HW #13 Solutions

26.18 Relativistic momentum must be conserved:
For total momentum to be zero after as it was before, we must have (with subscript 2 referring to
the heavier fragment, and subscript 1 to the lighter fragment): \( p_2 = p_1 \), or
\[
\gamma m_2 v_2 = \gamma m_1 v_1 = \frac{(2.50 \times 10^{-28} \text{ kg})(0.893c)}{\sqrt{1 - (0.893)^2}} = (4.96 \times 10^{-28} \text{ kg})c.
\]
This becomes
\[
\frac{(1.67 \times 10^{-27} \text{ kg})(v_2)}{\sqrt{1 - (v_2/c)^2}} = (4.96 \times 10^{-28} \text{ kg})c,
\]
or \( 3.37(v_2/c) = \sqrt{1 - (v_2/c)^2} \)
This yields \( v_2 = 0.284c \).

26.25 Using the velocity addition equation, the velocity of the students relative to the instructors (and
hence the clock) is found as follows
\[
v_{sE} = 0.60c = \text{velocity of students with respect to Earth.}
\]
\[
v_{IE} = -0.28c = \text{velocity of instructors with respect to Earth} = -v_{EI}
\]
\[
v_{sI} = \text{velocity of students relative to instructor.}
\]
\[
v_{sI} = \frac{(v_{sE}) + (v_{EI})}{1 + \frac{(v_{sE})(v_{EI})}{c^2}} = \frac{(0.60c) + (0.28c)}{1 + \frac{(0.60c)(0.28c)}{c^2}} = 0.75c
\]
(students relative to instructors)
(a) With a proper time interval of \( \Delta t_p = 50 \) minutes (measured in the instructor's rest frame),
the time interval measured by the students is: \( \Delta t = \gamma \Delta t_p \) with \( \gamma = \gamma = \frac{1}{\sqrt{1 - (0.75c/c)^2}} = 1.51. \)
Thus, the students measure the exam to last \( \Delta t = 1.51(50 \text{ min}) = 76 \text{ min.} \)
(b) The duration of the exam as measured by observers on Earth is:
\( \Delta t = \gamma \Delta t_p \) with \( \gamma = \frac{1}{\sqrt{1 - (0.28c/c)^2}} = 1.04, \)
so \( \Delta t = 1.04(50 \text{ min}) = 52 \text{ min.} \)

26.26 At \( v = 0.950c \), we find \( \gamma = 3.20. \)
(a) \( E_0 = mc^2 = (1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \)
\[
= 1.50 \times 10^{-10} \text{ J} = 939 \text{ MeV}
\]
(b) \( E = \gamma mc^2 = \gamma E_0 = (3.20)(939 \text{ MeV}) = 3.00 \times 10^9 \text{ eV} = 3.00 \text{ GeV} \)
(c) \( KE = E - E_0 = 3.00 \times 10^9 \text{ eV} - 939 \text{ MeV} = 2.06 \text{ GeV} \)

26.29 We are given that: \( KE = E_0. \) But, the total energy is \( E = E_0 + KE, \) so we find: \( E = 2E_0, \) or
\( \gamma mc^2 = 2mc^2. \) Thus, \( \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} = 2 \)
from which \( v = 0.866c \)
26.43 We have
\[ v_{qe} = 0.870c = \text{velocity of quasar with respect to earth} \]
\[ v_{mq} = -0.550c = \text{velocity of material with respect to the quasar.} \]
\[ v_{me} = \text{velocity of material relative to earth.} \]
\[ v_{me} = \frac{(v_{mq}) + (v_{qe})}{1 + \frac{(v_{mq})(v_{qe})}{c^2}} = \frac{(-0.550c) + (0.870c)}{1 + \frac{(-0.550c)(0.870c)}{c^2}} = 0.61c \]
or 0.61c away from earth.

27.7 The total energy radiated each second is:
\[ E = P_t = (150 \times 10^3 \text{ J/s})(1 \text{ s}) = 150 \times 10^3 \text{ J.} \]
The energy of each 99.7 MHz photon is:
\[ E_\gamma = hf = (6.63 \times 10^{-34} \text{ J s})(99.7 \times 10^6 \text{ s}^{-1}) = 6.61 \times 10^{-26} \text{ J.} \]
The number of photons emitted per second is therefore:
\[ n = \frac{E}{E_\gamma} = \frac{150 \times 10^3 \text{ J}}{6.61 \times 10^{-26} \text{ photons/s}} = 2.27 \times 10^{30} \text{ photons.} \]

27.13 (a) The photon energy is:
\[ E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{350 \times 10^{-9} \text{ m}} = 5.68 \times 10^{-19} \text{ J, or} \]
\[ E = 3.55 \text{ eV} \]
and \[ \phi = E - KE_{\text{max}} = 3.55 \text{ eV} - 1.31 \text{ eV} = 2.24 \text{ eV.} \]
(b) \[ \lambda_c = \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{(2.24 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} \]
\[ = 5.55 \times 10^{-7} \text{ m} = 555 \text{ nm} \]
(c) \[ f_c = \frac{c}{\lambda_c} = \frac{3.00 \times 10^8 \text{ m/s}}{5.55 \times 10^{-7} \text{ m}} = 5.41 \times 10^{14} \text{ Hz.} \]
The frequency of the 254 nm wavelength light is:

\[ f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{254 \times 10^{-9} \text{ m}} = 1.18 \times 10^{15} \text{ Hz} \]

Similarly, the frequency of the 436 nm light is \( 6.88 \times 10^{14} \text{ Hz} \).

The graph you draw should look somewhat like the sketch below. The desired quantities are read from the graph as indicated. You should find that: \( f_c = 4.77 \times 10^{14} \text{ Hz} \) and \( \phi = 2.03 \text{ eV} \).

\[
\begin{array}{c|c|c}
\hline
\text{KE (eV)} & \text{f (Hz)} \\
\hline
3.0 & 6.88 \times 10^{14} \\
0.9 & 11.8 \times 10^{14} \\
- \phi & f_c \\
\hline
\end{array}
\]

(a) From the Compton shift formula:
\[ \Delta \lambda = \lambda_c(1 - \cos \theta) = (0.00243 \text{ nm})(1 - \cos 37^\circ) = 4.89 \times 10^{-4} \text{ nm} \]

(b) The wavelength of the original photon was:
\[ \lambda_0 = \frac{hc}{E_{\gamma_0}} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{(300 \text{ keV})(1.6 \times 10^{-16} \text{ J/keV})} = 4.14 \times 10^{-12} \text{ m} = 4.14 \times 10^{-3} \text{ nm} \]

Thus, the scattered wavelength is:
\[ \lambda = \lambda_0 + \Delta \lambda = 4.14 \times 10^{-3} \text{ nm} + 4.89 \times 10^{-4} \text{ m} = 4.63 \times 10^{-3} \text{ nm} \]

and its energy is
\[ E_{\gamma} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{(4.63 \times 10^{-12} \text{ m})} = 4.29 \times 10^{-14} \text{ J} = 268 \text{ keV} \]

(c) The energy transferred to the recoiling electron is:
\[ KE = E_{\gamma_0} - E_{\gamma} = 32 \text{ keV} \]

(a) \( \lambda = \frac{h}{p} = \frac{h}{mv} \), so
\[ v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg})(500 \times 10^{-9} \text{ m})} = 1.46 \times 10^3 \text{ m/s} \]

(b) \( p = mv = (9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^7 \text{ m/s}) = 9.11 \times 10^{-24} \text{ kg m/s} \)

Thus, \( \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J s}}{9.11 \times 10^{-24} \text{ kg m/s}} = 7.28 \times 10^{-11} \text{ m} \)
This is a relativistic electron, so: \( E = E_0 + KE \),
where \( E_0 = m_0c^2 = (9.11 \times 10^{-31} \text{ kg })(3.00 \times 10^8 \text{ m/s})^2 = 8.20 \times 10^{-14} \text{ J} \)
Since, \( KE = 3.00 \text{ MeV} = 4.80 \times 10^{-13} \text{ J} \), we have \( E = E_0 + KE = 5.62 \times 10^{-13} \text{ J} \)
Also, \( p = \frac{1}{c} \sqrt{E^2 - E_0^2} \)
\[ = \frac{\sqrt{(5.62 \times 10^{-13} \text{ J})^2 - (8.20 \times 10^{-14} \text{ J})^2}}{3.00 \times 10^8 \text{ m/s}} = 1.85 \times 10^{-21} \text{ kg m/s} \]
Then, \( \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J s}}{1.85 \times 10^{-21} \text{ kg m/s}} = 3.58 \times 10^{-13} \text{ m} \)