1. (a) Construct a 3-dimensional orthonormal basis \{\hat{u}_1, \hat{u}_2, \hat{u}_3\} using (1,2,3) as one of the directions.

(b) Expand the vector (4,4,4) in your basis.

(c) Verify explicitly that your orthonormal basis is complete by showing that \[ \sum_{n=1}^{3} \hat{u}_n \hat{u}_n \]

is the identity matrix.

2. Consider the first full period of the sine function: \(\sin(x), 0 < x < 2\pi\).

(a) Expand this in a Fourier \textbf{cosine} series and list the first four non-zero Fourier coefficients. (This is not a trick question. You can expand any function outside its given range as either an even or an odd function.)

(b) Plot the original function and your four-term approximation using a computer for the range \(0 < x < 2\pi\).

(c) Plot the original function and your four-term approximation using a computer for the range \(-2\pi < x < 0\). Comment.

(d) Expand \(\sin(x), 0 < x < 2\pi\), in a Fourier sine series.