M. 11.11

**Friction Case**

\[ E = T + U = \text{const} \] (Gravity is only net force)

At \( \theta = 0 \): \( E = U = mg \frac{L}{\sqrt{2}} \)

At \( \theta = \theta_f \): \( E = \frac{1}{2} I \omega^2 + mg \frac{L}{2} \)

From Example 11.5 and Steiner's Parallel Axis Theorem:

\[ I = \frac{1}{2} M L^2 + M \frac{L^2}{2} = \frac{3}{2} M L^2 \]

So:

\[ mg \frac{L}{\sqrt{2}} = \frac{1}{2} M L^2 \omega^2 + mg \frac{L}{2} \]

\[ g \left( \frac{1}{\sqrt{2}} - \frac{1}{2} \right) = \frac{1}{3} L \omega^2 \]

\[ \omega^2 = \frac{3g}{2} \left( \frac{1}{\sqrt{2}} - \frac{1}{2} \right) \]
\[ y_{cm} = \frac{l}{\sqrt{2}} \sin \theta \]
\[ y_{cm} = \frac{L_i}{\sqrt{2}} \cos \theta \]
\[ \frac{L_i}{60^\circ} = \frac{L_i}{2} \]

Cons. of \( E \):
\[ mg \frac{l}{\sqrt{2}} = mg \frac{l}{2} + \frac{1}{2} M \frac{l^2}{2} \frac{l^2}{2} + \frac{1}{2} I \omega^2 \]
\[ = mg \frac{l}{2} + \frac{1}{4} M l^2 + \frac{1}{8} M l^2 \omega^2 \]
\[ g \cdot \left( \frac{l^2}{2} - \frac{l}{2} \right) = \frac{1}{8} l^2 \omega^2 + \frac{1}{8} l^2 \omega^2 \]
\[ = \frac{1}{8} \frac{3}{2} l^2 \omega^2 \]
\[ \omega^2 = \frac{24}{5} g \cdot \left( \frac{1}{2} - \frac{1}{2} \right) \]

Note that \( \omega_{\text{slide}} < \omega_{\text{friction}} \)
I_{11} = \sum_\alpha m_\alpha (r_\alpha^2 - x_\alpha^2) \\
I_{22} = \sum_\alpha m_\alpha (r_\alpha^2 - y_\alpha^2) \\
I_{33} = \sum_\alpha m_\alpha (r_\alpha^2 - z_\alpha^2) \\
I_{11} + I_{22} = \sum_\alpha m_\alpha \left[ (r_\alpha^2 - x_\alpha^2) + (r_\alpha^2 - y_\alpha^2) \right] \\
= \sum_\alpha m_\alpha \left( y_\alpha^2 + z_\alpha^2 + x_\alpha^2 + z_\alpha^2 \right) \\
= \sum_\alpha m_\alpha \left( r_\alpha^2 + z_\alpha^2 \right) \\
= \sum_\alpha m_\alpha \left[ (r_\alpha^2 - z_\alpha^2) + z_\alpha^2 \right] \\
= I_{33} + 2 \sum_\alpha m_\alpha z_\alpha^2 \\
\geq I_{33}

Hence, I_{33} \leq I_{11} + I_{22}

Cyclically permute other indices

I_{11} \leq I_{22} + I_{33}
I_{22} \leq I_{33} + I_{11}
I_{33} \leq I_{11} + I_{22}

\text{No principal moment exceeds sum of other two.}