Topics in Spin - Homework 2

March 18, 2014

Problem SS-4: The Raising and Lowering Operators, $L_+$ and $L_-$
Write the basis vectors in angular momentum space in terms of $\ell, m$ as $|\ell, m\rangle$. In this basis, determine the coefficients $C_\pm(\ell, m)$ in the following equations for the raising and lowering operators:

\[
\begin{align*}
L_+ |\ell, m\rangle &= C_+(\ell, m) |\ell, m + 1\rangle \\
L_- |\ell, m\rangle &= C_-(\ell, m) |\ell, m - 1\rangle
\end{align*}
\]

Problem SS-5: The Matrix Elements of the Orbital Angular Momentum Operators
Use what you have learned about the action of the operators $L^2$, $L_z$, $L_+$, and $L_-$ on the basis vectors to construct the matrix elements of $L^2$, $L_x, L_y$, and $L_z$. Do this for $\ell = 0, 1$. Reports results in the form of a $4 \times 4$ matrix. HINT: label your basis vectors $|1\rangle = |0, 0\rangle$, $|2\rangle = |1, 1\rangle$, $|3\rangle = |1, 0\rangle$, etc. if it helps you to think about how to start this problem.

Problem SS-6: Thinking Ahead: What if the electron were really spinning?
Let us pretend for a moment that “spin” corresponds to a real, mechanical property of the electron. The classical radius of the electron is given by equating the electrostatic potential energy of a sphere of charge, $e$, and radius, $r_0$, with the rest energy of the electron, $U = \frac{e^2}{2r_0} = m_e c^2$. One can then solve for the radius and, inserting the current values for the electron charge and mass, obtain, $r_0 = 2.818 \times 10^{-15}$ m.

Let us assume that, in fact, the electron is spinning and has total spin angular momentum $S^2 = \frac{3h^2}{16}$ and projection $S_z = \frac{1}{2} \hbar$. At what maximum linear speed is the “surface” of the electron charge sphere rotating? Please comment on your results.