

Quiz 1

Prove $\vec{A} \times (\nabla \times \vec{A}) = \frac{1}{2} \nabla(A^2) - (\vec{A} \cdot \nabla) \vec{A}$

$\vec{A} \times (\nabla \times \vec{A})$, here the ∇ applies to one \vec{A} .

temporarily write as $\vec{A}_1 \times (\nabla_2 \times \vec{A}_2)$, use the BAC-CAB rule

$$\vec{A}_1 \times (\nabla_2 \times \vec{A}_2) = \nabla_2 (\vec{A}_1 \cdot \vec{A}_2) - (\nabla_2 \cdot \vec{A}_1) \vec{A}_2$$

but $\vec{A}_1 = \vec{A}_2 = \vec{A}$, so $\nabla_2 (\vec{A}_1 \cdot \vec{A}_2) = \frac{1}{2} \nabla(A^2)$

$$\nabla_2 \cdot \vec{A}_1 = \vec{A}_1 \cdot \nabla_2, \text{ so } (\nabla_2 \cdot \vec{A}_1) \vec{A}_2 = (\vec{A}_1 \cdot \nabla_2) \vec{A}_2 = (\vec{A} \cdot \nabla) \vec{A}$$

so $\vec{A} \times (\nabla \times \vec{A}) = \frac{1}{2} \nabla(A^2) - (\vec{A} \cdot \nabla) \vec{A}$