Maxwell's Equations and Magnetism of Matter

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss' Law in electricity

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law in magnetism

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Faraday's Law of induction

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I_{\text{encircled}}$$

Ampere-Maxwell Law

Maxwell's Equations and Magnetism of Matter

$$\oiint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \oiiint \rho dV$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Gauss' Law in electricity

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Gauss' Law in magnetism

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law of induction

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \iint \vec{J} \cdot d\vec{S} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{E} \cdot d\vec{S}$$

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Ampere-Maxwell Law

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$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Ampere-Maxwell Law

Gauss' Law for Magnetic Particles

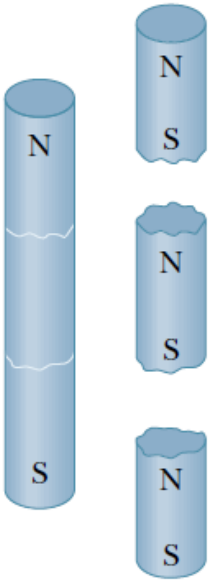
The net magnetic flux Φ_B through any closed Gaussian surface is zero.

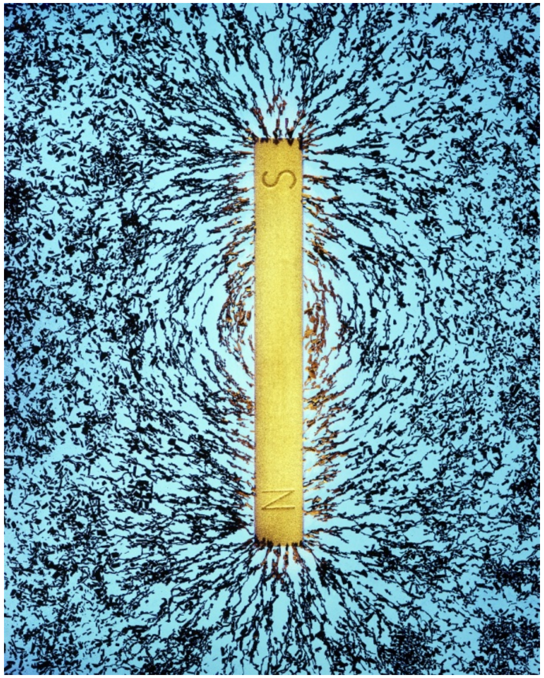
$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$ Gauss' law for magnetic fields

Compare with the Gauss' Law for electrical charges:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

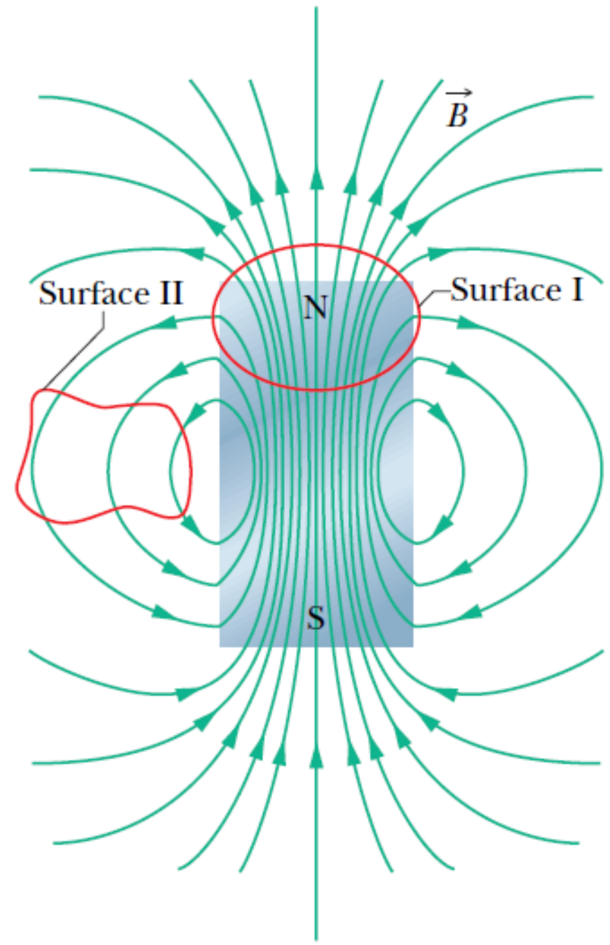
- Magnetic monopole does not exist (has not been found).





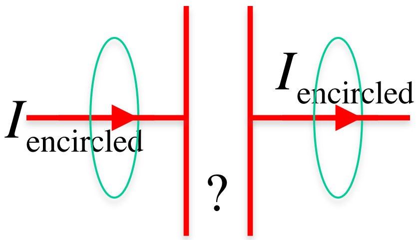
Gauss' Law for Magnetic Particles

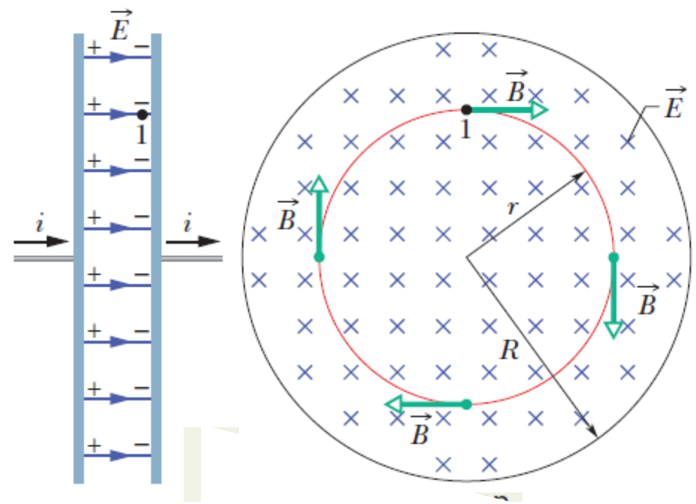
Gauss' law for magnetic fields holds for structures even if the Gaussian surface does not enclose the entire structure. Gaussian surface II near the bar magnet encloses no poles, and we can easily conclude that the net magnetic flux through it is zero.
For Gaussian surface I, it may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, magnetic field lines are loops and go from the south pole to the north pole inside the magnet.



Induced Magnetic Fields: Ampere-Maxwell's Law

Ampere's law $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{encircled}}$





Maxwell's law $\oint \vec{B}\cdot d\vec{s} = \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}$

Ampere-Maxwell's law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I_{\text{encircled}}$$

Displacement current

assume q is the charge on the plate,

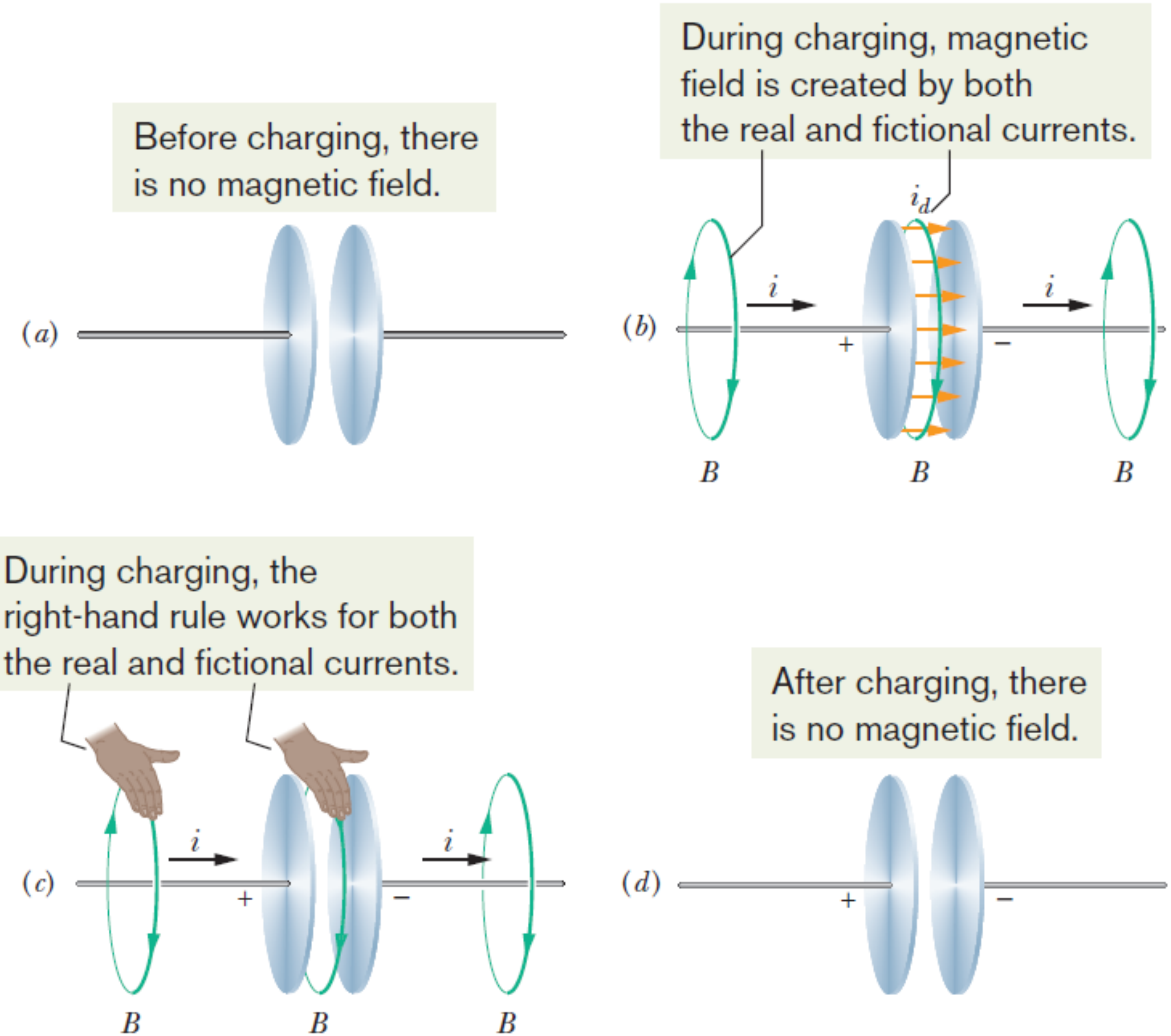
$$\text{from } q = CV = \left(\varepsilon_0 \frac{A}{d}\right)\cdot Ed = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

$$I_{\text{incircled}} = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_{\text{displacement}}$$

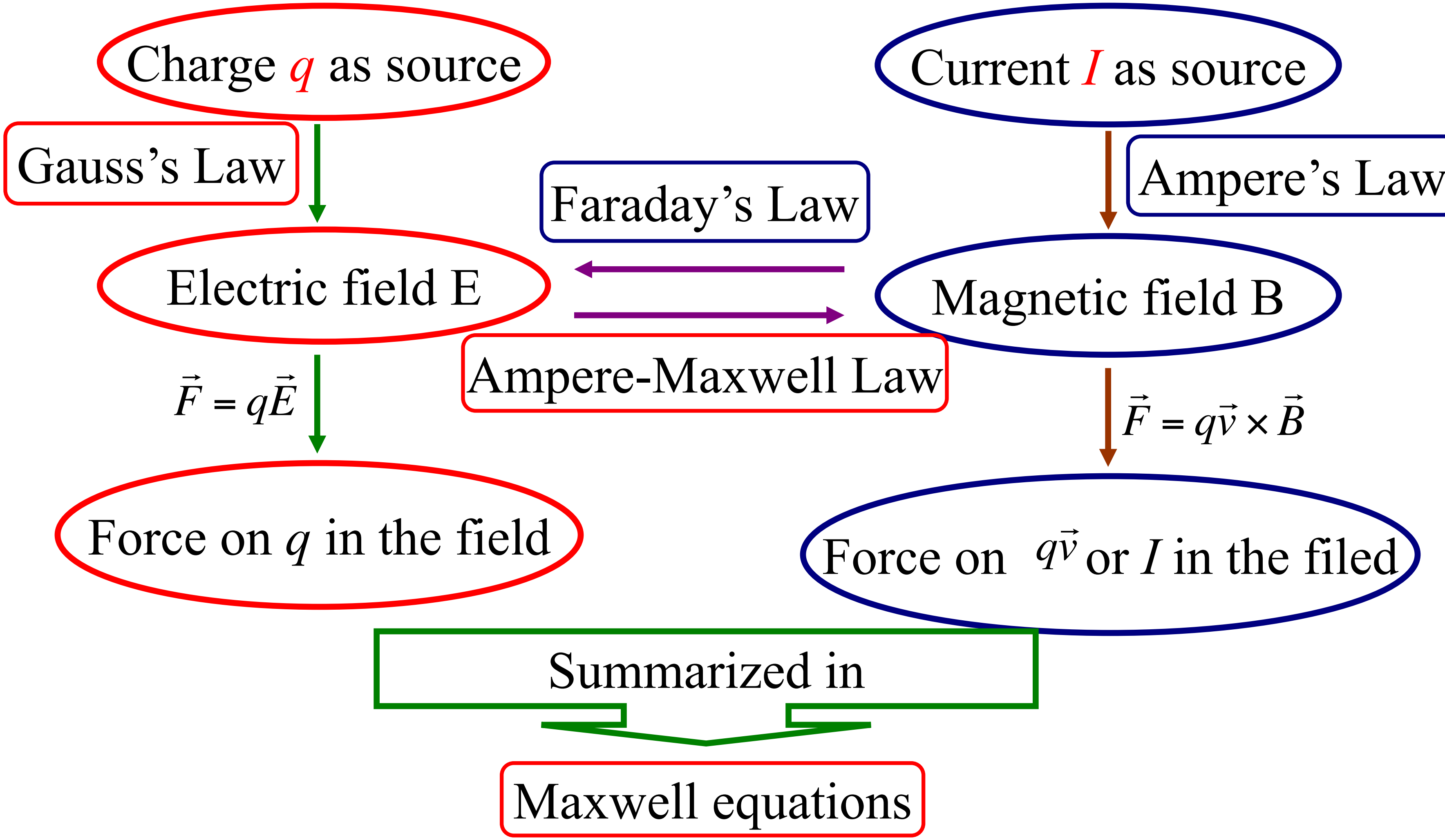
the associated magnectic field follows Ampere's law

$$B = \frac{\mu_0 I_{\text{displacement}}}{2\pi R^2} r \text{ when } r < R \text{ or } B = \frac{\mu_0 I_{\text{displacement}}}{2\pi r}$$

Displacement Current



Electricity & Magnetism, Maxwell’s Equations



Electricity & Magnetism, Maxwell's Equations

Charge q as source

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Electric field E

$$\vec{F} = q\vec{E}$$

Force on q in the field

Current I as source

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encircled}}$$

Magnetic field B

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

Force on $q\vec{v}$ or I in the filed

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I_{\text{encircled}}$$

Summarized in

Maxwell equations

Magnetism of Matter

There are three general types of magnetism:

1. Diamagnetism: Diamagnetic materials create an induced magnetic field in a direction opposite to an externally applied magnetic field, and are repelled by the applied magnetic field.

2. Paramagnetism: Paramagnetic materials are attracted by an externally applied magnetic field, and form internal, induced magnetic fields in the direction of the applied magnetic field.

3. Ferromagnetism is the basic mechanism by which certain materials (such as iron) form permanent magnets, or are attracted to magnets.

These magnetisms are due to the fact that electrons in atoms have orbital magnetic dipole moments and spin magnetic dipole moments. The vector sum of all these magnetic dipole moments determines the magnetic characteristics of a certain material.

Electron Spin Magnetic Dipole Moment

An electron has an intrinsic angular momentum called its **spin angular momentum** (or just **spin**), **S**; associated with this spin is an intrinsic spin magnetic dipole moment, $\boldsymbol{\mu}_s$. ("intrinsic" means that **S** and $\boldsymbol{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.)

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

in which *e* is the elementary charge (1.60 x10^{-19} C) and *m* is the mass of an electron (9.11 10^{31} kg).

1. Spin $\vec{S}$ itself cannot be measured. However, its component along any axis can be measured.
2. A measured component of $\vec{S}$ is *quantized*, which is a general term that means it is restricted to certain values. A measured component of $\vec{S}$ can have only two values, which differ only in sign.

Let us assume that the component of spin $\vec{S}$ is measured along the z axis of a coordinate system. Then the measured component S_z can have only the two values given by

$$S_z = m_s \frac{h}{2\pi}, \qquad \text{for } m_s = \pm\tfrac{1}{2}, \qquad (32\text{-}23)$$

where m_s is called the *spin magnetic quantum number* and h ($= 6.63 \times 10^{-34}\ \mathrm{J \cdot s}$) is the Planck constant, the ubiquitous constant of quantum physics.

Electron Spin Magnetic Dipole Moment

For an electron, the spin is opposite the magnetic dipole moment.

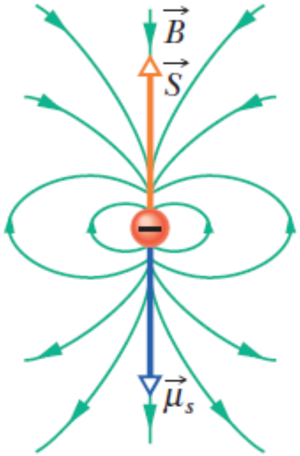


Fig. 32-10 The spin $\vec{S}$, spin magnetic dipole moment $\vec{\mu}_s$, and magnetic dipole field $\vec{B}$ of an electron represented as a microscopic sphere.

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

$$\Rightarrow \quad \mu_{s,z} = -\frac{e}{m}S_z. \quad \Rightarrow \quad \mu_{s,z} = \pm\frac{eh}{4\pi m},$$

$$\Rightarrow \quad \mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T} \qquad \text{(Bohr magneton).}$$

The orientation energy for the electron, when B_{ext} is the exterior magnetic field aligned along the z-axis.

$$U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z}B_{ext},$$

Electrons Orbital Magnetic Dipole Moment

When it is in an atom, an electron has an additional angular momentum called its orbital angular momentum, $\boldsymbol{L}_{orb}$. Associated with it is an orbital magnetic dipole moment, $\boldsymbol{\mu}_{orb}$; the two are related by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$

Only the component along any axis of the orbital angular momentum can be measured, and that component is quantized

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \qquad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit}),$$

in which is m_l called the orbital magnetic quantum number and "limit" refers to its largest allowed integer value.

Similarly, only the component of the magnetic dipole moment of an electron along an axis can be measured, and that component is quantized.

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_{\text{B}}.$$

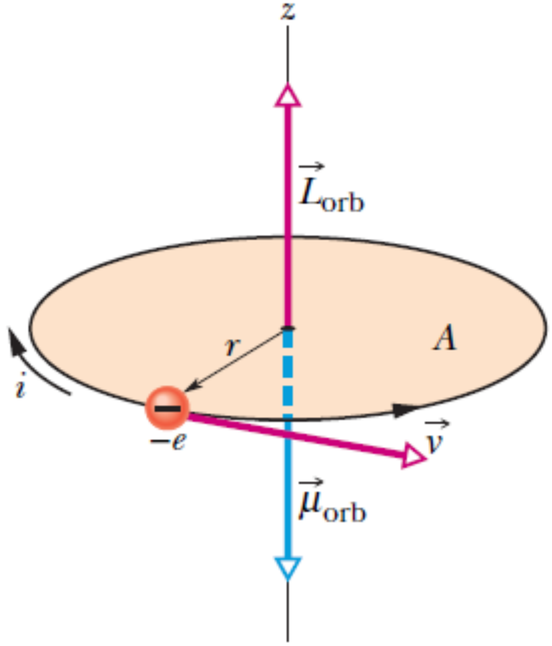
The orientation energy is:

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}},$$

where the z axis is taken in the direction of $\boldsymbol{B}_{ext}$.

Electrons Orbital Magnetic Dipole Moment

Fig. 32-11 An electron moving at constant speed v in a circular path of radius r that encloses an area A. The electron has an orbital angular momentum $\vec{L}_{orb}$ and an associated orbital magnetic dipole moment $\vec{\mu}_{orb}$. A clockwise current i (of positive charge) is equivalent to the counterclockwise circulation of the negatively charged electron.



The magnitude of the orbital magnetic dipole moment of the current loop shown is:

$$\mu_{orb} = iA,$$

Here A is the area enclosed by the loop. Since $i = \frac{\text{charge}}{\text{time}} = \frac{e}{2\pi r/v}.$

$$\mu_{orb} = \frac{e}{2\pi r/v}\pi r^2 = \frac{evr}{2}.$$

And since $L_{orb} = mrv \sin 90° = mrv.$

Therefore,

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb},$$