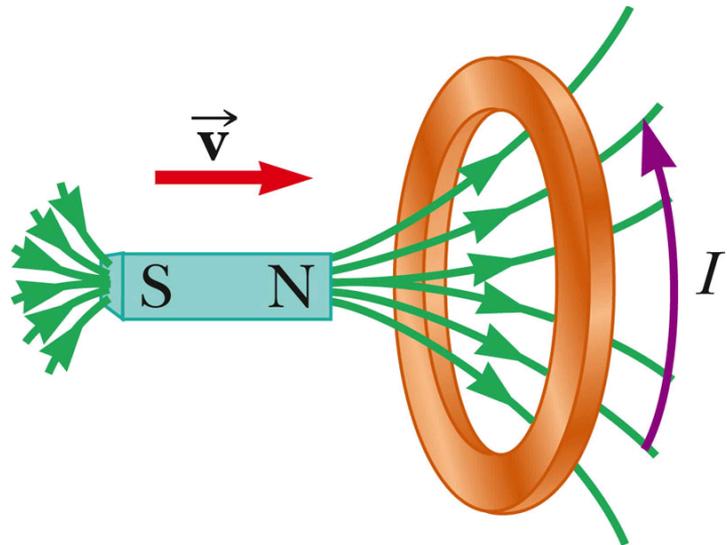


Lenz's Law

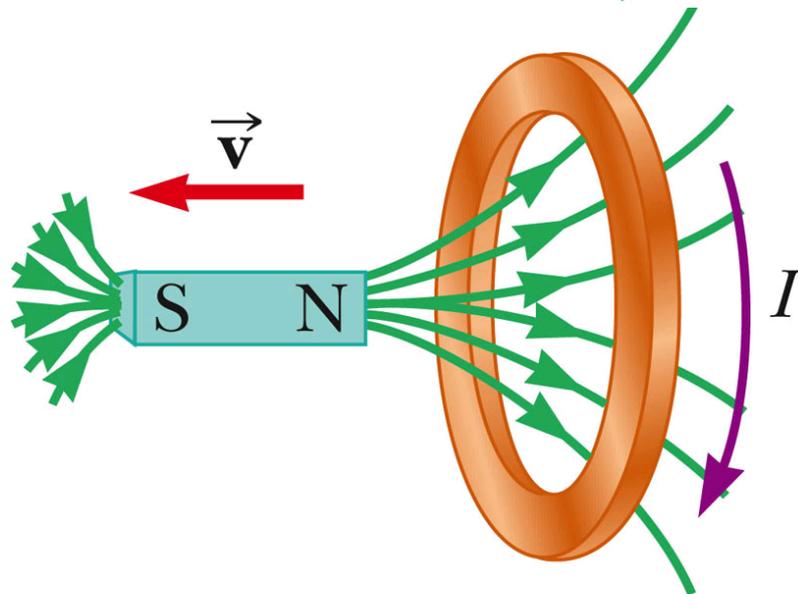
Lenz's Law, the direction of the induced emf

- **Lenz's law:** the induced current in a loop is in the direction that creates a magnetic field that **opposes the change** in magnetic flux through the area enclosed by the loop. The induced current tends to keep the original magnetic flux through the circuit from changing.
- The mathematical expression of this Law is the minus sign in Faraday's Law of induction.

Lenz's Law tells the induced current direction

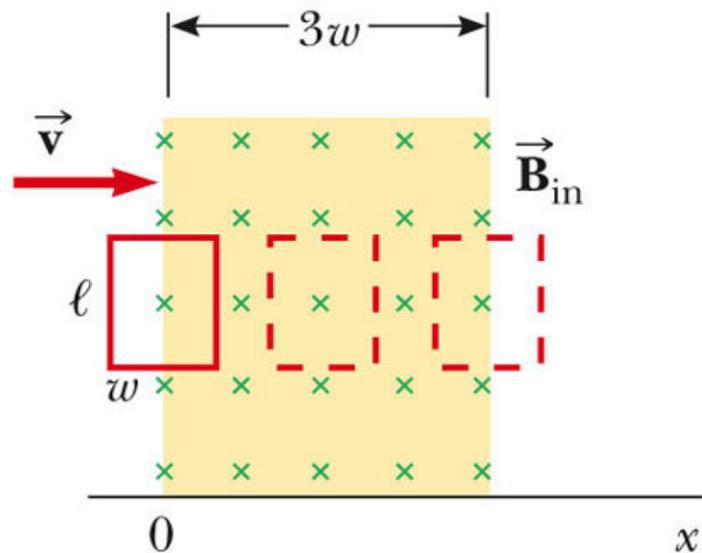


“source B” increases, so induced current generates a secondary B that opposes the **increase**.

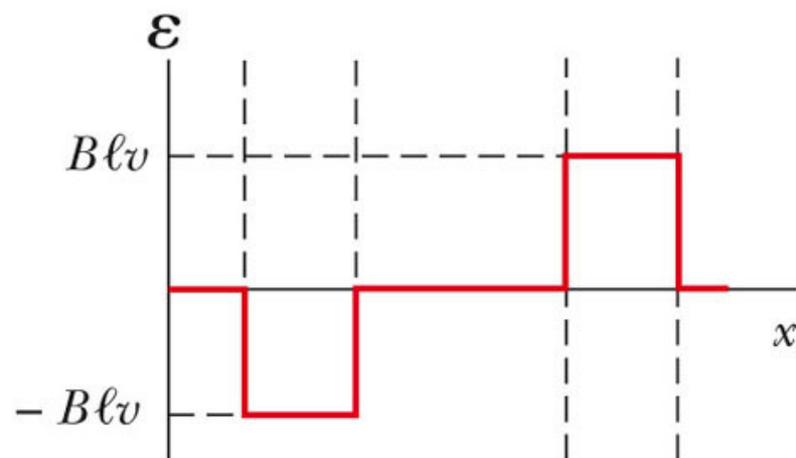


“source B” decreases, so induced current generates a secondary B that opposes/compensates the **decrease**.

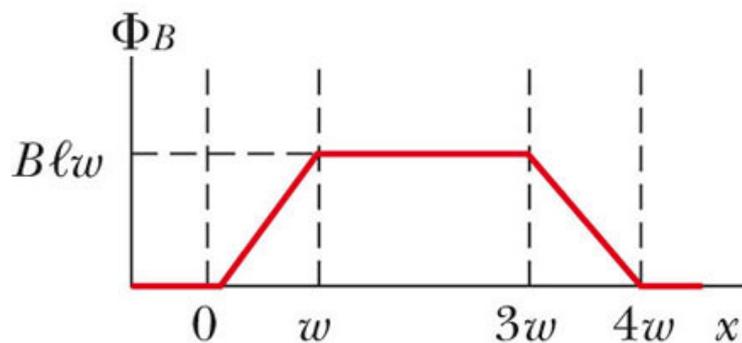
Lenz's Law, the concept of "change"



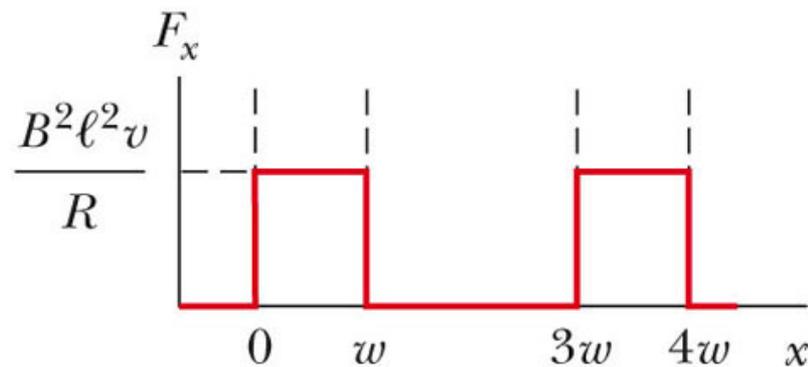
(a)



(c)



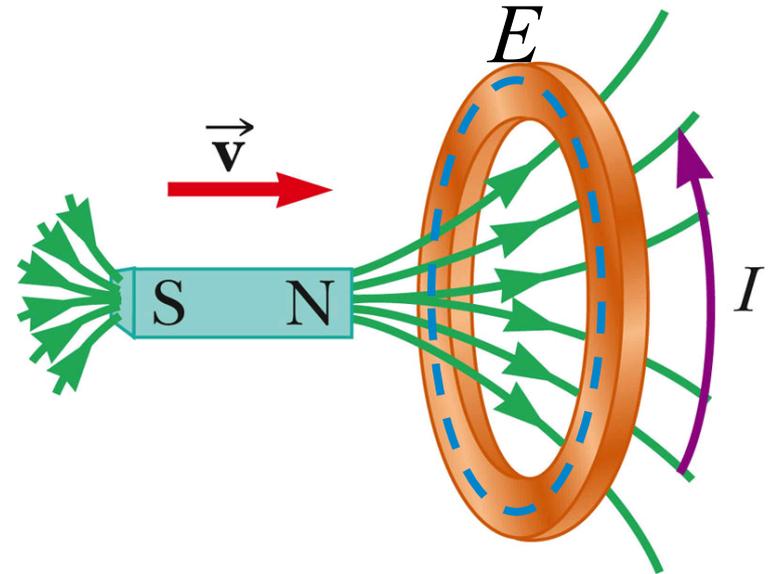
(b)



(d)

Induced emf and Electric Fields

- An electric field is created in the conductor as a result of the changing magnetic flux
- Even in the absence of a conducting loop, a changing magnetic field will generate an electric field in empty space



$$I = \frac{emf (= V)}{R}, \quad V = E \cdot d$$

$$-\frac{d\Phi_B}{dt} = emf = V = \oint_{circle} E \cdot ds$$

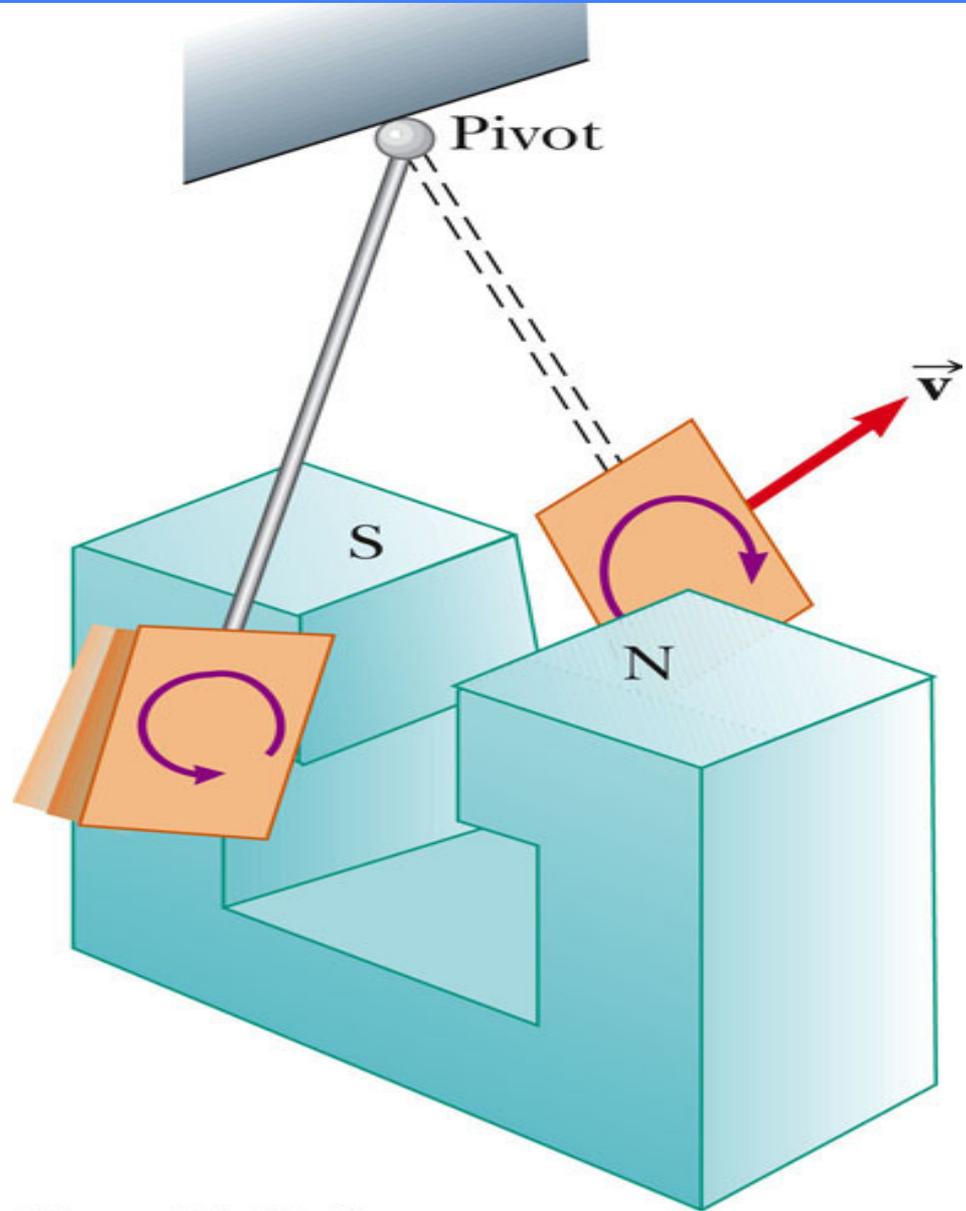
Induced emf and Electric Fields

- The emf for any closed path can be expressed as the line integral of $\vec{E} \cdot d\vec{s}$ over the path
- Faraday's law can be written in a general form:

$$\oint_{\text{circle}} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

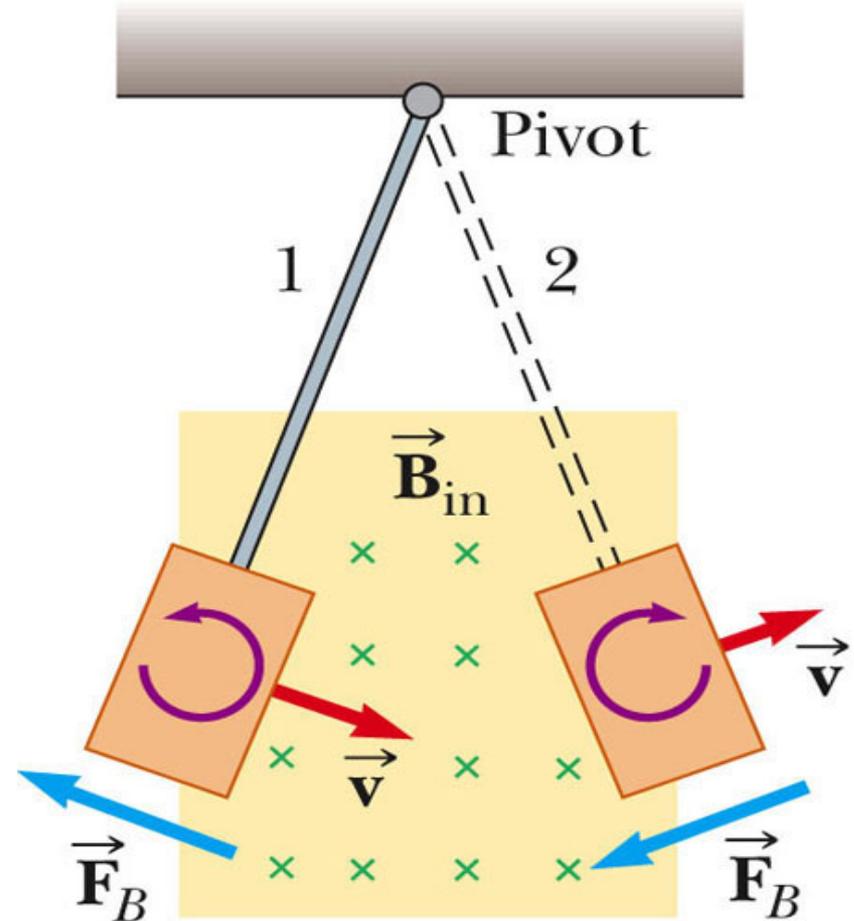
Eddy Currents

- Circulating currents called **eddy currents** are induced in bulk pieces of metal moving through a magnetic field
- The eddy currents are in opposite directions as the plate enters or leaves the field
- Eddy currents are often undesirable because they represent a transformation of mechanical energy into internal energy. But there are applications of Eddy currents as well.



Eddy Currents

- The magnetic field is directed into the page
- The induced eddy current is counterclockwise as the plate enters the field
- It is opposite when the plate leaves the field
- The induced eddy currents produce a magnetic retarding force and the swinging plate eventually comes to rest



Eddy Currents

- To reduce energy losses by the eddy currents, the conducting parts can
 - Be built up in thin layers separated by a nonconducting material
 - Have slots cut in the conducting plate
- Both prevent large current loops and increase the efficiency of the device



(Review on) Resistance, Capacitance and (introduce) **Inductance**

Ohm's Law defines **resistance**: $R \equiv \frac{\Delta V}{I}$

Resistors do not store energy, instead they transform electrical energy into thermo energy at a rate of:

$$P = \Delta V \cdot I = \frac{\Delta V^2}{R} = I^2 R$$

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Capacitance, the ability to hold charge: $C \equiv \frac{Q}{\Delta V}$

Capacitors store electric energy once charged:

$$\Delta U_E = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C (\Delta V)^2$$

(Review on) Resistance, Capacitance and (introduce) **Inductance**

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Inductance, the ability to “hold” current (moving charge).

Inductors store magnetic energy once “charged” with current, i.e., current flows through it.

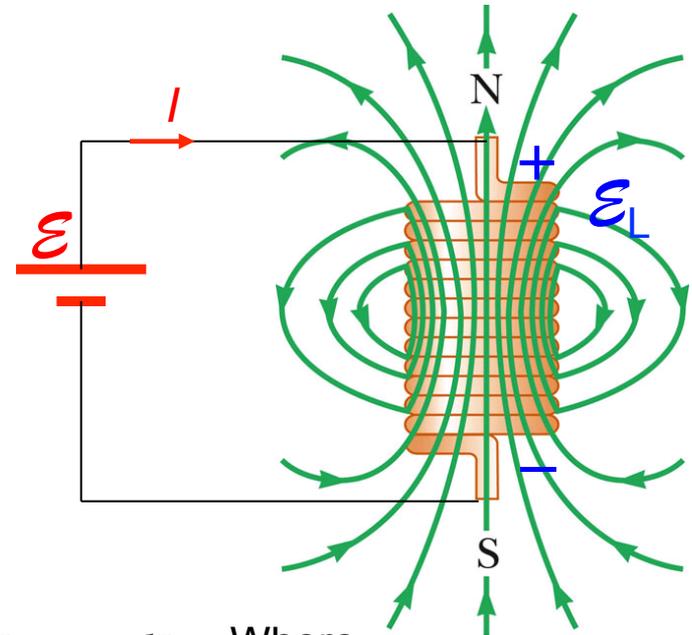
Inductance, the definition

When a current flows through a coil, there is magnetic field established. If we take the solenoid assumption for the coil: $B = \mu_0 nI$

When this magnetic field flux changes, it induces an emf, \mathcal{E}_L , called self-induction:

$$\mathcal{E}_L = -\frac{d\Phi_B}{dt} = -\frac{d(NAB)}{dt} = -\frac{d(NA\mu_0 nI)}{dt} = -\mu_0 n^2 V \frac{dI}{dt} \equiv -L \frac{dI}{dt}$$

or: $\mathcal{E}_L \equiv -L \frac{dI}{dt}$ For a solenoid: $L = \mu_0 n^2 V$



Where
 n : # of turns per unit length.
 N : # of turns in length ℓ .
 A : cross section area
 V : Volume for length ℓ .

This defines the inductance L , which is constant related only to the coil. The self-induced emf is generated by current flowing through a coil. According to Lenz Law, the emf generated inside this coil is always opposing the change of the current which is delivered by the original emf.

Inductor

We used a coil and the solenoid assumption to introduce the inductance. But the definition

$$L \equiv -\frac{\mathcal{E}_L}{dI/dt}$$

holds for all types of inductance, including a straight wire. **Any conductor has capacitance and inductance.** But as in the capacitor case, an inductor is a device made to have a sizable inductance.

An inductor is made of a coil. The symbol is 

Once the coil is made, its inductance L is defined. The self-induced emf over this inductor under a changing current I is given by:

$$\mathcal{E}_L = -L \frac{dI}{dt}$$

Unit for Inductance

- The SI unit for inductance is the **henry** (H)

$$1\text{H} = 1 \frac{\text{V} \cdot \text{s}}{\text{A}}$$

- Named for Joseph Henry:
 - 1797 – 1878
 - **American** physicist
 - First director of the Smithsonian
 - Improved design of electromagnet
 - Constructed one of the first motors
 - Discovered self-inductance



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Discussion about Some Terminology

- Use *emf* and *current* when they are caused by batteries or other sources
- Use *induced emf* and *induced current* when they are caused by changing magnetic fields
- When dealing with problems in electromagnetism, it is important to distinguish between the two situations

Example: Inductance of a coaxial cable

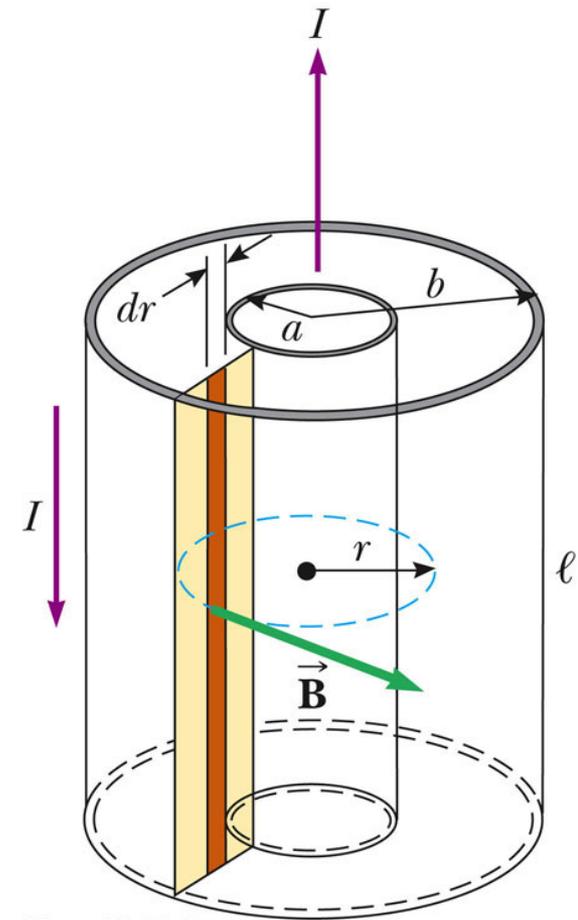
Start from the definition: $\text{emf} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$

We have $d\Phi_B = LdI$, or $\Phi_B = LI$

$$\begin{aligned}\Phi_B &= \int B dA = \int_a^b \frac{\mu_0 I}{2\pi r} l dr \\ &= \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}\end{aligned}$$

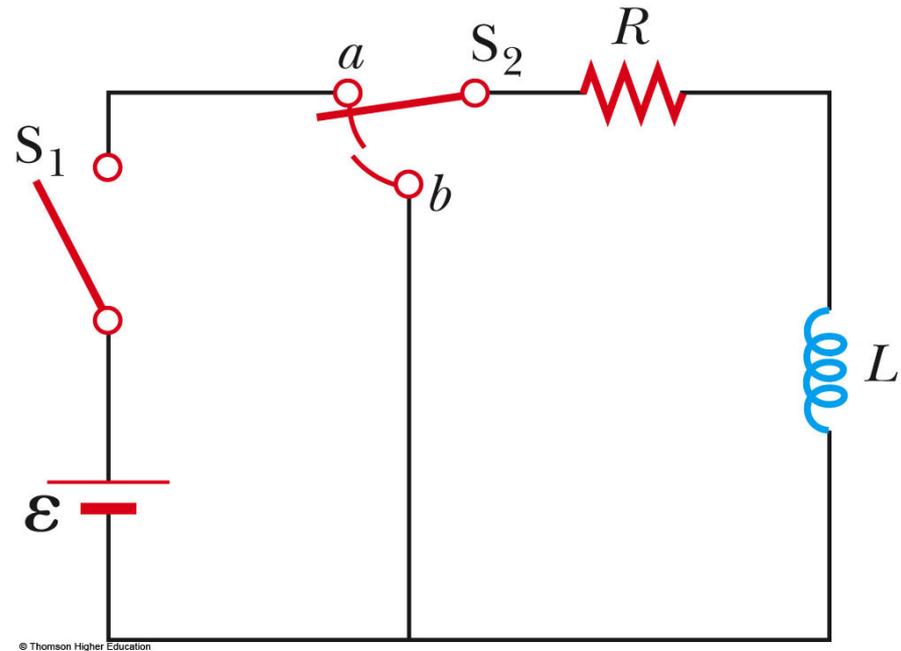
So the inductance is

$$L = \frac{\Phi_B}{I} = \frac{\mu_0 l}{2\pi} \ln \frac{b}{a}$$



Put inductor L to use: *the RL Circuit*

- An RL circuit contains a resistor R and an inductor L .
- There are two cases as in an RC circuit: charging and discharging. There are also two case in an RL circuit: except that here one charges with current, not electric charge.
- Charging:
 - When S_2 is connected to position a and when switch S_1 is closed (at time $t = 0$), the current begins to increase
- Discharging:
 - When S_2 is connected to position b .



RL Circuit, charging

- Applying Kirchhoff's loop rule to the circuit in the clockwise direction gives

$$\mathcal{E} - IR - L \frac{dI}{dt} = 0$$

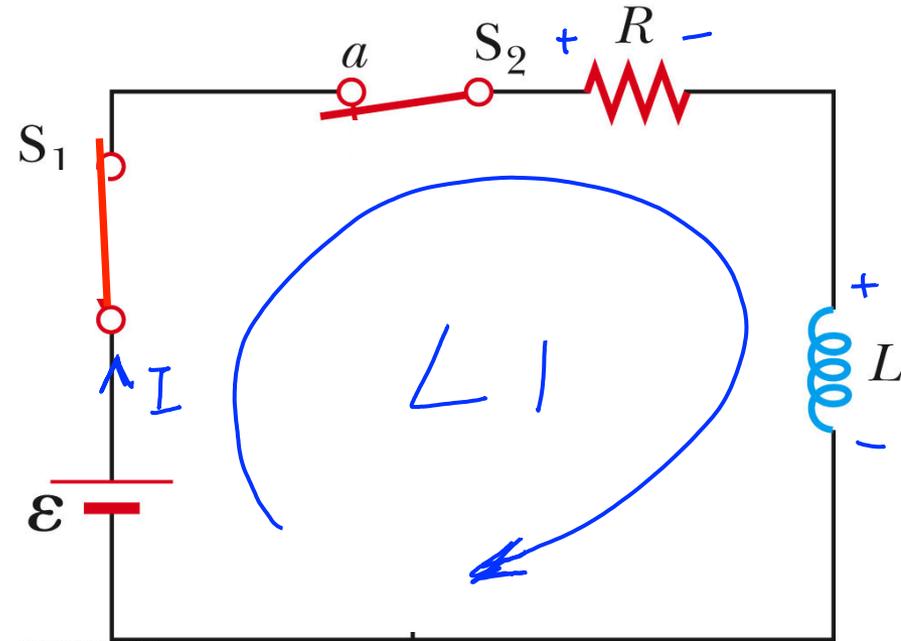
Here because the current is increasing, the induced emf has a direction that should oppose this increase.

- Solve for the current I , with initial condition that $I(t=0) = 0$, we find

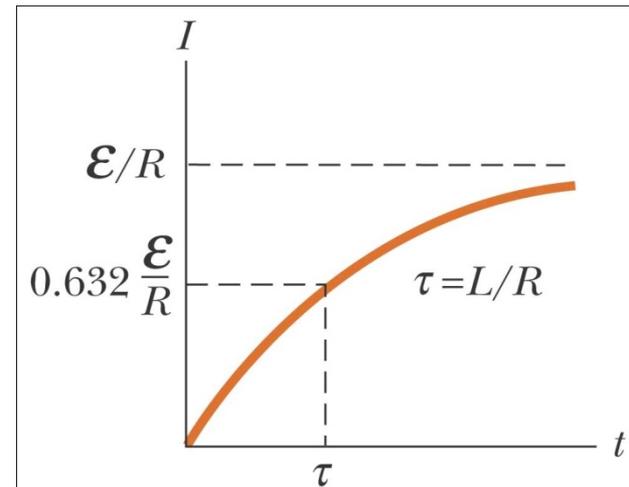
$$I = \frac{\mathcal{E}}{R} \left(1 - e^{-Rt/L}\right) \equiv \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau}\right)$$

- Where the time constant is defined as:

$$\tau = \frac{L}{R}$$



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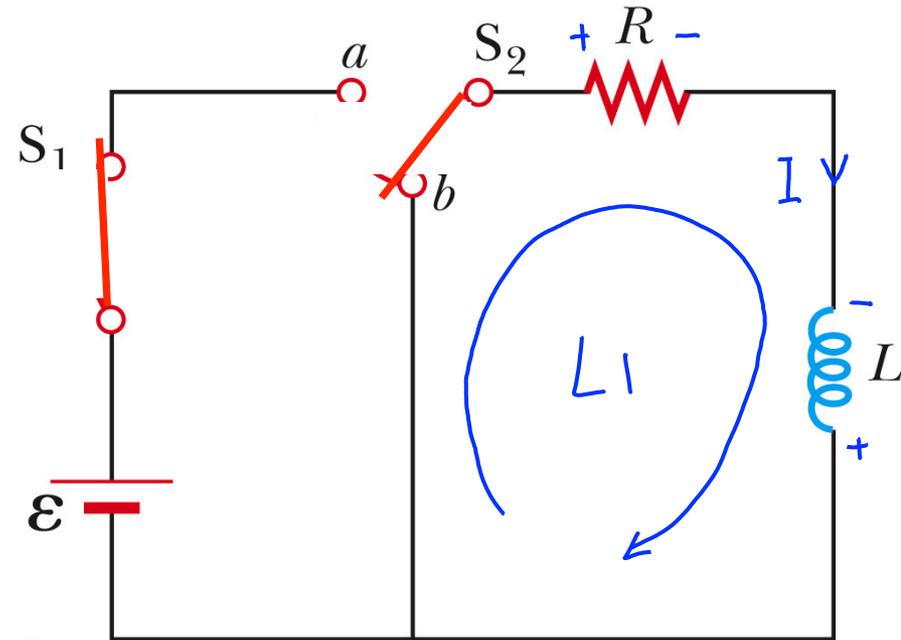
RL Circuit, discharging

- When switch S_2 is moved to position b , the original current disappears. The self-induced emf will try to prevent that change, and this determines the emf direction (Lenz Law).
- Applying Kirchhoff's loop rule to the previous circuit in the clockwise direction gives

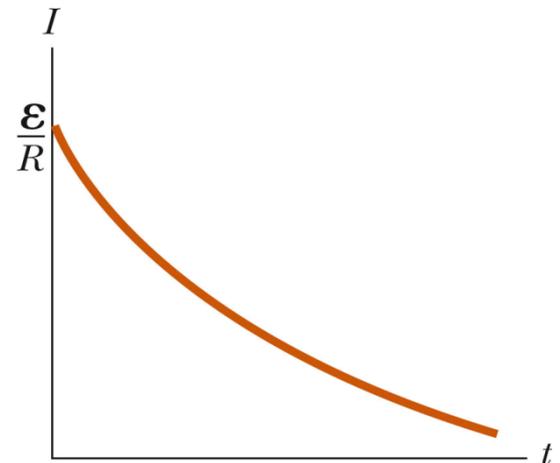
$$-IR + L \frac{dI}{dt} = 0$$

- Solve for the current I , with initial condition that $I(t=0) = \frac{\mathcal{E}}{R}$ we find

$$I = \frac{\mathcal{E}}{R} e^{-Rt/L} \equiv \frac{\mathcal{E}}{R} e^{-t/\tau}$$



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Energy stored in an inductor

In the charging case, the current I from the battery supplies power not only to the resistor, but also to the inductor.

From Kirchhoff's loop rule, we have

$$\mathcal{E} = IR + L \frac{dI}{dt}$$

Multiply both sides with I :

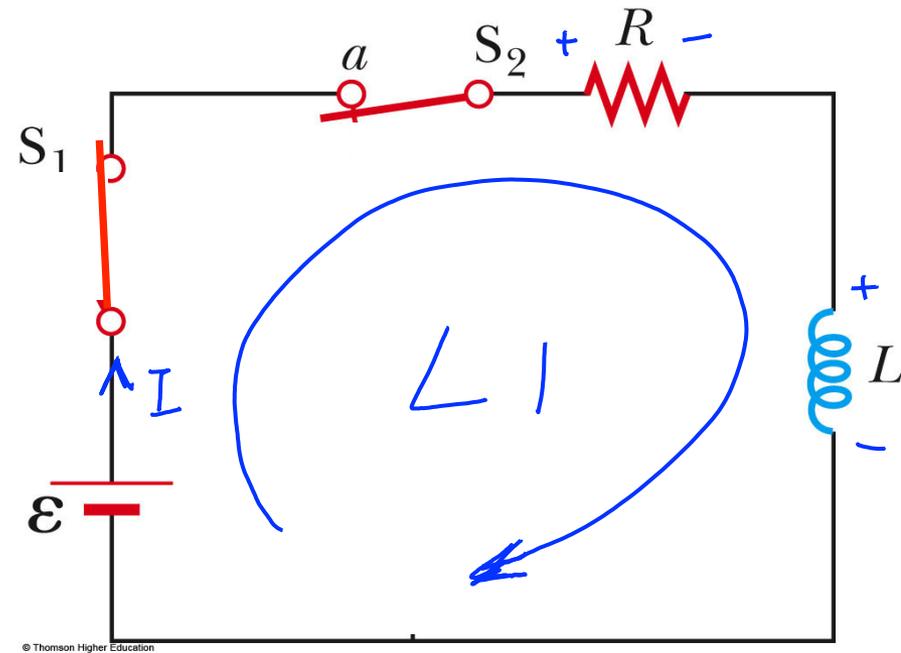
$$\mathcal{E}I = I^2 R + LI \frac{dI}{dt}$$

This equation reads: power_{battery} = power_R + power_L

So we have the energy increase in the inductor as:

$$\frac{dU_L}{dt} = LI \frac{dI}{dt}$$

$$\text{Solve for } U_L: U_L = \int_0^I LI dI = \frac{1}{2} LI^2$$



Stored energy type and the Energy Density of a Magnetic Field

- Given $U_L = \frac{1}{2}LI^2$ and assume (for simplicity) a solenoid with $L = \mu_0 n^2 V$

$$U_L = \frac{1}{2} \mu_0 n^2 V \left(\frac{B}{\mu_0 n} \right)^2 = \frac{B^2}{2\mu_0} V$$

So the energy stored in the solenoid volume V is magnetic (B) energy.

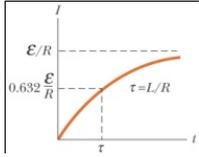
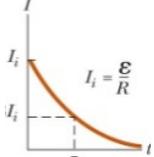
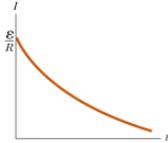
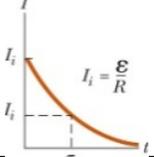
- Since V is the volume of the solenoid, the magnetic energy density, u_B is

$$u_B = \frac{U_L}{V} = \frac{B^2}{2\mu_0}$$

And the energy density is proportional to B^2 .

- This applies to any region in which a magnetic field exists (not just the solenoid)

RL and RC circuits comparison

	RL	RC
Charging	$I = \frac{\varepsilon}{R} \left(1 - e^{-Rt/L}\right)$ 	$I(t) = \frac{\varepsilon}{R} e^{-t/RC}$ 
Discharging	$I = \frac{\varepsilon}{R} e^{-Rt/L}$ 	$I(t) = \frac{Q}{RC} e^{-t/RC}$ 
Energy	$U_L = \frac{1}{2} LI^2$	$U_C = \frac{Q^2}{2C} = \frac{1}{2} C(\Delta V)^2$

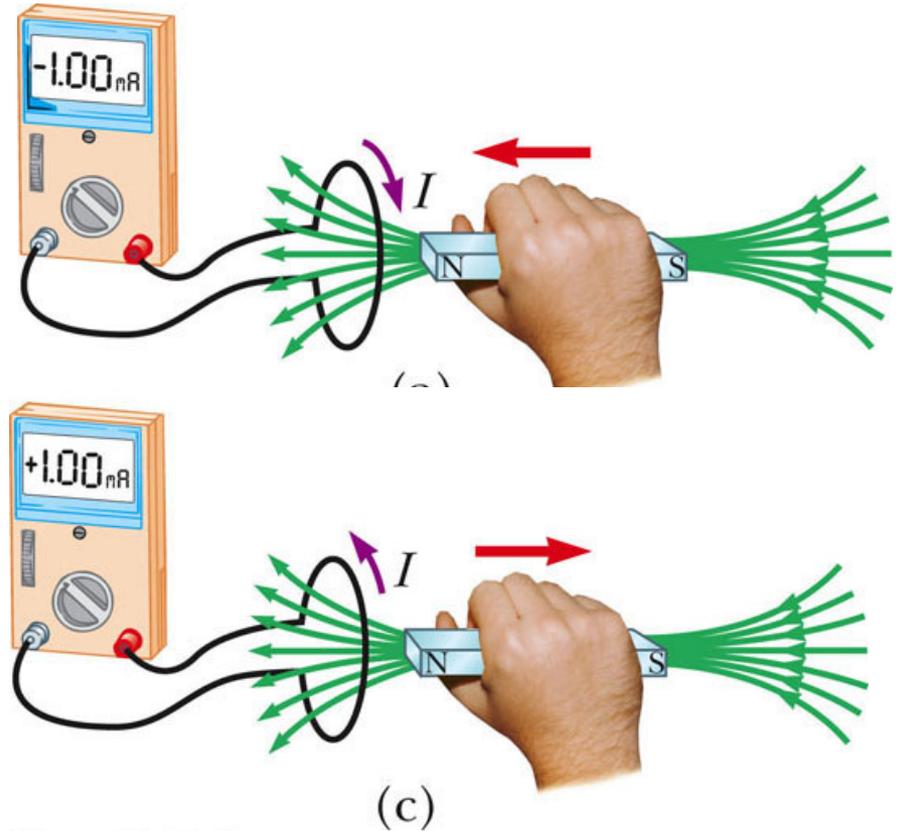
	Magnetic field	Electric field
Energy density	$u_B = \frac{B^2}{2\mu_0}$	$u_E = \frac{1}{2} \varepsilon_0 E^2$

Energy Storage Summary

- Inductor and capacitor store energy through different mechanisms
 - Charged capacitor
 - Stores energy as electric potential energy
 - When current flows through an inductor, it
 - Stores energy as magnetic potential energy
- A resistor does not store energy
 - Energy delivered is transformed into thermo energy

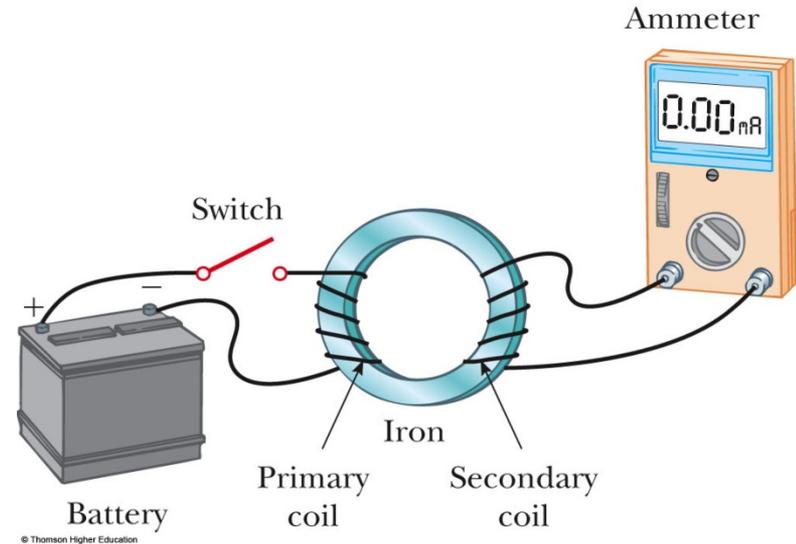
Example: EMF produced by a changing magnetic field

- A loop of wire is connected to a sensitive ammeter
- Determine the current in the loop and the magnet is being
 - Moved into the loop
 - Moved out of the loop
 - Held still inside the loop



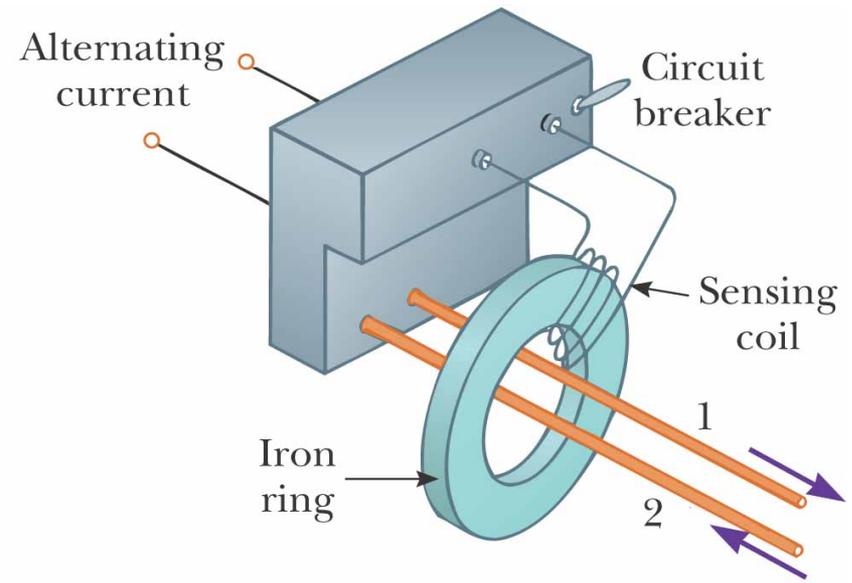
Example: a transformer

- A primary coil is connected to a switch and a battery
- The wire is wrapped around an iron ring
- A secondary coil is also wrapped around the iron ring
- There is no battery present in the secondary coil
- The secondary coil is not directly connected to the primary coil
- Close the switch and observe the current readings given by the ammeter



Applications of Faraday's Law – GFI

- A GFI (ground fault indicator) protects users of electrical appliances against electric shock
- When the currents in the wires are in opposite directions, the flux is zero
- When the return current in wire 2 changes, the flux is no longer zero
- The resulting induced emf can be used to trigger a circuit breaker

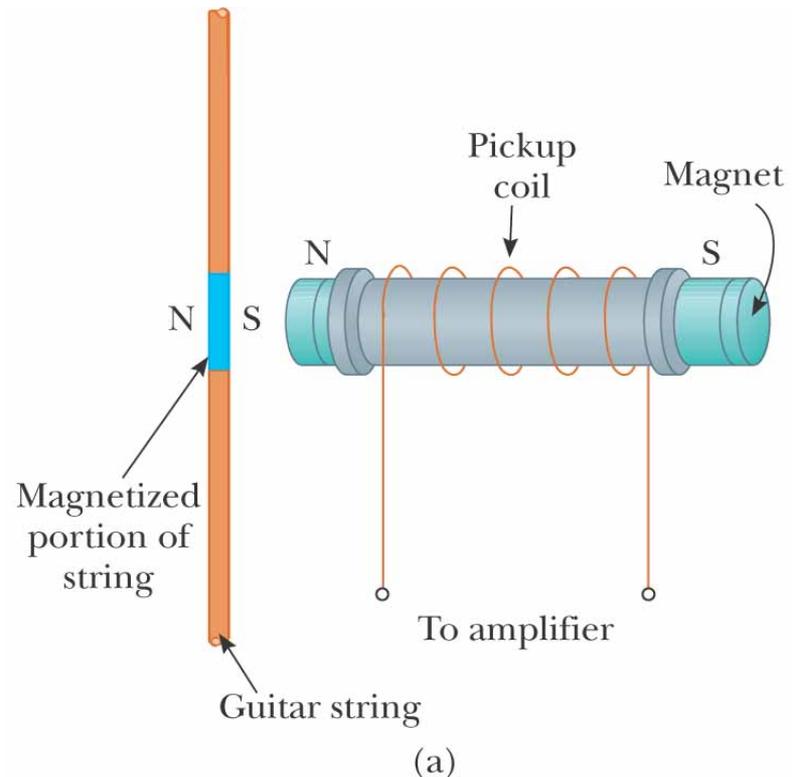


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Applications of Faraday's Law

– Pickup Coil

- The pickup coil of an electric guitar uses Faraday's law
- The coil is placed near the vibrating string and causes a portion of the string to become magnetized
- When the string vibrates at some frequency, the magnetized segment produces a changing flux through the coil
- The induced emf is fed to an amplifier



Rotating Loop

- Assume a loop with N turns, all of the same area rotating in a magnetic field
- The flux through the loop at any time t is

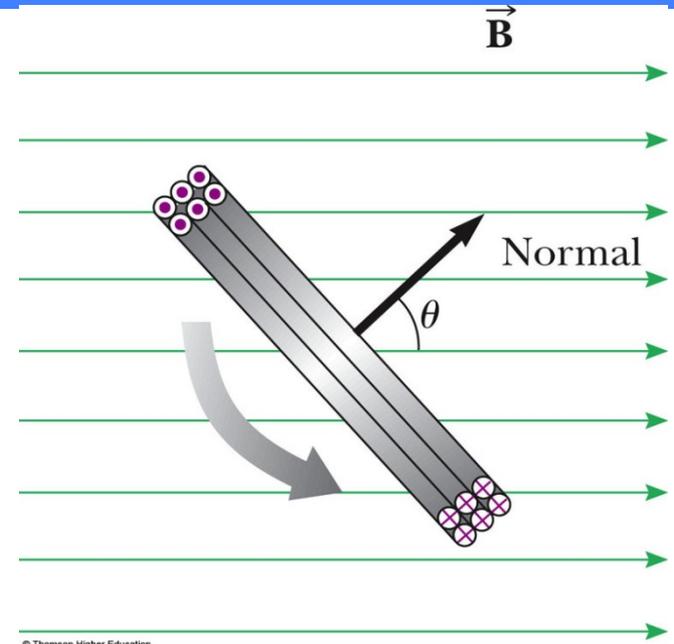
$$\Phi_B = BA \cos \theta$$

$$= BA \cos \omega t$$

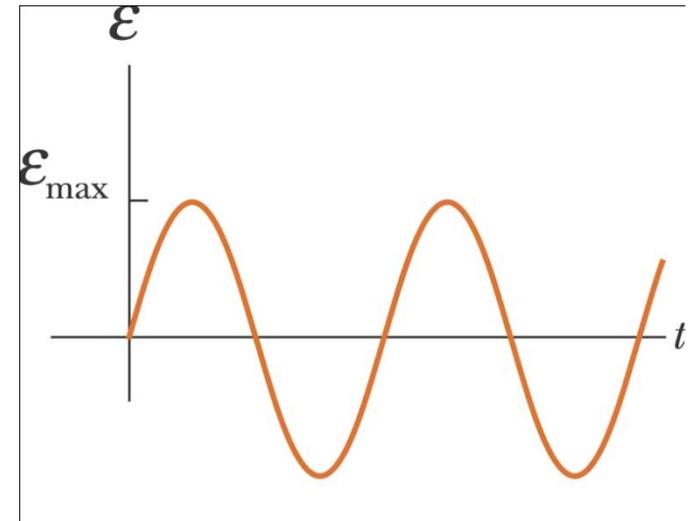
So

$$\text{emf} = -N \frac{d\Phi_B}{dt} = NBA\omega \sin \omega t$$

The *emf* is a sin wave: AC.

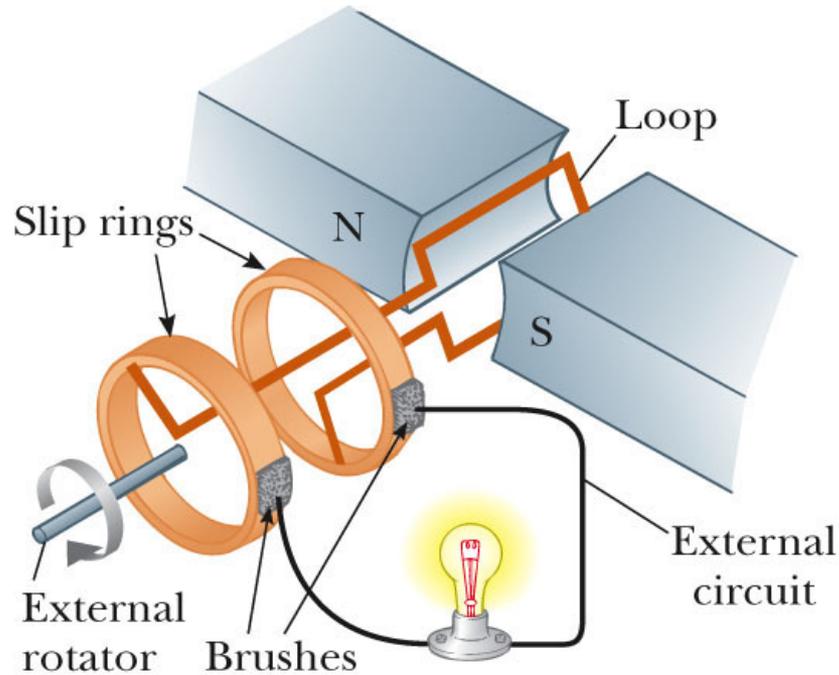


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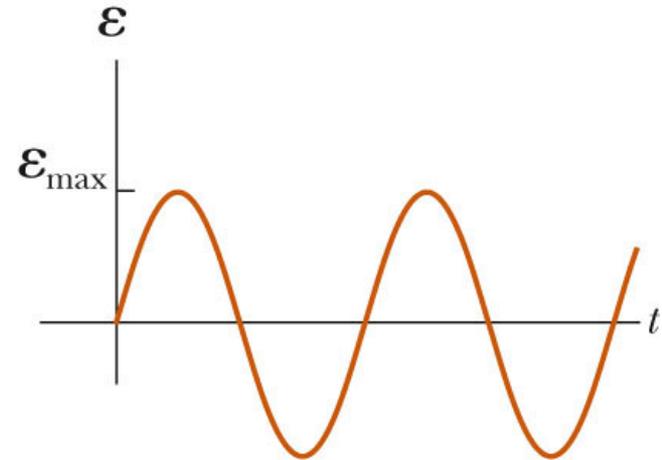


Generators

- Electric generators take in energy by work and transfer it out by electrical transmission
- The AC generator consists of a loop of wire rotated by some external means in a magnetic field



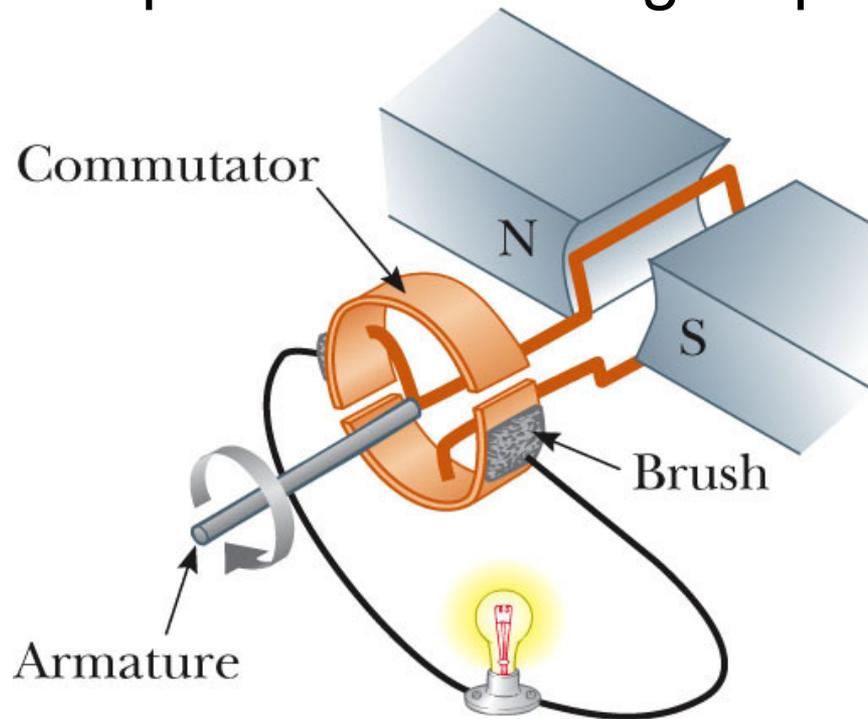
(a)



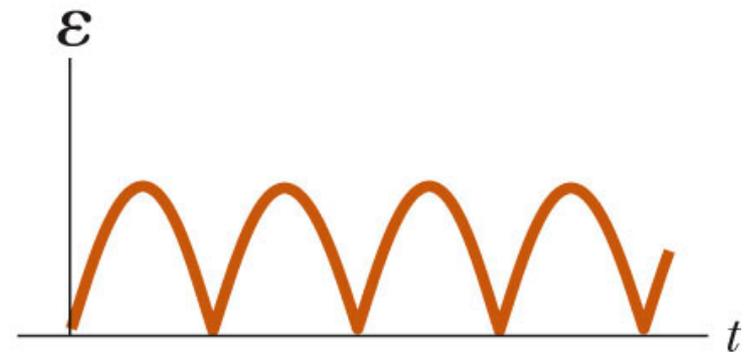
(b)

DC Generators

- The DC (direct current) generator has essentially the same components as the AC generator
- The main difference is that the contacts to the rotating loop are made using a split ring called a *commutator*



(a)



(b)

Motors

- Motors are devices into which energy is transferred by electrical transmission while energy is transferred out by work
- A motor is a generator operating in reverse
- A current is supplied to the coil by a battery and the torque acting on the current-carrying coil causes it to rotate

Motors

- Useful mechanical work can be done by attaching the rotating coil to some external device
- However, as the coil rotates in a magnetic field, an emf is induced
 - This induced emf always acts to reduce the current in the coil
 - The back emf increases in magnitude as the rotational speed of the coil increases

Motors

- The current in the rotating coil is limited by the back emf
 - The term *back emf* is commonly used to indicate an emf that tends to reduce the supplied current
- The induced emf explains why the power requirements for starting a motor and for running it are greater for heavy loads than for light ones

Reading material and Homework assignment

Please watch this video (about 50 minutes each):
http://videolectures.net/mit802s02_lewin_lec16/ and
http://videolectures.net/mit802s02_lewin_lec17/

Please check wileyplus webpage for homework assignment.