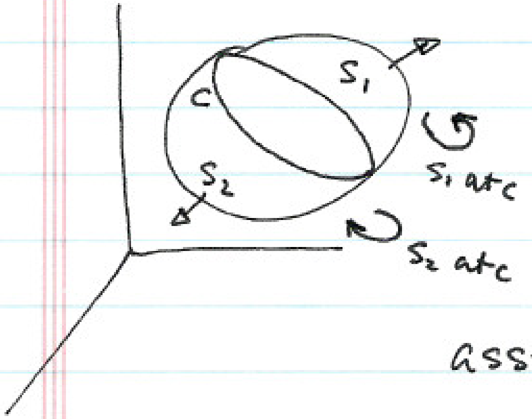

Prove $\int_S \nabla \times \vec{v}\, d\vec{\sigma} = 0$

when S is a closed surface.

assume $S = S_1 + S_2$, and c is the place,

the line where S_1 and S_2 meet.

So $\int_{S_1 \text{ at } c} \nabla \times \vec{v}\, d\vec{\sigma} = \oint_{S_1 \text{ at } c} \nabla \cdot d\vec{\lambda}$

$$\int_{S_2 \text{ at } c} \nabla \times \vec{v}\, d\vec{\sigma} = \int_{S_2 \text{ at } c} \nabla \cdot d\vec{\lambda} = -\int_{S_1 \text{ at } c} \nabla \cdot d\vec{\lambda}$$

So $\int_S \nabla \times \vec{v}\, d\vec{\sigma} = \int_{S_1} \nabla \times \vec{v} \cdot d\vec{\sigma} + \int_{S_2} \nabla \times \vec{v}\, d\vec{\sigma} = 0$

There is another way, but not as simple.