Electrostatics

\[ \rho(\mathbf{r}) \]

\[ \mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\varepsilon_0} \]

\[ \mathbf{E}(\mathbf{r}) = \nabla \phi(\mathbf{r}) \]

\[ \oint_{\Sigma} \mathbf{E} \cdot d\mathbf{a} = -\int_{\Omega} \mathbf{J} \cdot d\mathbf{V} \]

\[ \mathbf{E}(\mathbf{r}) = -\nabla \phi(\mathbf{r}) \]

MKS

\[ \nabla \times \mathbf{E} = 0 \]

\[ \varepsilon_0 \mathbf{E} = \frac{\rho(\mathbf{r})}{\varepsilon_0} \]

\[ \frac{\partial}{\partial t} \mathbf{E}(\mathbf{r}) + \nabla \times \mathbf{B} = 0 \]
Magnetostatics

\[ \vec{A}(\vec{r}) = \frac{1}{4\pi} \iiint \frac{\vec{B}(\vec{r}') \times (\vec{r} - \vec{r}')}{(\vec{r} - \vec{r}')^3} \, dV' \]

\[ \vec{B}(\vec{r}) = \vec{\nabla} \times \vec{A}_0(\vec{r}) \]

\[ \vec{A}_0(\vec{r}) = \frac{1}{4\pi} \iiint \frac{\vec{B}(\vec{r}') \times (\vec{r} - \vec{r}')}{(\vec{r} - \vec{r}')^3} \, dV' \]

\[ \vec{\nabla} \cdot \vec{A}_0(\vec{r}) = 0 \]

\[ \vec{\nabla} \times \vec{A}(\vec{r}) = \frac{\lambda_0}{\mu_0} \vec{J}(\vec{r}) \]

Ampère's Law:

\[ \nabla \times \vec{B} = \vec{J} \]

Poisson's Law:

\[ \nabla^2 \vec{A} = \vec{J} \]
38. Since \( \nabla \cdot \vec{J}(\vec{r}) = 0 \), we can write the current density \( \vec{J}(\vec{r}) \) as the curl of a vector field \( \vec{\psi}(\vec{r}) \). Use \( \vec{J}(\vec{r}) = \nabla \times \vec{\psi}(\vec{r}) \) to show that

\[
\int dV \left[ x_i J_j(\vec{r}) + x_j J_i(\vec{r}) \right] = 0.
\]

\[
I = \int dV \left\{ x_e J_j(\vec{r}) + x_j J_e(\vec{r}) \right\}
\]

\[
= \int dV \left\{ x_e \left[ \nabla \times \vec{\psi}(\vec{r}) \right]_j + x_j \left[ \nabla \times \vec{\psi}(\vec{r}) \right]_e \right\}
\]

\[
= \int dV \sum_{i=1}^{3} \sum_{k=1}^{3} \left\{ x_e \varepsilon_{ijk} \frac{\partial}{\partial x_i} \psi_k(\vec{r}) + x_j \varepsilon_{ijk} \frac{\partial}{\partial x_e} \psi_k(\vec{r}) \right\}
\]

Integrate each term by parts and use the following:

\[
\int dV x_e \frac{\partial}{\partial x_i} \psi_k(\vec{r}) = \int dV \frac{\partial}{\partial x_i} [x_e \psi_k(\vec{r})] - \int dV \psi_k(\vec{r}) \frac{\partial x_e}{\partial x_i}
\]

\[
= -\int dV \psi_k(\vec{r}) \delta_{ei}
\]

The total derivative term becomes a surface integral which vanishes by the standard arguments.

\[
I = -\sum_{i=1}^{3} \sum_{k=1}^{3} \int dV \left\{ \varepsilon_{ijk} \delta_{ei} + \varepsilon_{ijk} s_{ij} \right\} \psi_k(\vec{r})
\]

\[
= -\sum_{k=1}^{3} \int dV \left\{ \varepsilon_{ijk} + \varepsilon_{jik} \right\} \psi_k(\vec{r}) = 0
\]

since \( \varepsilon_{ijk} \) is totally antisymmetric.