Lecture #1
Read Mauer Ch. 1

**def** scalar - a quantity that does not change when the coordinate system is rotated (reflections later), e.g. \( T, P, m, \# \text{ oranges, ..., t, } 1/t, \alpha, \beta \)

\( x \) to my right, \( y \) in front of me, standing at origin \( T \) at student position, now turn, new \( T' \) is the same.

**def** vector - a quantity that changes like displacement \( \overrightarrow{F} \) under a rotation of coordinates.

\( \text{student is at } \overrightarrow{r} = (x, y) = (0, 3) \text{ meters, now turn, } \overrightarrow{r}' = \overrightarrow{r} \)

e.g. other vectors: \( \overrightarrow{x} = \overrightarrow{F} \) (\( \overrightarrow{x} \) is Mauer's notation)
\( \overrightarrow{v} = \frac{\partial \overrightarrow{F}}{\partial t} \), \( \overrightarrow{\rho} \), ... \n\[ \text{[not } \overrightarrow{r} \text{ angular momentum, see reflections]} \]

**Notation**
\[ \overrightarrow{F} = \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \begin{pmatrix} \frac{\partial F_x}{\partial x} \\ \frac{\partial F_y}{\partial y} \end{pmatrix} \text{ Cartesian} \]
\[ \overrightarrow{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \text{ column} \]
\[ \overrightarrow{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \text{ row} \]
\[ v_i \text{ one index } \]
\# indices \( \equiv \) rank

Vectors are rank 1 objects, scalars have rank 0.

\( i = 1, \ldots, d \) where \( d \) = dimension of "space" (\( i = 3 \) for now).

\[ \text{Not } \overrightarrow{3 \text{ oranges}} \text{ does not change like } \overrightarrow{F} \text{ under coordinate rotations } \Rightarrow \text{ not a vector.} \]
def rank n tensor - a quantity that changes like the exterior product of n position vectors

$T_{ij}$ changes under rotations like $R_{ij}$ (or $R_{ij}$)

Rank 2

$T_{ij}$ = $R_{ij}$, $i, j = 1, \ldots, 3$ (in general)

$T_{ij}$ can be represented as a matrix, but not all matrices are rank 2 tensors. In particular, the transformation matrix that relates $\mathbf{r}$ to $\mathbf{r}'$ is not a tensor.

E.g. (10) identity matrix is not a rank 2 tensor.

proof:

\[
\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix}
\]

$r_{11} = 1$ $\Rightarrow r_{1}, \neq 0$

$r_{22} = 1$ $\Rightarrow r_{2}, \neq 0$

but $r_{12} = 0$. Impossible.

E.g. rank 2 tensors: $\delta_{ij}$ energy-momentum tensor

$T_{ij}$ moment of inertia tensor

(by the way, $I$ = mass, $I_i$ = displacement vector at $I_i$ = center of mass

$\rho$ = mass density

$\epsilon_{ij}$ dielectric tensor

$T_{ijk}$ changes like $\mathbf{r}_i \mathbf{r}_j \mathbf{r}_k$

Rank 3

Hold it! I thought that the moment of inertia about the center of mass for a disk (for example) was $\frac{1}{2} m R^2$. How is this a tensor?

\[
\begin{align*}
I_{xx} & = I_{yy} = \frac{1}{2} m R^2 \\
I_{xx} & = I_{yy} \neq 0 \\
I_{xy} & = 0 \\
I_{x} & = 0 \\
I_{y} & = 0 \\
I_{z} & = 0
\end{align*}
\]
Examples:

\( I = \int \rho \, dV = \text{mass} = m \)

\( I_i = \int x_i \rho \, dV = m \langle x_{\text{cm}} \rangle_i \)

\( \mathbf{I} = \int \mathbf{x} \rho \, dV = \int \mathbf{r} \rho \, dV = m \langle \mathbf{x}_{\text{cm}} \rangle \)

\( I_{ij} = \int x_i x_j \rho \, dV \)

\( \mathbf{I} \) is not standard notation

\( I_i \) is not standard notation

\( \mathbf{I} \) is not standard notation

\( I_{ij} \) is standard notation for moment of inertia tensor